DBD-NME in the QRPA method: a critical review

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*some of the results presented in this talk have been obtained in collaboration with J. Suhonen. Univ. of Jyvaskyla .

Topics and Motivations

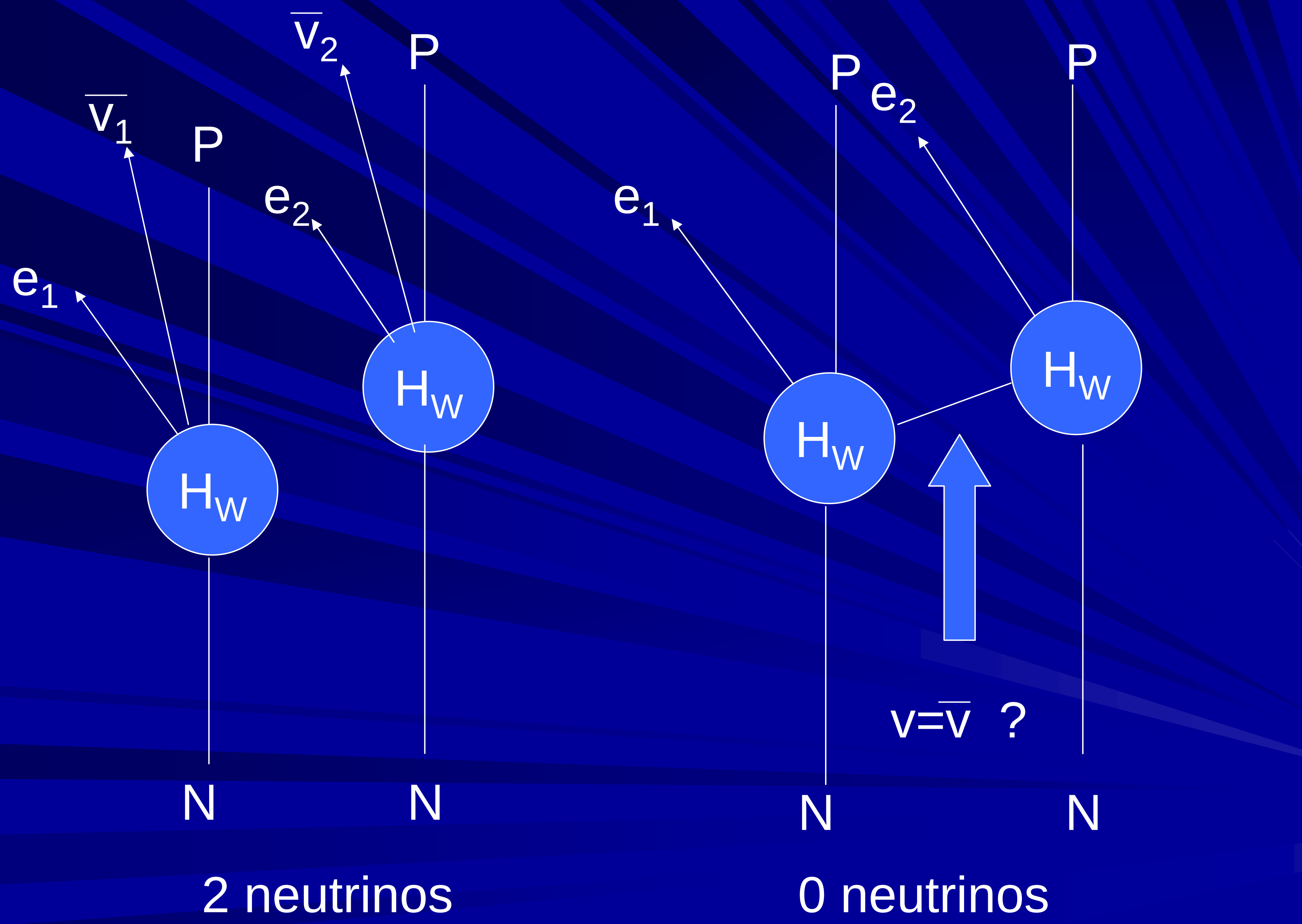
- Basic notions about the QRPA
- Nuclear matrix elements in the QRPA
- Parameters and dependence upon them
- Beyond the QRPA
- Some results (symmetry aspects)
- Conclusions

-Nuclear structure effects on the matrix elements governing the mass sector of the neutrinoless double beta decay (g_pp-dependence)

-Effective parameters: single-beta decay and double beta decay?
two-neutrino or zero-neutrino modes?

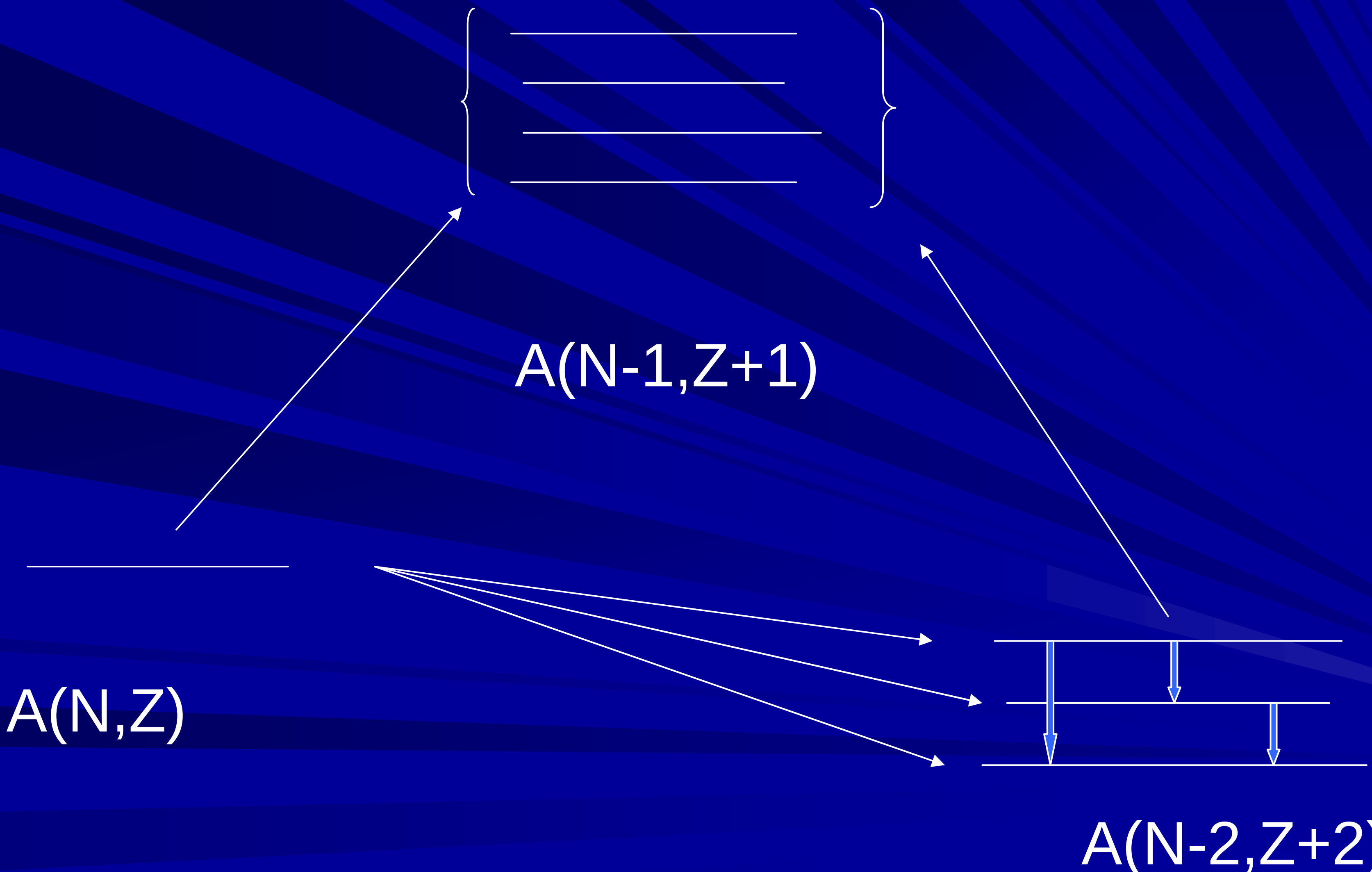
-Universality (if any) of the matrix elements

* as a rule:
if a certain approximation does not work in a simple model then there is certainly a problem with it in realistic cases.



$\bar{\nu}_1$
$\bar{\nu}_2$
P
e_1
e_2
H_W
N
P
e_1
e_2
H_W
$\nu=\bar{\nu}$?
N
2 neutrinos
0 neutrinos

Schematic view



Recommended values (Barabash)

- ^{48}Ca - $(4.2\ ^{+2.1}_{-1.0})\cdot 10^{19}$ y
- ^{76}Ge – $(1.5 \pm 0.1)\cdot 10^{21}$ y
- ^{82}Se – $(0.92 \pm 0.07)\cdot 10^{20}$ y
- ^{96}Zr – $(2.0 \pm 0.3)\cdot 10^{19}$ y
- ^{100}Mo – $(7.1 \pm 0.4)\cdot 10^{18}$ y
- ^{100}Mo – ^{100}Ru (0^+_1) – $(6.8 \pm 1.2)\cdot 10^{20}$ y
- ^{116}Cd – $(3.1 \pm 0.2)\cdot 10^{19}$ y
- ^{128}Te(geo) – $(2.5 \pm 0.3)\cdot 10^{24}$ y

  ^{130}Te(geo) – $(0.9 \pm 0.1)\cdot 10^{21}$ y
- ^{150}Nd - $(7.8 \pm 0.7)\cdot 10^{18}$ y
- ^{150}Nd – ^{150}Sm (0^+_1) – $(1.4^{+0.5}_{-0.4})\cdot 10^{20}$ y
- ^{238}U(rad) - $(2.0 \pm 0.6)\cdot 10^{21}$ y

ECEC(2ν):

- ^{130}Ba(geo) - $(2.2 \pm 0.5)\cdot 10^{21}$ y

2-neutrino mode

A(N,Z) $\longrightarrow$ A(N-2,Z+2)+2 electrons +2 neutrinos

$$\left[t_{1/2}^{(2\nu)}(0_I^+ \rightarrow 0_F^+)\right]^{-1} = G^{(2\nu)} \left|M_{\mathrm{GT}}^{(2\nu)}\right|^2$$

a) Lepton number is conserved

b) Suppressed by kinematics (four leptons in the final state)

c) Independent of neutrino properties

$$\left[t_{1/2}^{(2\nu)}(0_I^+ \to 0_F^+)\right]^{-1} = G^{(2\nu)} \left|M_{\mathrm{GT}}^{(2\nu)}\right|^2$$

$$M_{\mathrm{GT}}^{(2\nu)} = \sum_n \frac{(0_F^+ \parallel \sum_j \sigma(j) t_j^- \parallel 1_n^+)(1_n^+ \parallel \sum_j \sigma(j) t_j^- \parallel 0_I^+)}{(\frac{1}{2}Q_{\beta\beta} + E_n - M_I)/m_{\mathrm{e}} + 1}$$

0-neutrino mode

A(N,Z) $\longrightarrow$ A(N-2,Z+2)+2 electrons

$$\left[t_{1/2}^{0\nu}(J_f)\right]^{-1} = C_{mm}^{(0\nu)} \frac{< m_\nu >^2}{{m_e}^2}$$

$$C_{mm}^{(0\nu)} = G_1^{(0\nu)} \left[({M_{GT}}^{(0\nu)})(1-\chi_F)\right]^2$$

a) lepton number is not conserved

b) not suppressed by kinematics (two leptons in the final state)

$$\left[t^{0\nu}_{1/2}(J_f)\right]^{-1} = C^{(0\nu)}_{mm}\frac{< m_\nu >^2}{{m_e}^2}$$

$$C^{(0\nu)}_{mm} = G^{(0\nu)}_1\left[({M_{GT}}^{(0\nu)})(1-\chi_F)\right]^2$$

$$\chi_F = \frac{M^{(0\nu)}_F}{M^{(0\nu)}_{GT}}$$

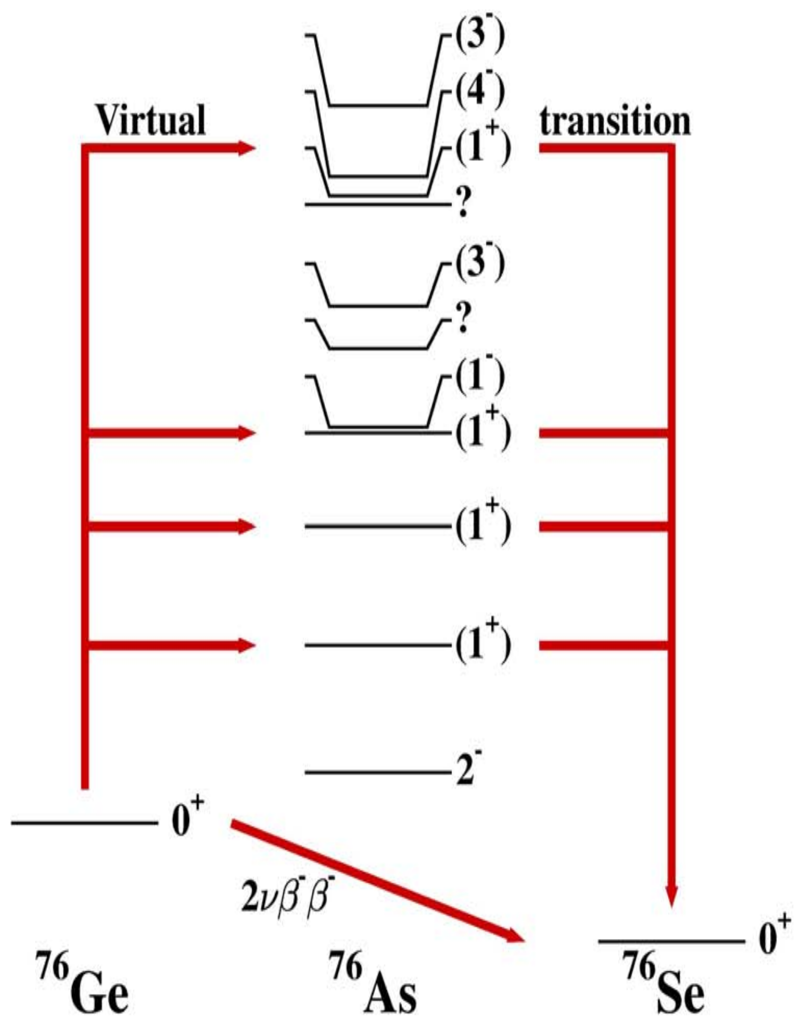
$$M^{(0\nu)}_{\mathrm{GT}} = (m_{\mathrm{e}}R)^{-2}$$

$$\sum_{ij}\sum_a < 0^+_F \parallel h_+(r_{ij},E_a)\sigma(i)\sigma(j)\tau(i)^-\tau(j)^- \parallel 0^+_I >$$

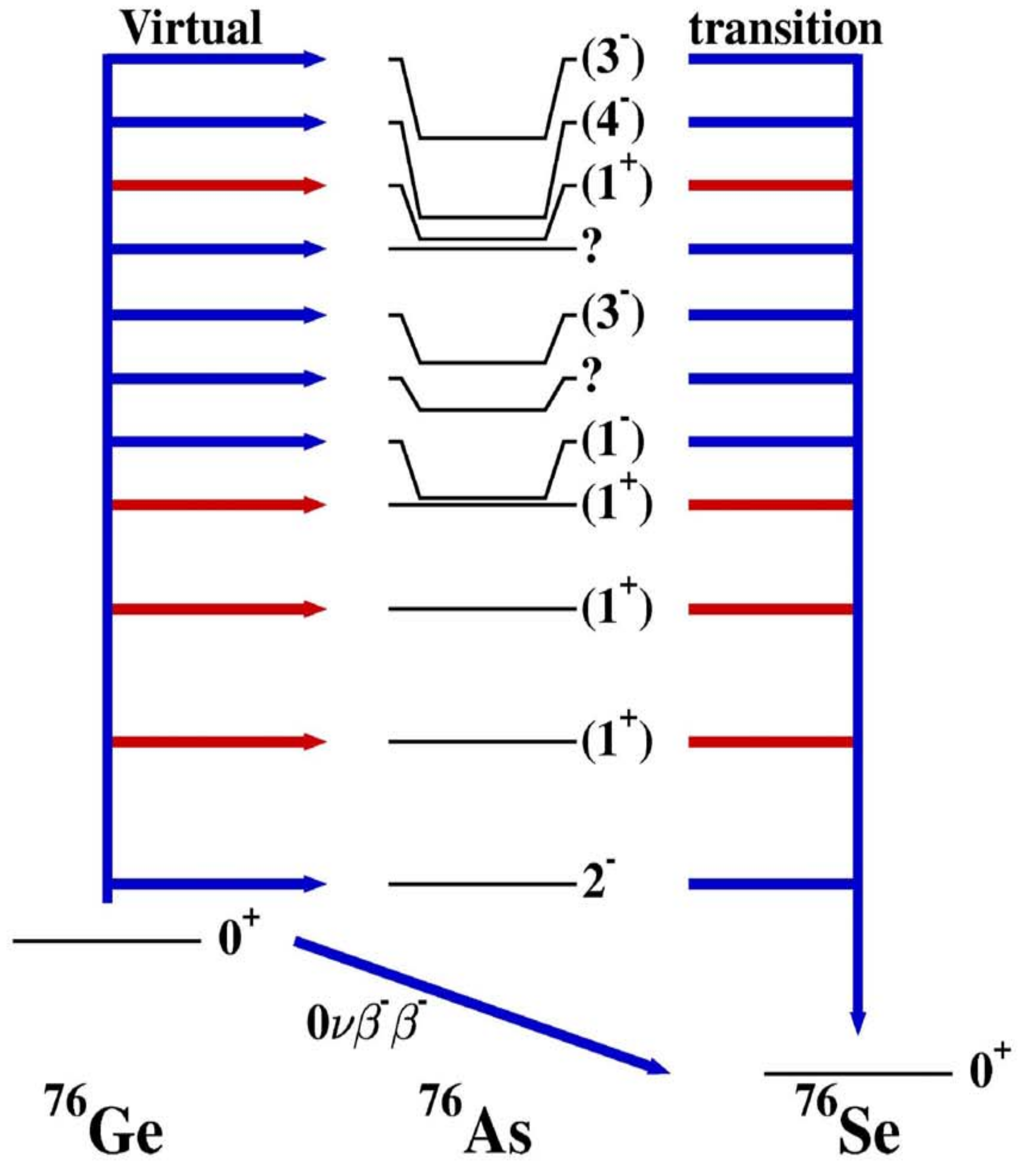
Basic Information (consistency test)

- Single beta decay and EC transitions
- Energy spectra, electromagnetic transitions
- Particle transfer and charge exchange (3He,t),(p,n) data.

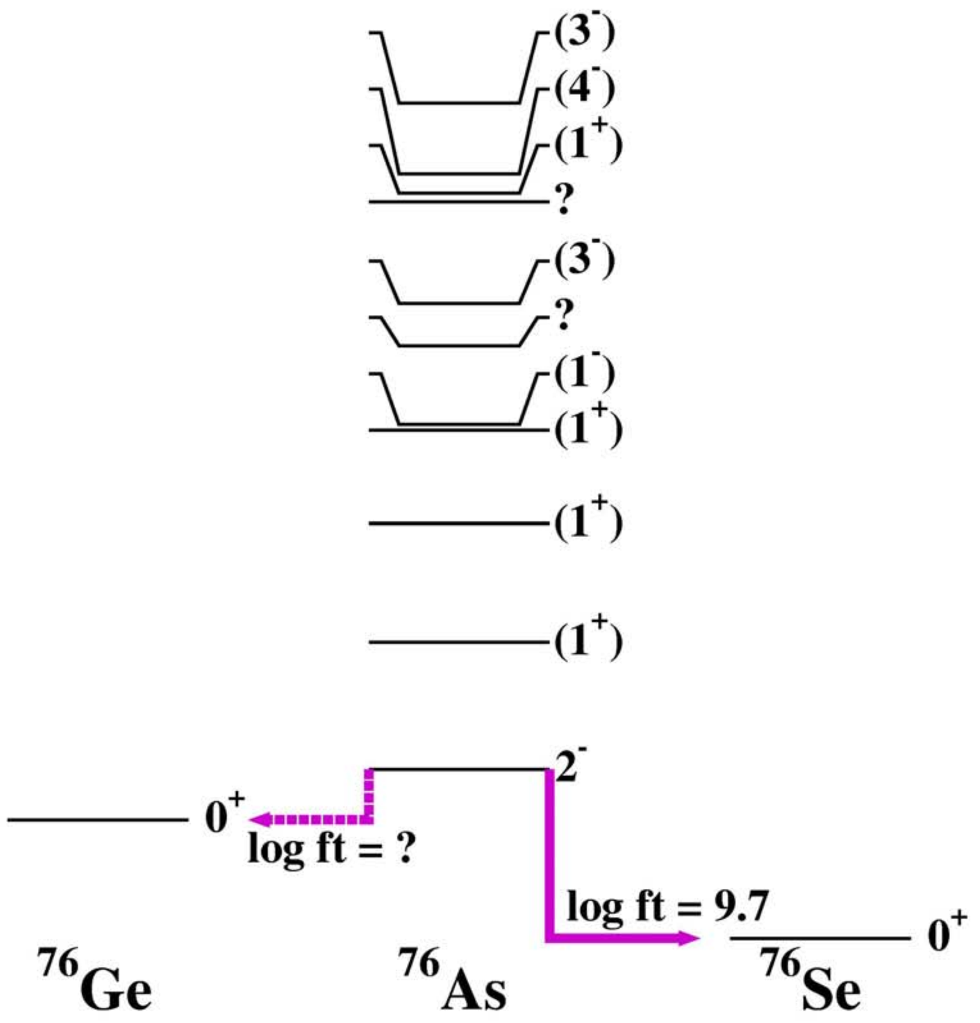
Practical rule: The information obtained by "theoretical reproducing" a single DBD transition does not mean anything, unless these related observables are also reproduced.



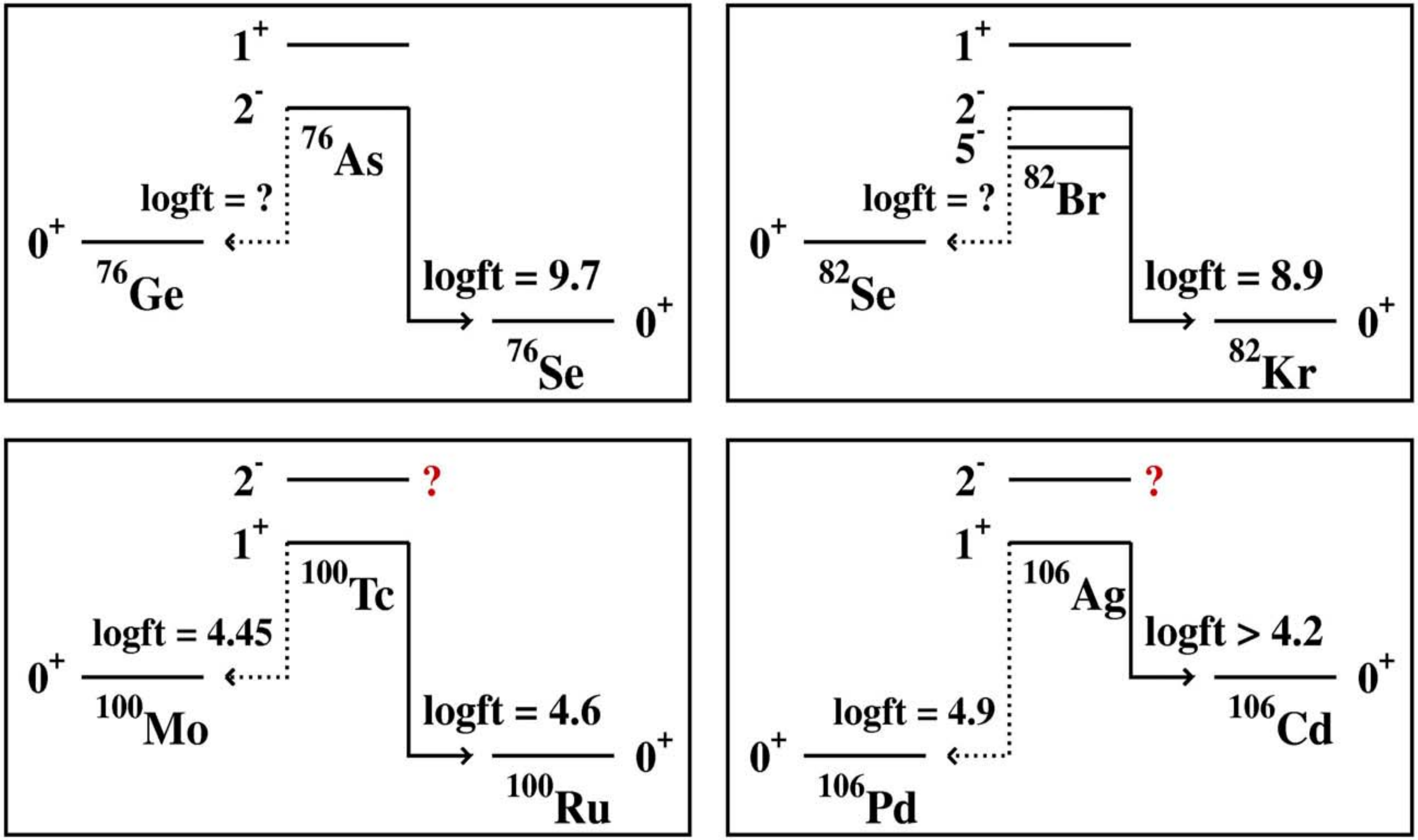
Virtual
transition
(3⁻)
(4⁻)
(1⁺)
?
(3⁻)
?
(1⁻)
(1⁺)
(1⁺)
(1⁺)
2⁻
0⁺
$2\nu\beta^-\beta^-$
0⁺
^{76}Ge
^{76}As
^{76}Se



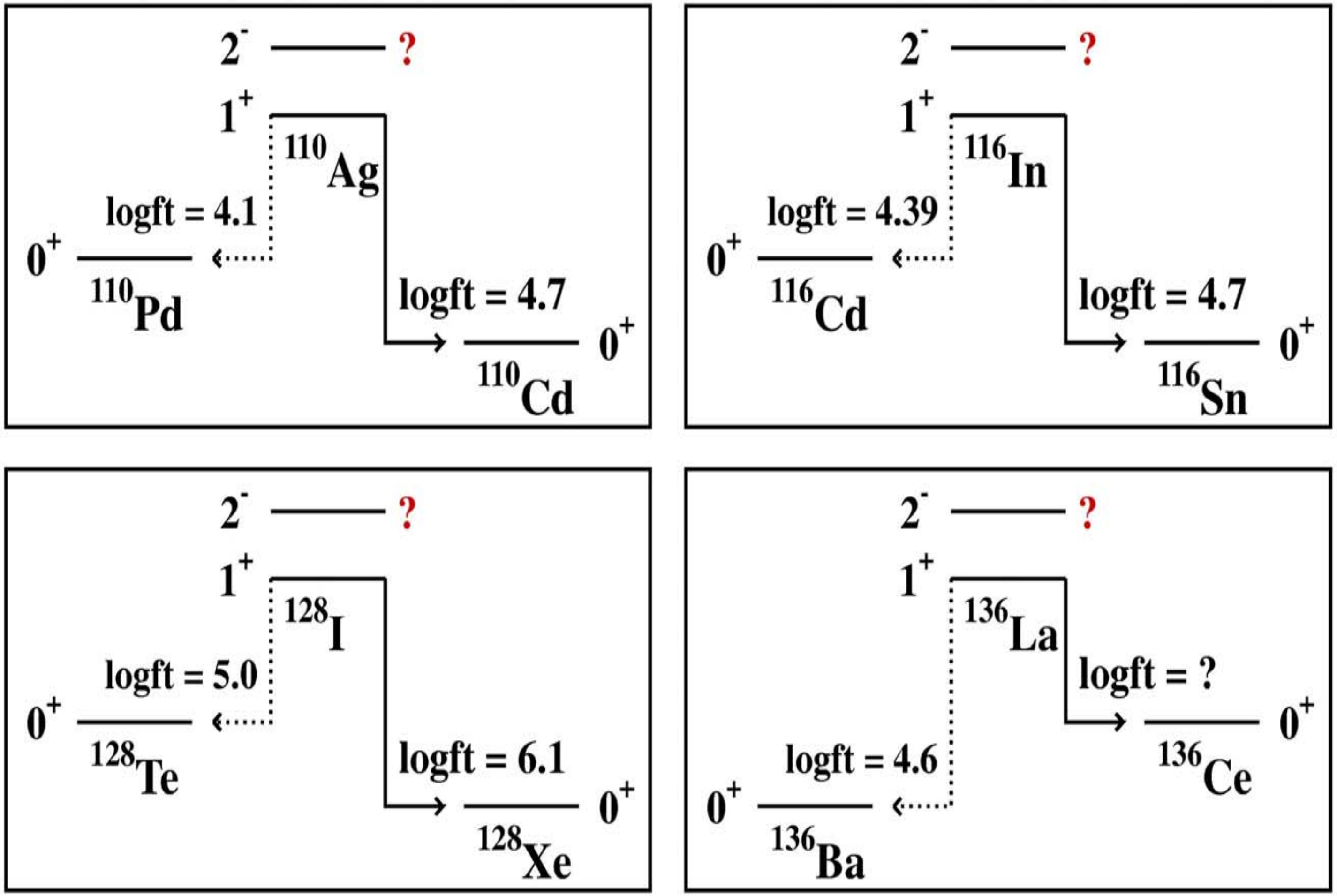
Virtual
transition
(3-)
(4-)
(1+)
?
(3-)
?
(1-)
(1+)
(1+)
(1+)
2-
0+
0νβ-β-
0+
76Ge
76As
76Se



(3-)
(4-)
(1+)
?
(3-)
?
(1-)
(1+)
(1+)
(1+)
2-
0+
log ft = ?
log ft = 9.7
0+
76Ge
76As
76Se



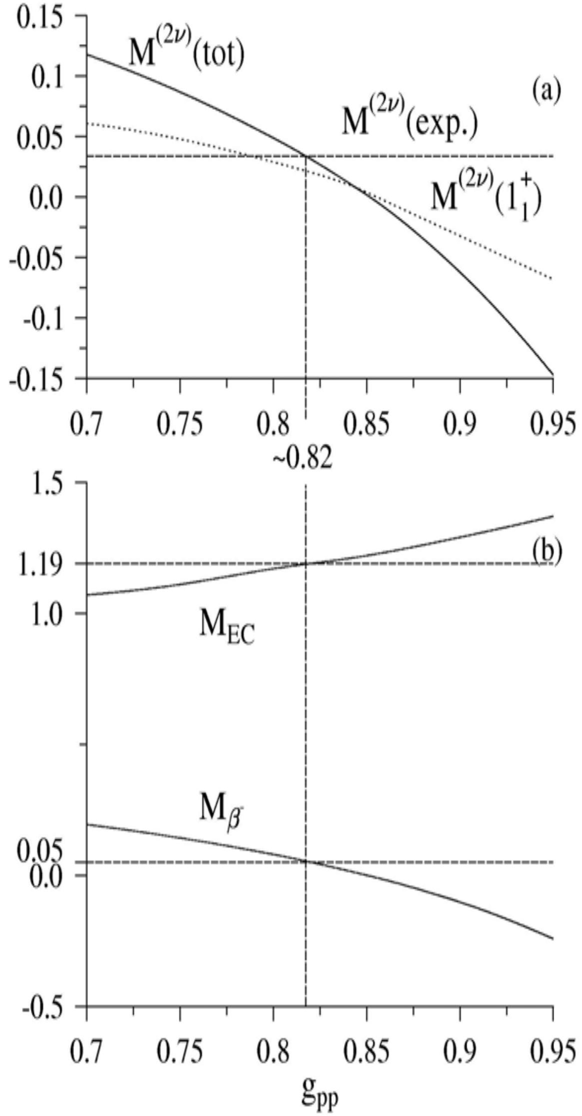
1+
2-
76As
logft = ?
0+
76Ge
logft = 9.7
0+
76Se
1+
2-
5-
82Br
logft = ?
0+
82Se
logft = 8.9
0+
82Kr
2-
?
1+
100Tc
logft = 4.45
0+
100Mo
logft = 4.6
0+
100Ru
2-
?
1+
106Ag
logft > 4.2
0+
106Cd
logft = 4.9
0+
106Pd


2-
?
1+
110Ag
logft = 4.1
0+
110Pd
logft = 4.7
0+
110Cd
2-
?
1+
116In
logft = 4.39
0+
116Cd
logft = 4.7
0+
116Sn
2-
?
1+
128I
logft = 5.0
0+
128Te
logft = 6.1
0+
128Xe
2-
?
1+
136La
logft = ?
0+
136Ce
logft = 4.6
0+
136Ba

G_pp dependence

- Transformation from the single-particle to the quasi-particle basis (particle number is lost, isospin symmetry is lost).
- In open shells the Inclusion of pair terms of the residual proton-neutron interaction induces attractive pairing-like effects which compite with repulsive spin-dependent terms. This effect is also present in shell model results.
- The renormalization of the particle-particle interactions may induced a symmetry breaking, which invalidates the qrpa approach
- Mean field terms and residual interactions (they do not talk to each other)

$$
\begin{aligned}
H_{pn} \;=\; & \frac{1}{2J+1} \sum_{pn,M} < p||\emptyset(J)||n >< p'||\emptyset(J)||n' >^{*} \\
& \Big\{ \chi \left([a_p^{\dagger} a_{\overline{n}}]^{JM} [a_{p'}^{\dagger} a_{\overline{n'}}]^{\dagger \overline{JM}} + [a_p^{\dagger} a_{\overline{n}}]^{\dagger \overline{JM}} [a_{p'}^{\dagger} a_{\overline{n'}}]^{JM} \right) \\
& -\kappa \left([a_p^{\dagger} a_{\overline{n}}^{\dagger}]^{JM} [a_{p'}^{\dagger} a_{\overline{n'}}^{\dagger}]^{\dagger \overline{JM}} + [a_p^{\dagger} a_{\overline{n}}^{\dagger}]^{\dagger \overline{JM}} [a_{p'}^{\dagger} a_{\overline{n'}}^{\dagger}]^{JM} \right) \Big\},
\end{aligned}
$$



0.15
0.1
0.05
0.0
-0.05
-0.1
-0.15
M^(2ν)(tot)
(a)
M^(2ν)(exp.)
M^(2ν)(1_1^+)
0.7
0.75
0.8
0.85
0.9
0.95
~0.82
1.5
1.19
1.0
(b)
M_EC
M_β-
0.05
0.0
-0.5
0.7
0.75
0.8
0.85
0.9
0.95
g_pp

Beyond the QRPA

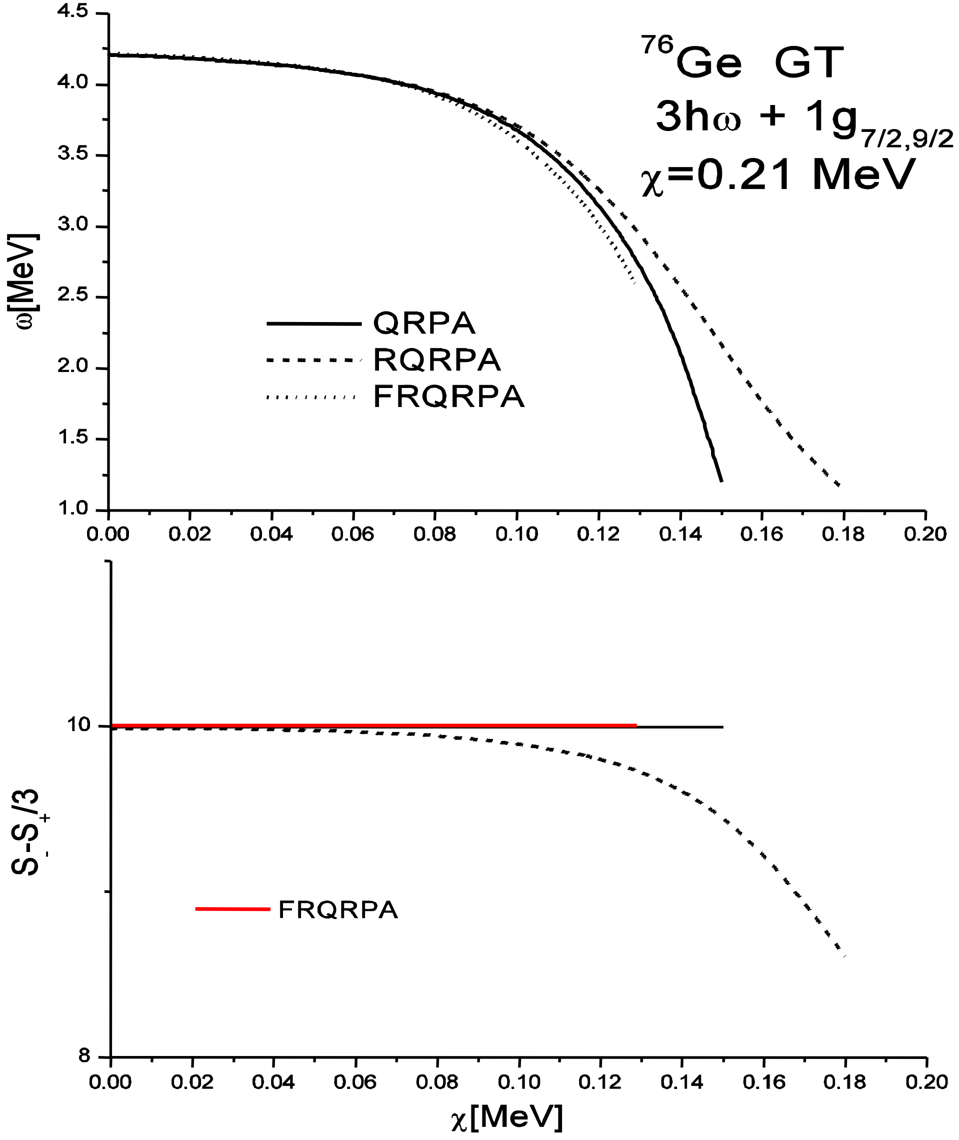
-Inclusion of terms which go beyond the quasi-boson approximation: the procedure is not supported by self-consistency requirements and it introduces spurious effects.

-Among the attempts: “fully renormalized qrpa”

$$
\begin{aligned}
H &= E_p N_p + E_n N_n \\
&+ \lambda_1 A^\dagger A \\
&+ \lambda_2 (A^\dagger A^\dagger + AA) \\
&- \lambda_3 (A^\dagger B + B^\dagger A) \\
&- \lambda_4 (A^\dagger B^\dagger + BA) \\
&+ \lambda_5 B^\dagger B \\
&+ \lambda_6 (B^\dagger B^\dagger + BB)
\end{aligned}
$$

$$\Gamma = X\ \tilde{A}^{\dagger} - Y\ \tilde{A}$$

$$\begin{aligned} \tilde{A}^{\dagger} &= D^{-1/2}\ A^{\dagger}, \\ \tilde{A} &= D^{-1/2}\ A, \\ \left[\tilde{A}, \tilde{A}^{\dagger}\right] &= 1, \\ D &= <[A, A^{\dagger}]> \ = 1 - < \frac{N_p + N_n}{2\ \Omega} > . \end{aligned}$$

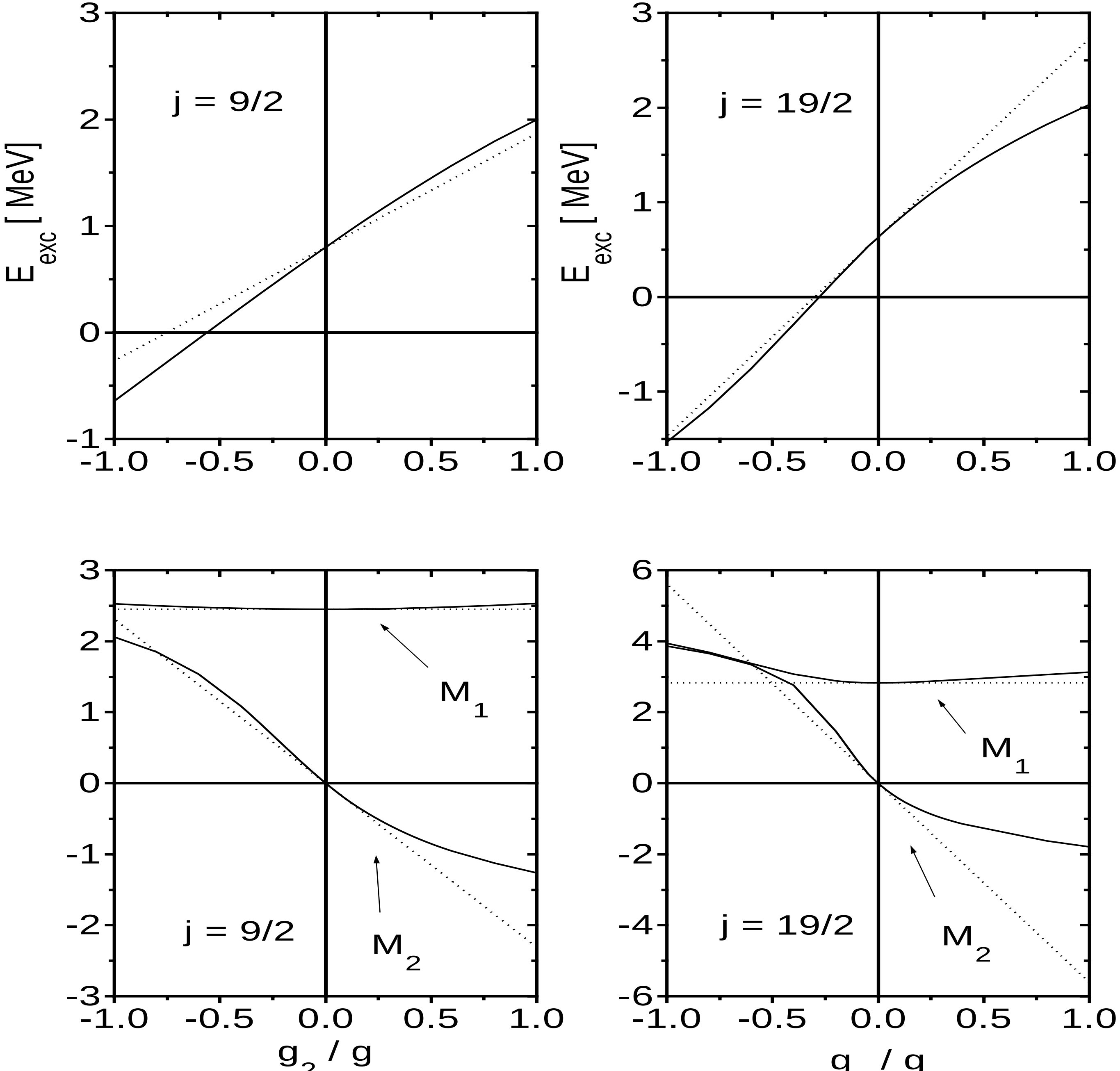


^{76}Ge GT
3hω + 1g$_{7/2,9/2}$
χ=0.21 MeV
ω[MeV]
QRPA
RQRPA
FRQRPA
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16 0.18 0.20
S-S$_-$S$_+$/3
10
8
FRQRPA
0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16 0.18 0.20
χ[MeV]

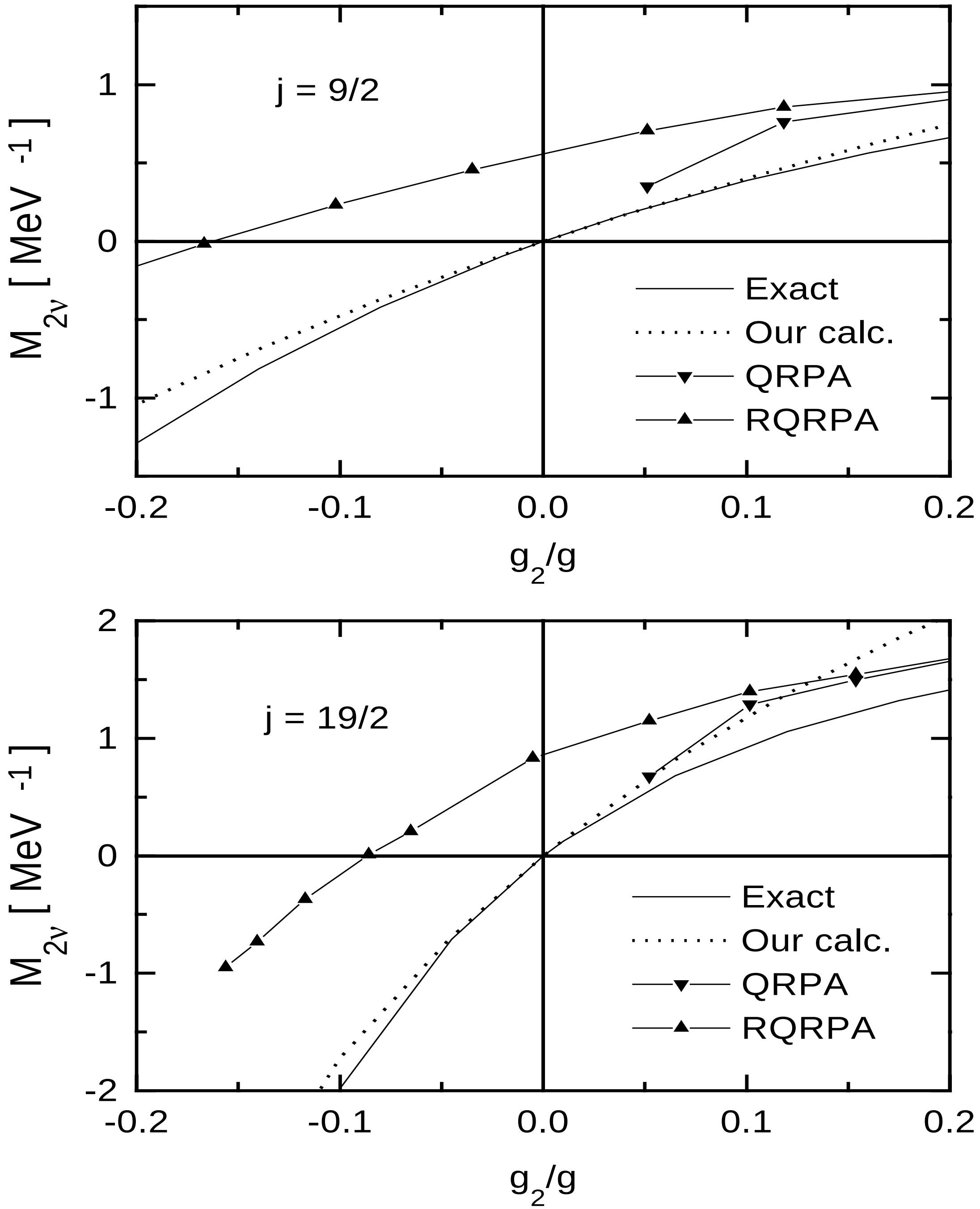
Symmetries

Isospin violations (mean field effects)

1) Mean field approximations (like BCS) break isospin and number symmetries.
2) The QRPA also break these symmetries, but in a different way.
3) Both mechanisms are not coupled.



j = 9/2
j = 19/2
E_{exc} [MeV]
E_{exc} [MeV]
M 1
M 2
j = 9/2
M 1
M 2
j = 19/2
g_2 / g
g_2 / g



j = 9/2
M2ν [MeV -1]
g2/g
Exact
Our calc.
QRPA
RQRPA
j = 19/2
M2ν [MeV -1]
g2/g
Exact
Our calc.
QRPA
RQRPA

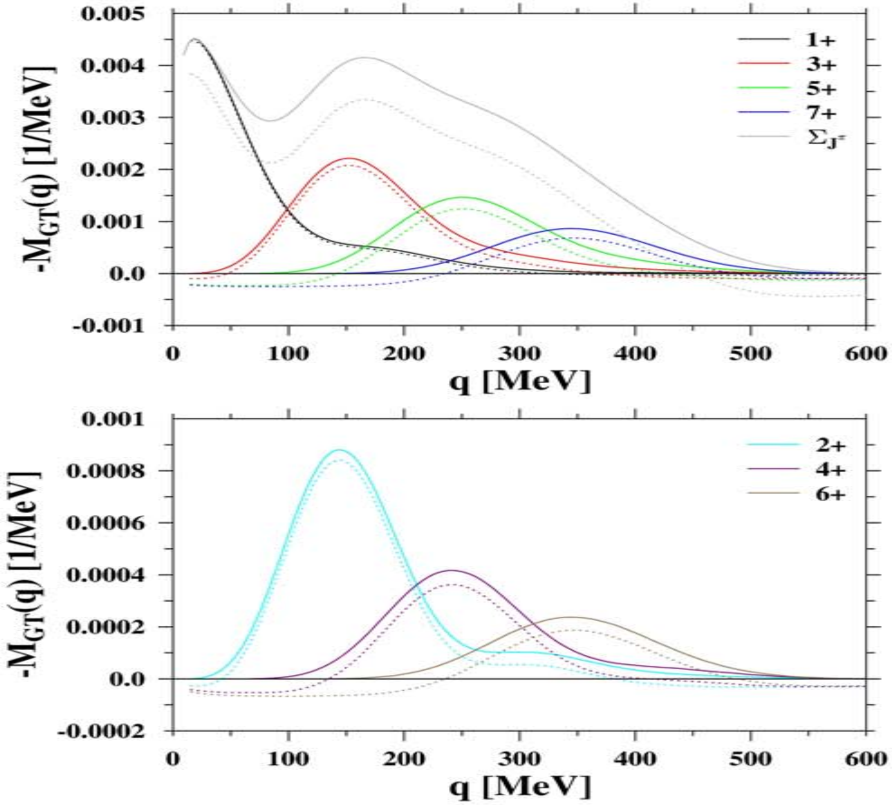
Spurious roots

The QRPA do have spurious roots (a zero energy root) which originates in the quasiparticle pair terms of symmetry operators (like number, isospin, or angular momentum in the deformed case). They represent collective rotations in the space of the symmetry variables.

The root is coupled to the “intrinsic” roots, sometimes it is mistaken as a real intrinsic root.

Short Range Correlations

- Effects due to the finite size of the nucleons, with reference to the momentum q (neutrino momentum)
- Since q is or the order of 150-200 MeV/c short range corelation effects are expected to be operative at distances of the order of 1fm
- Abnormaly large correcctions (of the order of two) have been reported in the literature, in contrast to smaller effects found in shell model calculations.



1+
3+
5+
7+
-M_GT(q) [1/MeV]
q [MeV]
2+
4+
6+
-M_GT(q) [1/MeV]
q [MeV]

Results

Standard elements are:

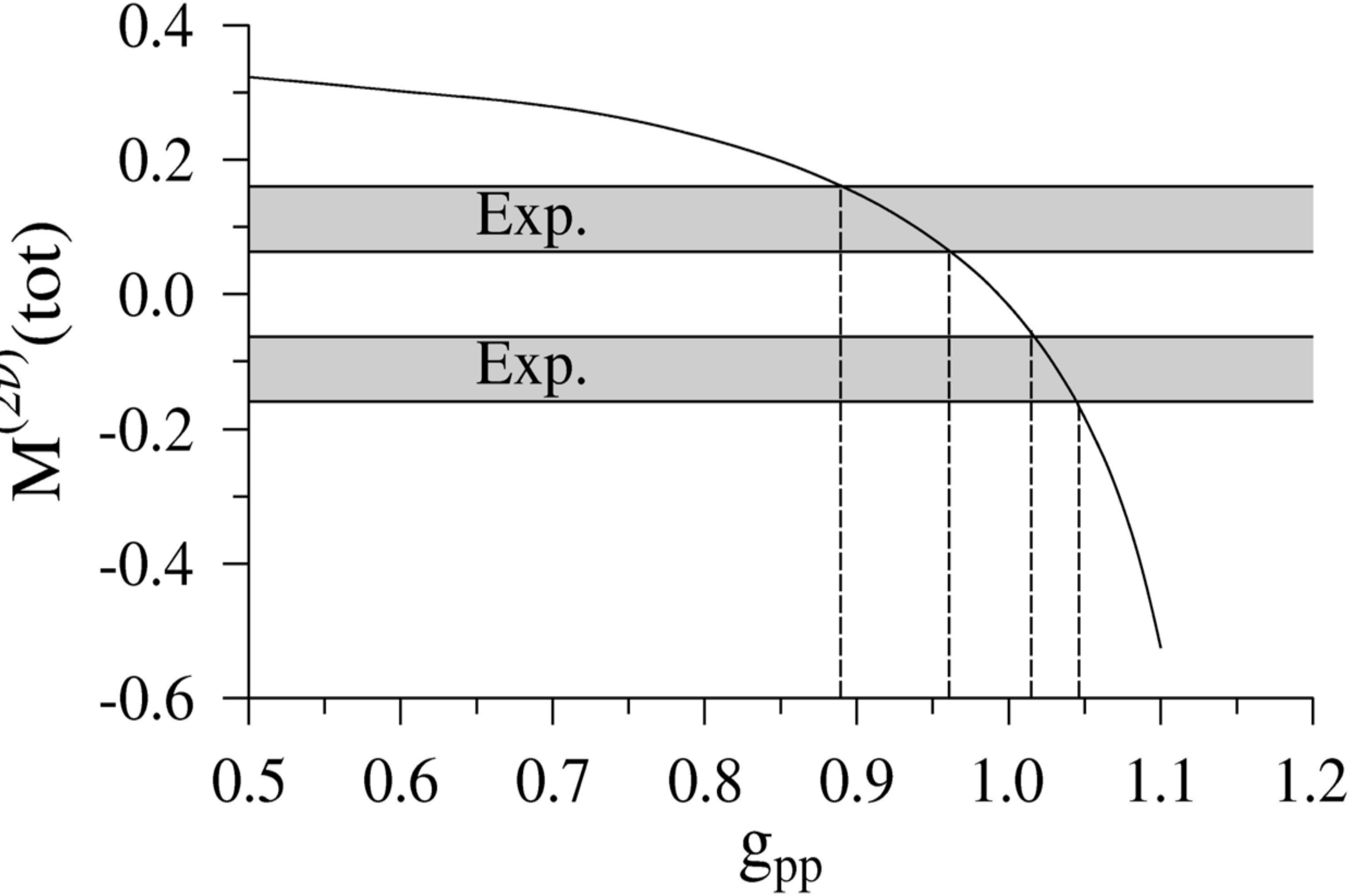
- Saxon-Woods or harmonic oscillator potentials.
- Effective interactions constructed from one pion exchange potentials.
- Gap parameters adjusted to reproduce odd-even mass differences around the double beta decay systems.
- Residual interactions adjusted to reproduced the Gamow-Teller resonance and other giant states if their energies are known.

$$F_N = C_{mm}^{(0\nu)} T_{1/2}$$

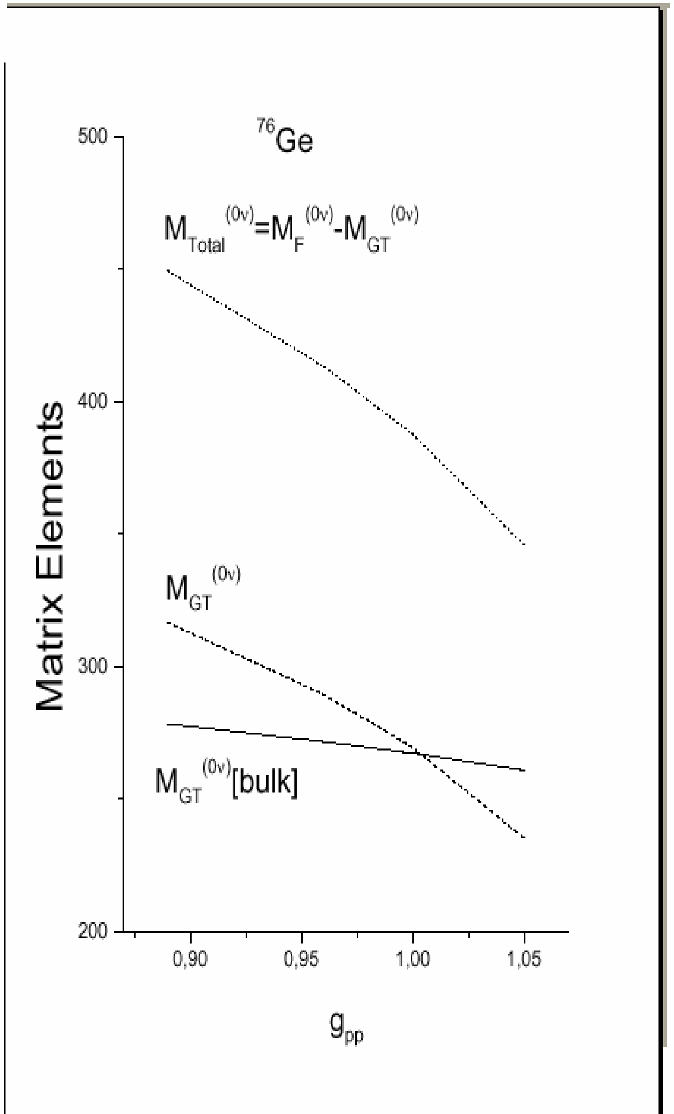
$$\left[t_{1/2}^{0\nu}(J_f)\right]^{-1} = C_{mm}^{(0\nu)} \frac{< m_\nu >^2}{{m_e}^2}$$

$$C_{mm}^{(0\nu)} = G_1^{(0\nu)} \left[({M_{GT}}^{(0\nu)})(1 - \chi_F)\right]^2$$

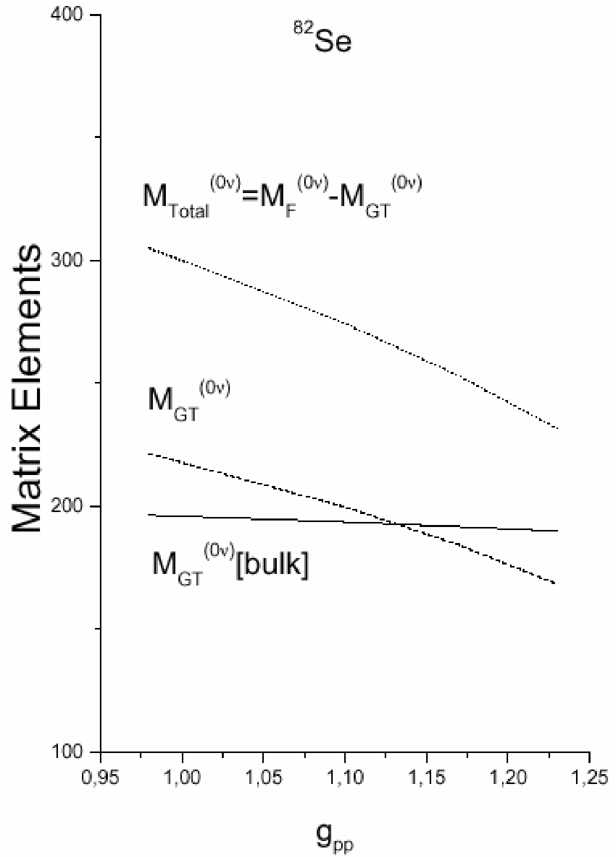
$C^{(0)}_{mm}$	$F_N \times 10^{-12}$	Theory
1.12×10^{-13}	2.80	pnQRPA
6.97×10^{-14}	1.74	pnQRPA
7.51×10^{-14}	1.88	pnQRPA (proj.)
7.33×10^{-14}	1.83	pnQRPA
1.42×10^{-14}	0.35	pnQRPA+ pn pairing
1.18×10^{-13}	2.95	pnQRPA
8.27×10^{-14}	2.07	pnQRPA
2.11×10^{-13}	5.27	RQRPA
6.19×10^{-14}	1.55	RQRPA+ q-dep.
$1.8 - 2.2 \times 10^{-14}$	0.45-0.55	pnQRPA
$5.5 - 6.3 \times 10^{-14}$	1.37-1.57	RQRPA
$2.7 - 3.2 \times 10^{-15}$	0.07-0.08	SCRQRPA
1.85×10^{-14}	0.46	pnQRPA
1.21×10^{-14}	0.30	RQRPA
3.63×10^{-14}	0.91	full-RQRPA
6.50×10^{-14}	1.62	SQRPA
2.88×10^{-13}	7.20	VAMPIR
1.58×10^{-13}	3.95	Shell Model
1.90×10^{-14}	0.47	Shell Model



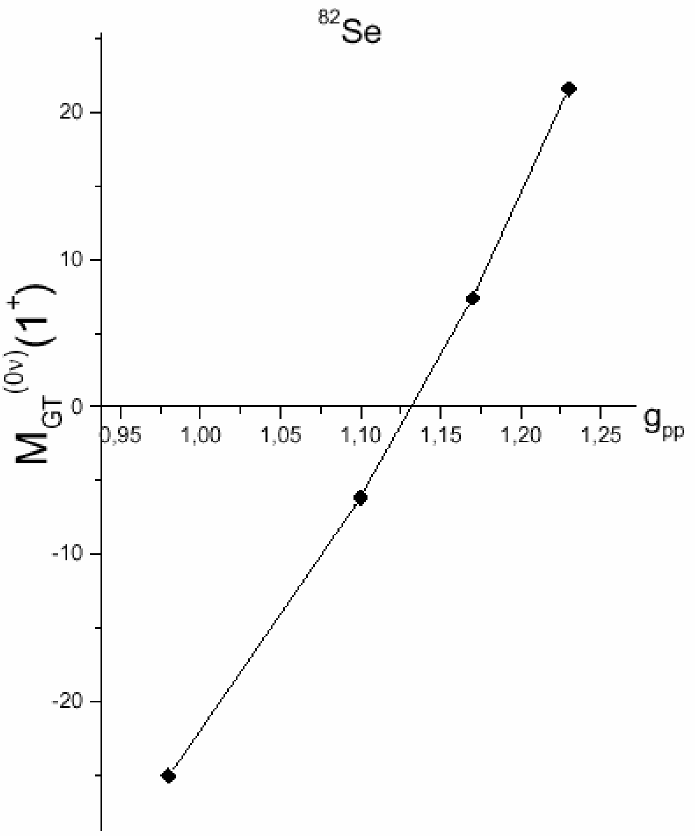
0.4
0.2
0.0
-0.2
-0.4
-0.6
$M^{(2\nu)}$(tot)
Exp.
Exp.
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
g_{pp}



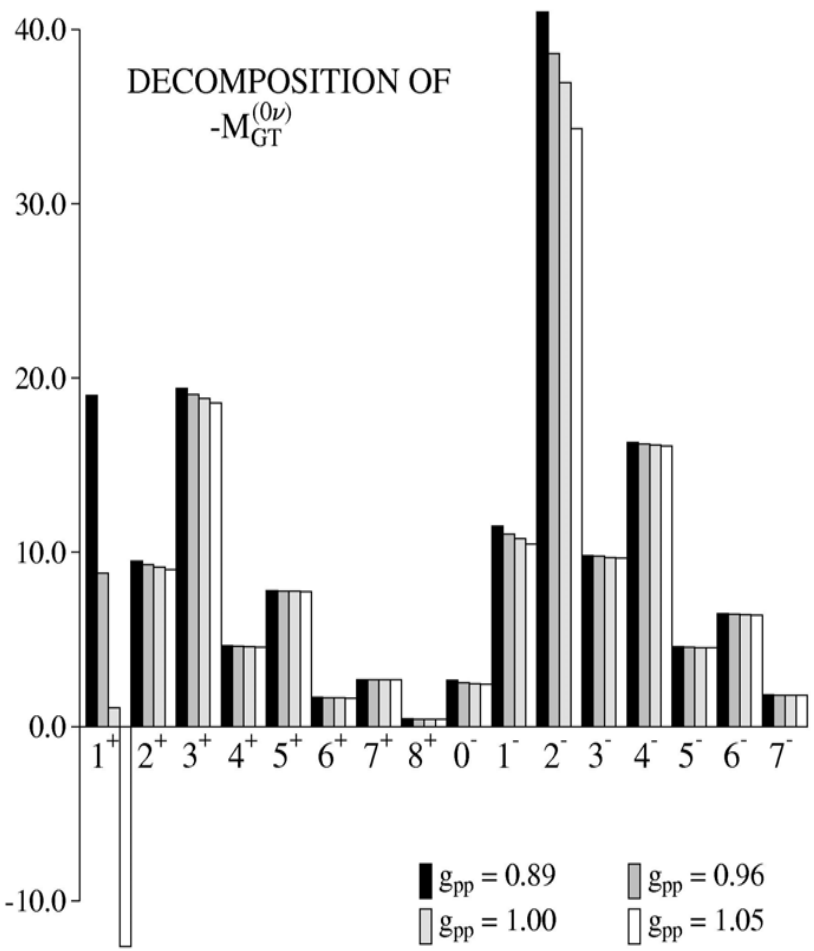
76Ge
M_Total^(0v)=M_F^(0v)-M_GT^(0v)
M_GT^(0v)
M_GT^(0v)[bulk]
Matrix Elements
500
400
300
200
0,90
0,95
1,00
1,05
g_pp



82Se
Matrix Elements
400
300
200
100
M_Total^(0ν)=M_F^(0ν)-M_GT^(0ν)
M_GT^(0ν)
M_GT^(0ν)[bulk]
0,95
1,00
1,05
1,10
1,15
1,20
1,25
g_pp



82Se
20
10
0
-10
-20
0,95
1,00
1,05
1,10
1,15
1,20
1,25
g_pp
M_GT^(0ν)(1+)



DECOMPOSITION OF
$-M_{GT}^{(0\nu)}$
40.0
30.0
20.0
10.0
0.0
-10.0
1^+ 2^+ 3^+ 4^+ 5^+ 6^+ 7^+ 8^+ 0^- 1^- 2^- 3^- 4^- 5^- 6^- 7^-
$g_{pp} = 0.89$
$g_{pp} = 0.96$
$g_{pp} = 1.00$
$g_{pp} = 1.05$

Case	N.M.E.(extracted)	N.M.E.	$< m_\nu >$
^{48}Ca	1.08-2.38		8.70-19.0
^{76}Ge	2.98-4.33	3.33	0.30-0.43
^{82}Se	2.53-3.98	3.44	4.73-7.44
^{96}Zr	2.74	3.55	19.1-24.7
^{100}Mo	0.77-4.67	2.97	1.38-8.42
^{116}Cd	1.09-3.46	3.75	2.37-8.18
^{128}Te	2.51-4.58		9.51-17.4
^{130}Te	2.10-3.59	3.49	1.87-3.20
^{136}Xe	1.61-1.90	4.64	0.79-2.29

SUMMARY

- *Results of the extensions of the QRPA and of the QRPA are not comparable.*
- *The effect of renormalized 1^+ contributions to the NME of the neutrinoless mode is minor.*
- *The bulk of the NME, for the neutrinoless mode, is nearly insensitive to g_pp*
- *Single beta decay, particle transfer, (3He,t) and (p.n) data are proper tools to extract g_pp values.*
- *It may be that g_pp is enterely fixed by symmetry, i.e. : not need of "ad-hoc" procedures which go beyond the mean field + qrpa approach.*