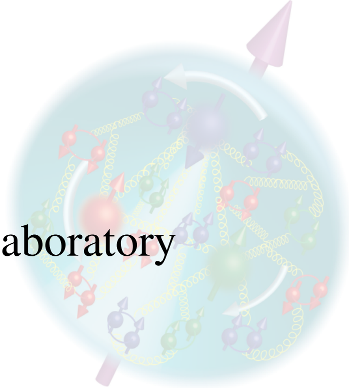
Unravelling the Structure of the Pion



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Argonne National Laboratory

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Sentiments from Theorists

- Craig is on a trip to Trento and Huelva but sends his best wishes to Roy
- This work started when Roy walked into Craig's office in the late 90s and asked if he could calculate the pion's quark distribution function

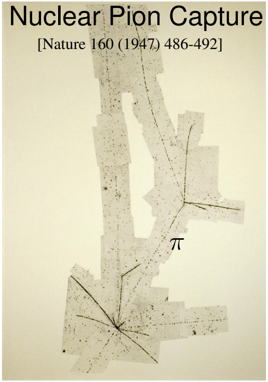
- General sentiments from theorists:

  I gave a theory talk about *X* – thinking it would never be measured – but then Roy came up to me afterwards and said I think I know how to measure this

  "He is a very stimulating colleague who knows the value of close interchange with theorists – if only there were more like him!"

The Pion – Nature's strong messenger

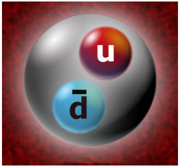
- Hideki Yukawa in 1935 postulated a strongly interacting particle of mass $\sim 100\,\text{MeV}$
  - Yukawa called this particle a "meson"
- Cecil Powell in 1947 discovered the π-meson from cosmic ray tracks in a photographic emulsion – a technique Cecil developed



- Cavendish Lab had said method is incapable of "*reliable and reproducible precision measurements*"
- The measured *pion* mass was $\sim 130 - 150\,\text{MeV}$
- Fittingly, both Yukawa & Powell received Nobel Prize – in 1949 and 1950 respectively
- Discovery of pion beginning of particle physics; before long there was the particle *zoo*

The Pion in QCD

- Today the pion is understood as both a bound state of a *dressed-quark* and a *dressed-antiquark* in QFT and the Goldstone mode associated with DCSB in QCD



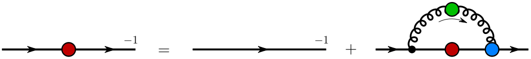
- This dichotomous nature has numerous ramifications, e.g.:

$$m_\rho/2 \sim M_N/3 \sim 350\,\text{MeV} \qquad \text{however} \qquad m_\pi/2 \simeq 0.2 \times 350\,\text{MeV}$$

- The pion is unusually light, the key is dynamical chiral symmetry breaking
  - curiously in coming to understand the pion's mass, DCSB has been exposed as the origin of more than 98% of the mass in the visible Universe
- QCD is characterized by two emergent phenomena: *confinement & DCSB*
  - it is also the only known example in nature of a fundamental QFT that is innately non-perturbative
- *In the quest to understand QCD must discover the origin of confinement, its relationship to DCSB and understand how these phenomenon influence hadronic observables*

QCDs Dyson-Schwinger Equations

- The equations of motion of QCD $\Longleftrightarrow$ QCDs Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - must implement a symmetry preserving truncation
- The most important DSE is QCDs gap equation $\Longrightarrow$ quark propagator

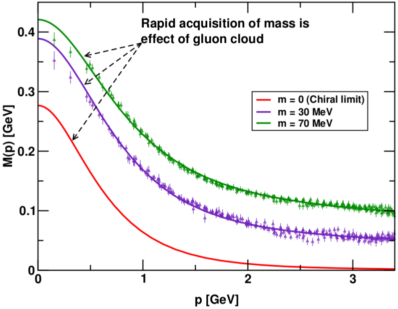


  - ingredients – dressed gluon propagator & dressed quark-gluon vertex

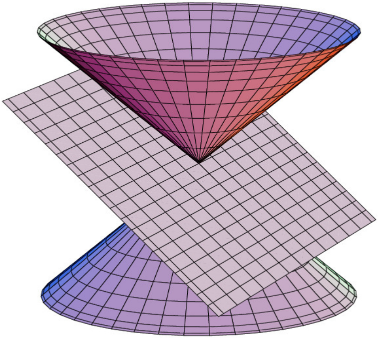
$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

- $S(p)$ has correct perturbative limit
- $M(p^2)$ exhibits dynamical mass generation $\Longleftrightarrow$ DCSB
- $S(p)$ has complex conjugate poles
  - no real mass shell $\Longleftrightarrow$ confinement

[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



Light-Front Wave Functions

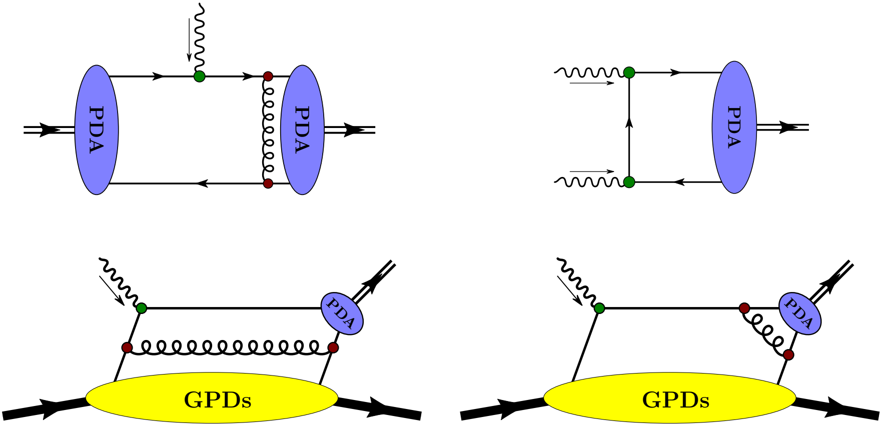


- In equal-time quantization a hadron wave function is a frame dependent concept
  - boost operators are dynamical, that is, they are interaction dependent
- In high energy scattering experiments particles move at near speed of light
  - natural to quantize a theory at equal light-front time: $\tau = (t+z)/\sqrt{2}$
- Light-front quantization $\Longrightarrow$ light-front WFs; many remarkable properties:
  - frame-independent; probability interpretation – as close as QFT gets to QM
  - boosts are kinematical – *not dynamical*
- Parton distribution amplitudes (PDAs) are (almost) observables & are related to light-front wave functions

$$\varphi(x) = \int d^2\vec{k}_\perp \; \psi(x, \vec{k}_\perp)$$

Pion's Parton Distribution Amplitude

- pion's PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*
  - it's a function of the light cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2



- PDAs enter numerous hard exclusive scattering processes

Pion's Parton Distribution Amplitude

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  - it's a function of the lightcone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2
- The pion's PDA is defined by

$$f_\pi \, \varphi_\pi(x) = Z_2 \int \frac{d^4 k}{(2\pi)^2} \, \delta\left(k^+ - x\, p^+\right) \mathrm{Tr}\left[\gamma^+ \gamma_5 \, S(k) \, \Gamma_\pi(k,p) \, S(k-p)\right]$$

- $S(k)\,\Gamma_\pi(k,p)\,S(k-p)$ is the pion's Bethe-Salpeter wave function
  - in the non-relativistic limit it corresponds to the Schrodinger wave function
- $\varphi_\pi(x)$: is the axial-vector projection of the pion's Bethe-Salpeter wave function onto the light-front [pseudo-scalar projection also non-zero]
- Pion PDA is an essentially nonperturbative quantity whose asymptotic form is known; in this regime governs, e.g., Q^2 dependence of pion form factor

$$Q^2 \, F_\pi(Q^2) \overset{Q^2 \to \infty}{\longrightarrow} 16\,\pi\, f_\pi^2 \, \alpha_s(Q^2) \qquad \Longleftrightarrow \qquad \varphi_\pi^{\text{asy}}(x) = 6\, x\, (1-x)$$

QCD Evolution & Asymptotic PDA

- ERBL (Q^2) evolution for pion PDA [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu} \varphi(x,\mu) = \int_0^1 dy\, V(x,y)\, \varphi(y,\mu)$$

- This evolution equation has a solution of the form

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x) \left[1 + \sum_{n=2,\,4,\ldots} a_n^{3/2}(Q^2)\, C_n^{3/2}(2x-1)\right]$$

  - $\alpha = 3/2$ because in $Q^2 \to \infty$ limit QCD is invariant under the collinear conformal group $SL(2;\mathbb{R})$
  - Gegenbauer-$\alpha = 3/2$ polynomials are irreducible representations $SL(2;\mathbb{R})$

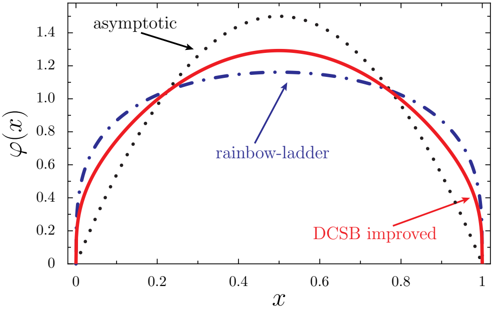
- The coefficients of the Gegenbauer polynomials, $a_n^{3/2}(Q^2)$, evolve logarithmically to zero as $Q^2 \to \infty$: $\varphi_\pi(x) \to \varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$

- At what scales is this a good approximation to the pion PDA?

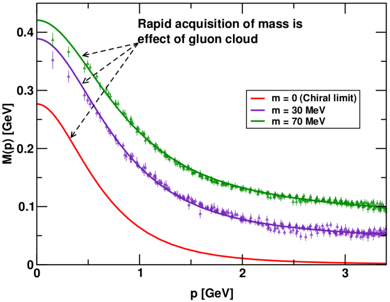
- E.g., AdS/QCD find $\varphi_\pi(x) \sim x^{1/2}\,(1-x)^{1/2}$ at $Q^2 = 1\,\text{GeV}^2$; expansion in terms of $C_n^{3/2}(2x-1)$ convergences slowly: $a_{32}^{3/2} / a_2^{3/2} \sim 10\,\%$

Pion PDA from the DSEs

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)]



[C.D. Roberts, Prog. Part. Nucl. Phys. 61 50 (2008)]



- Both DSE results, each using a different Bethe-Salpeter kernel, exhibit a pronounced broadening compared with the asymptotic pion PDA
  - scale of calculation is given by renormalization point $\zeta = 2\,\mathrm{GeV}$
- Broading of the pion's PDA is directly linked to DCSB
- As we shall see the dilation of pion's PDA will influence the Q^2 evolution of the pion's electromagnetic form factor

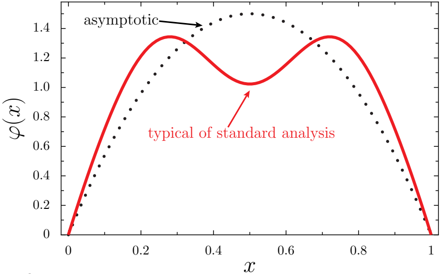
Pion PDA from lattice QCD

- Lattice QCD can only determine one non-trivial moment

$$\int_0^1 dx\,(2\,x-1)^2\varphi_\pi(x) = 0.27 \pm 0.04$$

[V. Braun *et al.*, Phys. Rev. D **74**, 074501 (2006)]

  - scale is $Q^2 = 4\,\mathrm{GeV}^2$



- Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1+\sum\nolimits_{n=2,\,4,\ldots} a_n^{3/2}(Q^2)\,C_n^{3/2}(2x-1)\right]$$

  - however this expansion is guaranteed to converge rapidly only when $Q^2 \to \infty$
  - this procedure results in a *double-humped* pion PDA

- Advocate using a *generalized expansion*

$$\varphi_\pi(x,Q^2) = N_\alpha\,x^{\alpha-1/2}(1-x)^{\alpha-1/2}\left[1+\sum\nolimits_{n=2,\,4,\ldots} a_n^{\alpha}(Q^2)\,C_n^{\alpha}(2x-1)\right]$$

- Find $\varphi_\pi \simeq x^\alpha(1-x)^\alpha,\ \alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \simeq 0.30$

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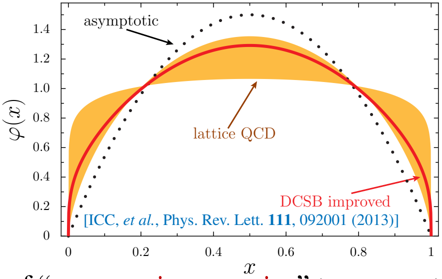

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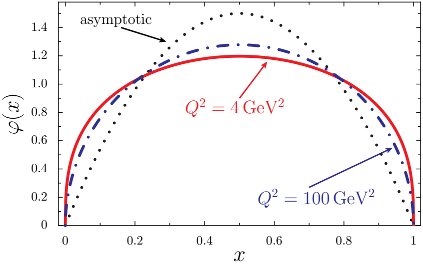
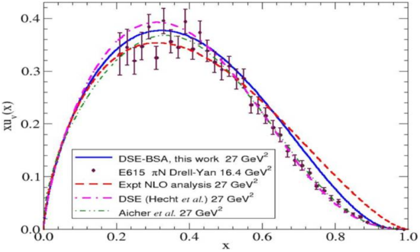
When is the Pion's PDA Asymptotic

[I. C. Cloët, *et al.*, Phys. Rev. Lett. **111**, 092001 (2013)]

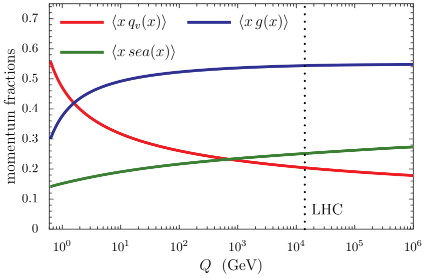


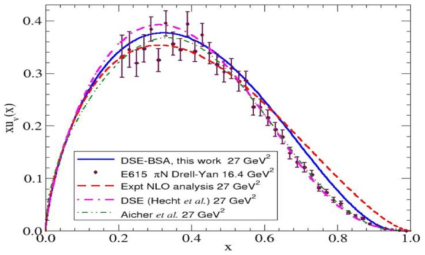
[T. Nguyen, *et al.*, Phys. Rev. C **83**, 062201 (2011)]



- Under leading order Q^2 evolution the pion PDA remains broad to well above $Q^2 > 100\,\text{GeV}^2$, compared with $\varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$
- *Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors*
- Importantly, $\varphi_\pi^{\text{asy}}(x)$ is only guaranteed be an accurate approximation to $\varphi_\pi(x)$ when pion valence quark PDF satisfies: $q_v^\pi(x) \sim \delta(x)$
- This is far from valid at forseeable energy scales

When is the Pion's Valence PDF Asymptotic





- LO QCD evolution of momentum fraction carried by valence quarks

$$\langle x\,q_v(x)\rangle\,(Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{qq}^{(0)2}/(2\beta_0)} \langle x\,q_v(x)\rangle\,(Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0$$

  - therefore, as $Q^2 \to \infty$ we have $\langle x\,q_v(x)\rangle \to 0$ implies $q_v(x) = \delta(x)$

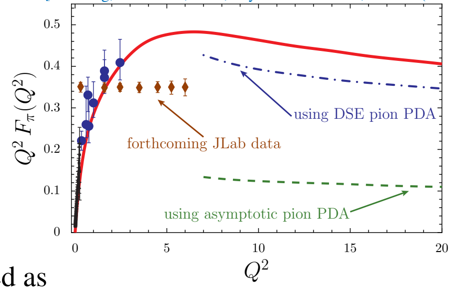
- At LHC energies valence quarks still carry 20% of pion momentum
  - the gluon distribution saturates at $\langle x\,g(x)\rangle \sim 55\%$

- *Asymptotia is a long way away!*

Pion Elastic Form Factor

[L. Chang, I. C. Cloët, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]

- Extended the pre-experiment DSE prediction to $Q^2 > 4\,\mathrm{GeV}^2$
- Predict max at $Q^2 \approx 6\,\mathrm{GeV}^2$; within domain accessible at JLab12
- Magnitude directly related to DCSB



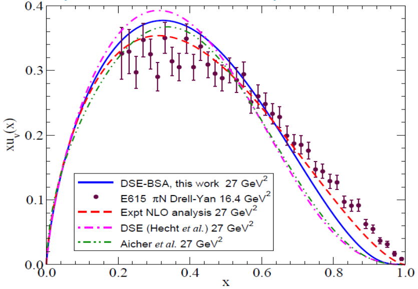
- The QCD prediction can be expressed as

$$Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda^2_{\rm QCD}}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_\pi(x)$$

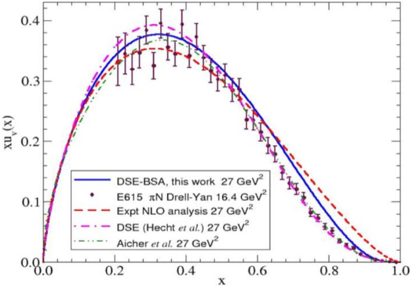
- Using $\varphi_\pi^{\rm asy}(x)$ significantly underestimates experiment
- Within DSEs there is consistency between the direct pion form factor calculation and that obtained using the DSE pion PDA
  - 15% disagreement explained by higher order/higher-twist corrections
- *We predict that QCD power law behaviour sets in at $Q^2 \sim 8\,\mathrm{GeV}^2$*

Pion PDF

[J. S. Conway et al. (E615 Collaboration), Phys. Rev. D 39, 92 (1989)]



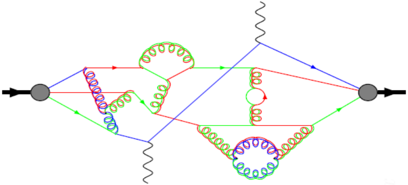
[M. Aicher, *et al.*, Phys. Rev. Lett. **105**, 252003 (2010)]



- Need for *QCD-based* calculation is emphasized by story of pion's valence quark distribution function:
  - 1989: $u_v^\pi \overset{x\to 1}{\sim} (1-x)^1$ – inferred from LO-Drell-Yan & disagrees with QCD
  - *Roy talks to Craig about the pion PDF*
  - 2001: DSEs predicts $u_v^\pi \overset{x\to 1}{\sim} (1-x)^2$ – argues that distribution inferred from data can't be correct
  - 2010: new NLO reanalysis – including soft-gluon resummation – inferred distribution agrees with DSE-QCD

PDFs and lattice QCD

- PDFs enter DIS cross-sections & are critial components of hadron structure
  - PDFs – e.g. $q(x, Q^2)$ – are Lorentz invariant and are functions of the light cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2
  - $q(x, Q^2)$: *probability to strike a quark of flavour q with light cone momentum fraction x of the target momentum*
- PDFs represent parton correlations along the light cone and are inherently Minkowski space objects
  - lattice QCD, which is definied in Euclidean space, cannot directly calculate PDFs
  - further, since lattice only possesses hypercubic symmetry, only the first few moments of a PDF can be accessed in contemporary simulations



$$q(x, Q^2) = \int \frac{d\xi^-}{2\pi}\, e^{ip^+\xi^- x} \times \langle P|\overline{\psi}_q(0)\,\gamma^+\,\psi_q(\xi^-)|P\rangle$$

PDFs and Quasi-PDFs

- In *PRL 110 (2013) 262002* Xiangdong Ji proposed a method to access PDFs on the lattice via Quasi-PDFs
  - may people where already aware of this idea but Ji put it on a firmer footing theoretically
- Quasi-PDFs represent parton correlations along the z-direction $[\tilde{x} = \frac{k_z}{p_z}]$

$$\tilde{q}(\tilde{x}, Q^2, p_z) = \int \frac{d\xi_z}{2\pi}\, e^{ip_z\,\xi_z\,\tilde{x}} \langle P|\overline{\psi}_q(0)\,\gamma_z\,\psi_q(\xi_z)|P\rangle$$

$$c.f. \quad q(x, Q^2) = \int \frac{d\xi^-}{2\pi}\, e^{ip^+\,\xi^-\,x} \langle P|\overline{\psi}_q(0)\,\gamma^+\,\psi_q(\xi^-)|P\rangle$$

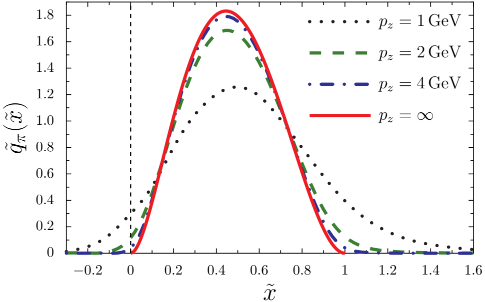
  - in limit $p_z \to \infty$ then $\tilde{q}(\tilde{x}, Q^2, p_z) \to q(x, Q^2)$; corrections $\mathcal{O}\left[\frac{M^2}{p_z^2}, \frac{\Lambda_{\text{QCD}}^2}{p_z^2}\right]$
- $\tilde{q}$ depends on p_z & is therefore not a Lorentz invariant; $\tilde{x}$ not bounded by p_z:

$$-\infty < \tilde{x} = \frac{k_z}{p_z} < \infty; \qquad \text{c.f.} \qquad 0 < x = \frac{k^+}{p^+} < 1$$

- Need to put fast moving hadron on a lattice; but when is p_z large enough?

Pion Quasi-PDFs from DSEs

- Using the DSEs we can determine both the PDFs and Quasi-PDFs
  - can then infer how large p_z must be to have $\tilde{q}(\tilde{x}, Q^2, p_z) \simeq q(x, Q^2)$
- For $p_z \lesssim 1\,\text{GeV}$ find that *quark* distribution has sizeable support for $\tilde{x} < 0$
  - this is in constrast to PDFs, however it is natural since k_z can be negative
- For $p_z \simeq 4\,\text{GeV}$ find that the pion PDF and quasi-PDF are very similar
  - pion likely best case scenario, e.g., nucleon likely has large $\frac{M^2}{p_z^2}$ corrections
- Quasi-PDFs do not give parton momentum fractions [Y. Ma & J. Qiu - arXiv:1404.6860]



All results in chiral limit

$$\langle \tilde{x}\, \tilde{q}_z(x) \rangle_{p_z = 1\,\text{GeV}} = 0.53 \quad (14\%)$$

$$\langle \tilde{x}\, \tilde{q}_z(x) \rangle_{p_z = 2\,\text{GeV}} = 0.49 \quad (5\%)$$

$$\langle \tilde{x}\, \tilde{q}_z(x) \rangle_{p_z = 4\,\text{GeV}} = 0.48 \quad (3\%)$$

$$\langle \tilde{x}\, \tilde{q}_z(x) \rangle_{p_z = \infty} = 0.47$$

Conclusion

- QCD and therefore hadron physics is unique:
  - must confront a fundamental theory in which the elementary degrees-of-freedom are confined and only hadrons reach detectors
- A solid understanding of the pion is critical
- Both DSEs and lattice QCD agree that the pion PDA is significantly broader than the asymptotic result
  - using LO evolution find dilation remains significant for $Q^2 > 100\,\mathrm{GeV}^2$
  - asymptotic form of pion PDA only guaranteed to be valid when $q_v^\pi(x) \propto \delta(x)$
- Determined the pion form factor for all spacelike momenta
  - $Q^2 F_\pi(Q^2)$ peaks at $6\,\mathrm{GeV}^2$, with maximum directly related to DCSB
  - predict that QCD power law behaviour sets in at $Q^2 \sim 8\,\mathrm{GeV}^2$
- *Thanks to Roy's question to Craig – about 15 years ago – we have now developed a deep understanding of pion structure!*