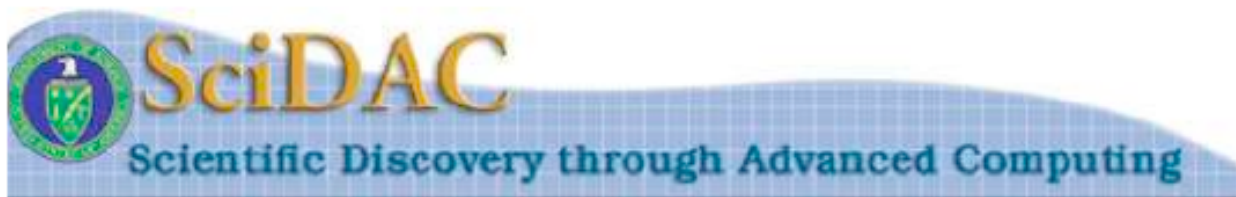


# Non-empirical energy density functionals

S.K. Bogner (NSCL/MSU)

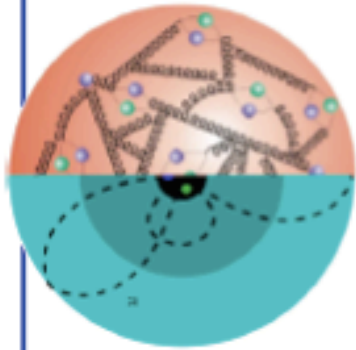


# Outline

- 1) Renormalization group methods
- 2) Microscopically-constrained Skyrme functionals
- 3) Other efforts towards non-empirical functionals

# $\Lambda$ / Resolution dependence of nuclear forces

with high-energy probes:  
quarks+gluons



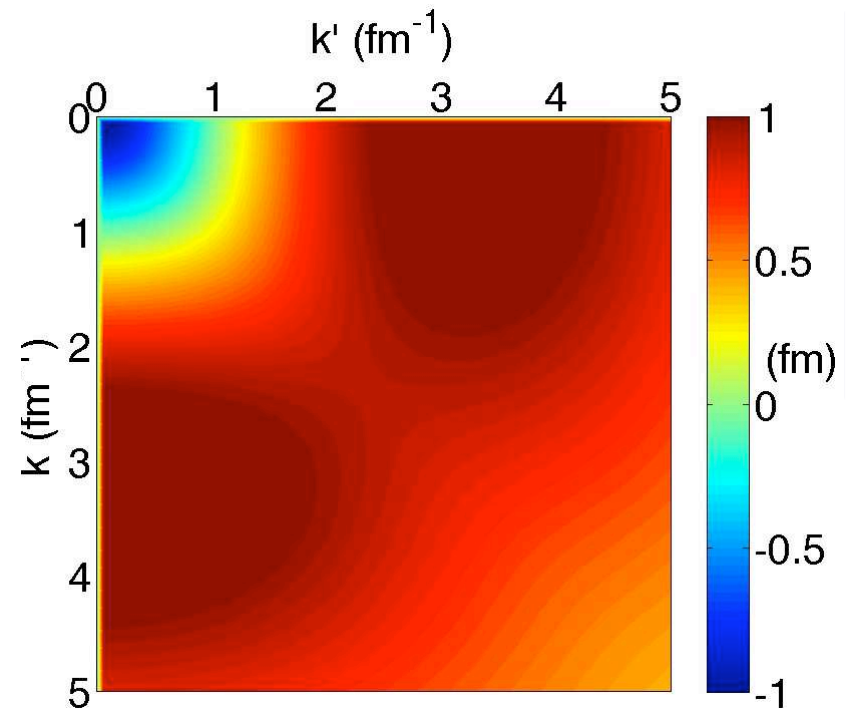
Effective theory for NN, 3N, many-N interactions and electroweak operators: resolution scale/ $\Lambda$ -dependent

$$H(\Lambda) = T + V_{\text{NN}}(\Lambda) + V_{\text{3N}}(\Lambda) + V_{\text{4N}}(\Lambda) + \dots$$

$\Lambda \gg m_\pi, k_F$  in typical interactions

# of sp states for A-body  $\sim \Lambda^3 A$

Strong correlations, non-perturbative

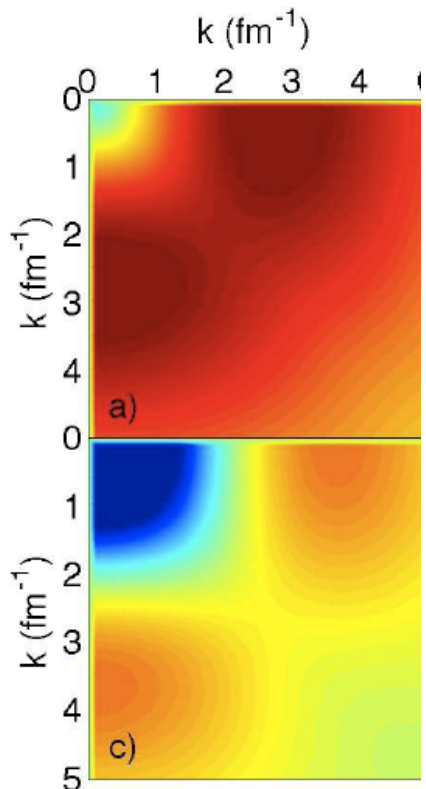


# Why large $\Lambda$ 's are complicated: ab initio DFT

Ab initio DFT (OEP/effective action) corresponds to MBPT with

$$\begin{aligned}
 H &= (T + U) + (V - U) \\
 &= H_{KS} + H_1
 \end{aligned}$$

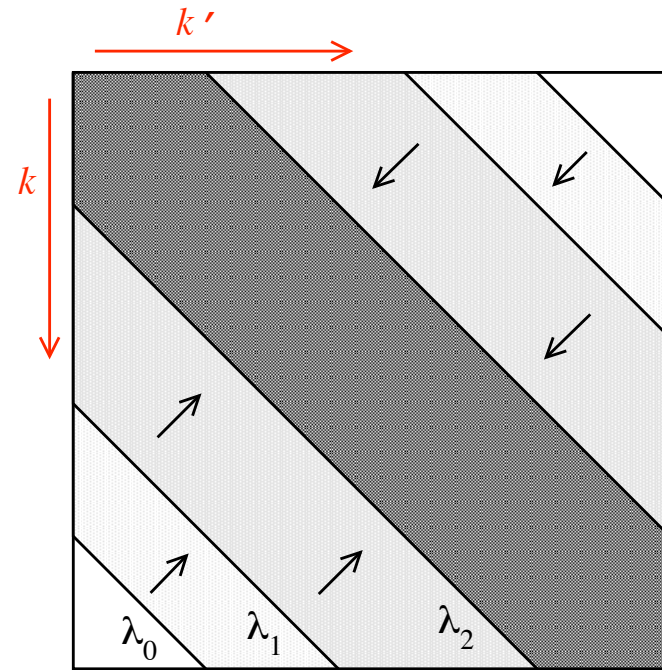
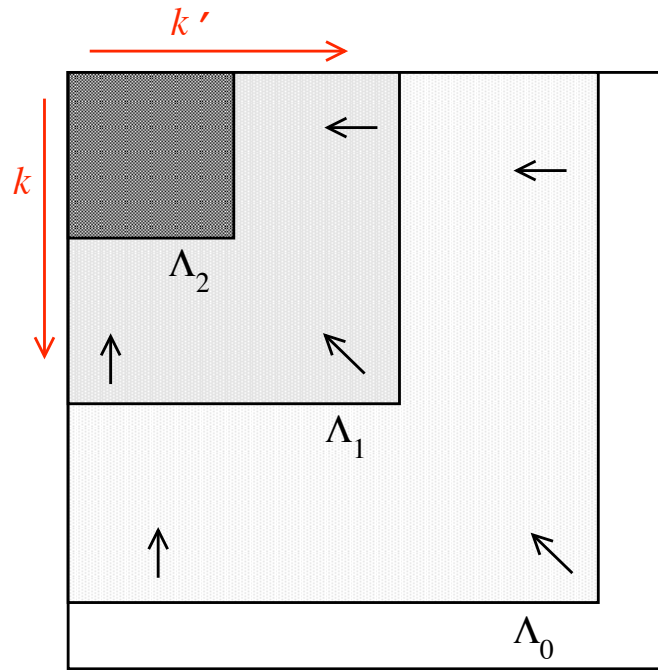
Want freedom to chose U such that corrections to density beyond  $H_{KS}$  vanish



Large  $\Lambda$   $V_{NN}$  strongly couples low/high  $k$

coupling persists even with G matrix resummation  $\implies$  non-perturbative in G and convergence of hole-line expansion **strongly depends on U**

## 2 Types of Renormalization Group Transformations



“ $V_{\text{low } k}$ ”

integrate-out high  $k$  states

preserves observables for  $k < \Lambda$

“Similarity RG”

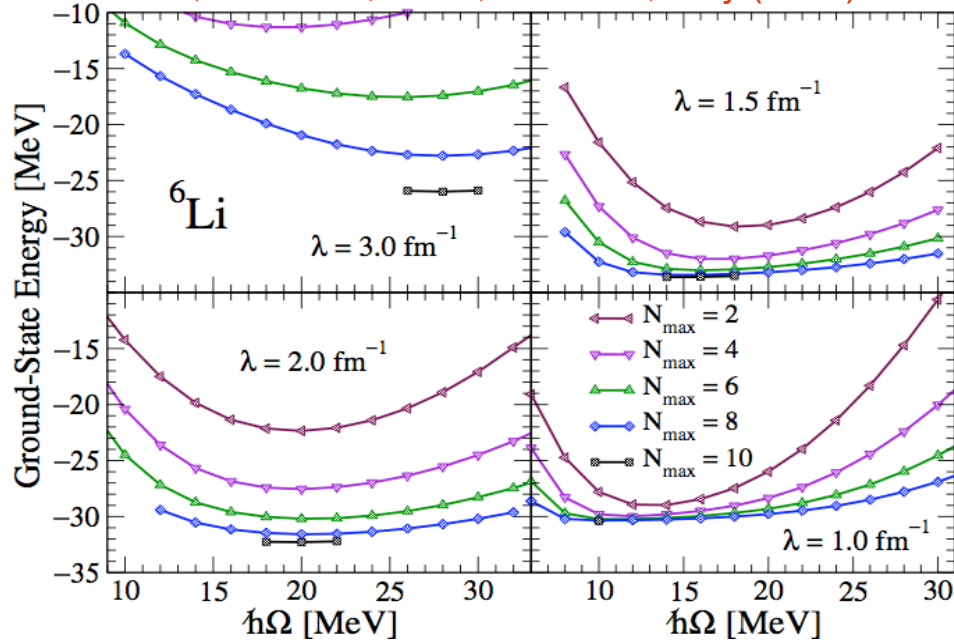
eliminate far off-diagonal coupling

preserves “all” observables

Very similar consequences despite differences in appearance  
(low and high momentum decoupled)

# RG-Improved Convergence in ab-initio calculations

SKB, Furnstahl, Maris, Schwenk, Vary (2008)



Li-6 diagonalization in HO basis

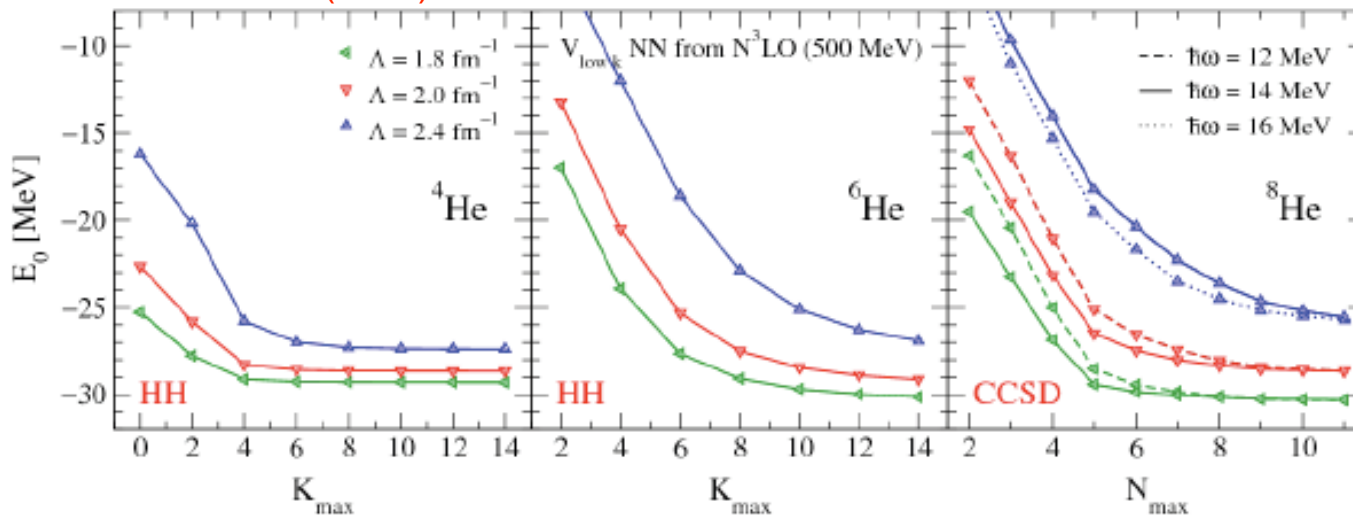
$10^3$  states for  $N_{\text{max}} = 2$

versus

$10^7$  states for  $N_{\text{max}} = 10$

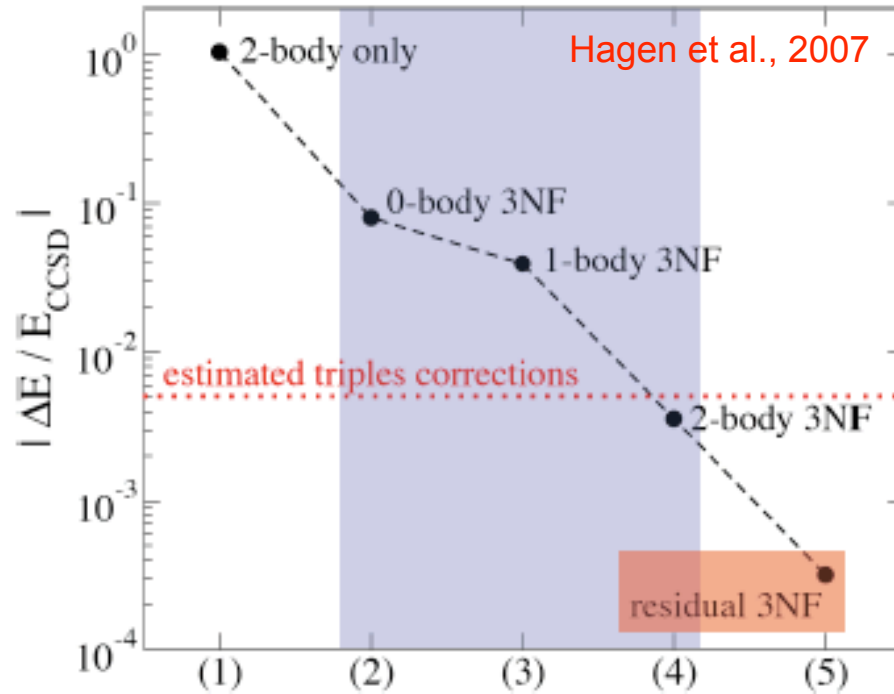
Helium Halo Nuclei

Bacca et al. (2009)



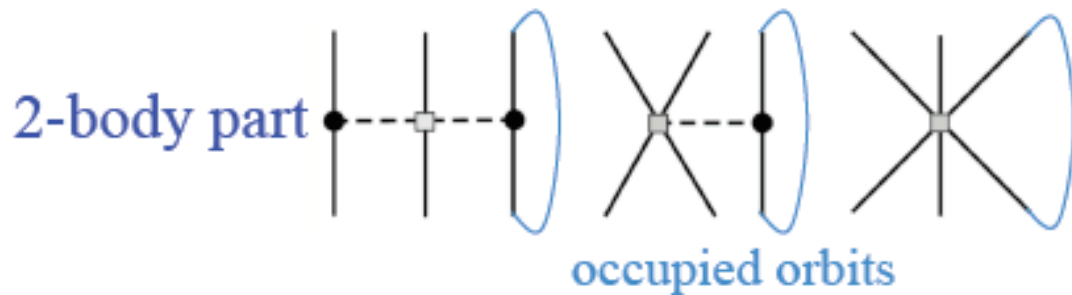
Ab-initio calculations of heavier nuclei accessible...

# Towards including 3N interactions in medium mass nuclei



coupled-cluster  
calculations of  
closed-shell nuclei

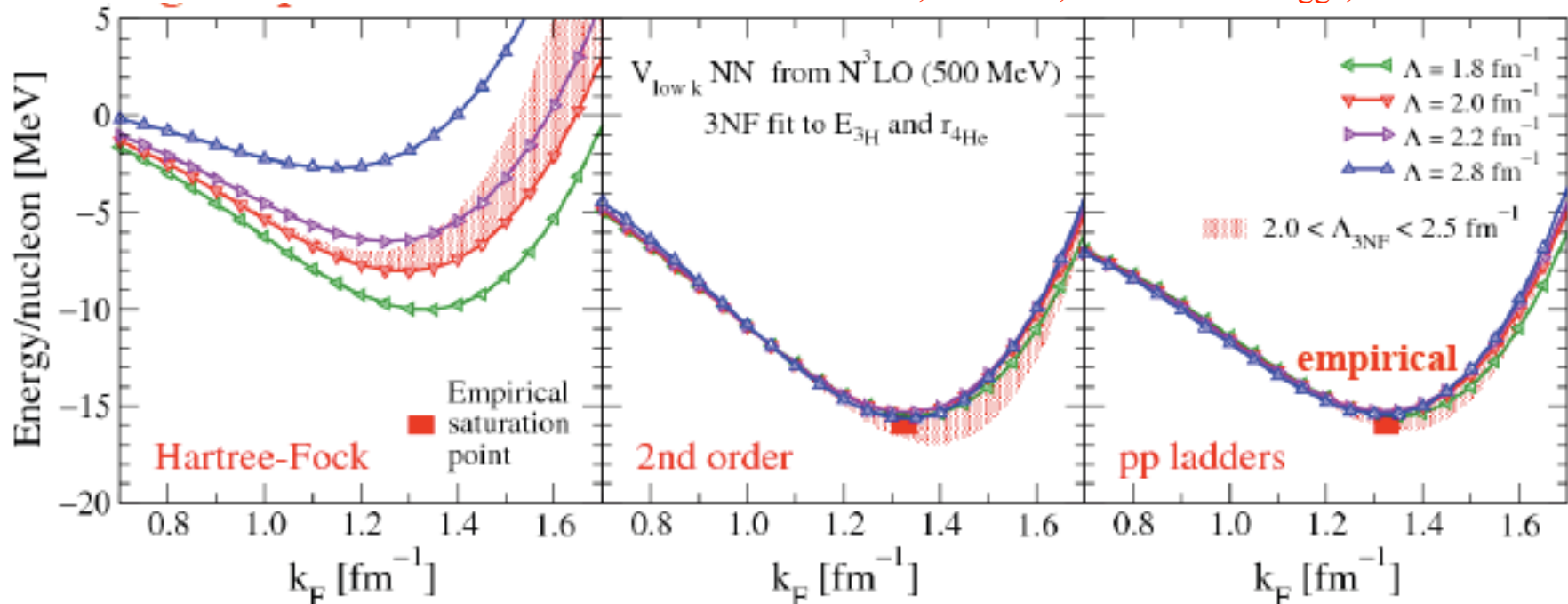
normal-ordered 0-, 1- and 2-body parts of 3N interaction dominate



residual 3N interaction can be  
neglected: very promising

# Perturbative Nuclear Matter with chiral EFT + RG?

SKB, Furnstahl, Schwenk and Nogga, 0903.3366



HF bound and saturates, converged at  $\approx$  2nd order MBPT  
 3N drives saturation, theoretical error bands

Empirical saturation lies in theoretical error bands w/out fine-tuning

Is a solution to a 50 year old problem in reach?

Promising for a microscopic nuclear Density Functional Theory (DFT)?



# The Similarity Renormalization Group

Wegner, Glazek and Wilson

Unitary transformation on an initial  $H = T + V$

$$H_\lambda = U(\lambda) H U^\dagger(\lambda) \equiv T + V_\lambda \quad \lambda = \text{continuous flow parameter}$$

Differentiating with respect to  $\lambda$ :

$$\frac{dH_\lambda}{d\lambda} = [\eta(\lambda), H_\lambda] \quad \text{with} \quad \eta(\lambda) \equiv \frac{dU(\lambda)}{d\lambda} U^\dagger(\lambda)$$

Engineer  $\eta$  to do different things as  $\lambda \Rightarrow 0$

$$\eta(\lambda) = [\mathcal{G}_\lambda, H_\lambda]$$

$\mathcal{G}_\lambda = T \Rightarrow H_\lambda$  driven towards diagonal in  $k$  – space

$\mathcal{G}_\lambda = P H_\lambda P + Q H_\lambda Q \Rightarrow H_\lambda$  driven to block – diagonal

⋮

## Normal Ordered Hamiltonians

$$H = \sum t_i a_i^\dagger a_i + \frac{1}{4} \sum V_{ijkl}^{(2)} a_i^\dagger a_j^\dagger a_l a_k + \frac{1}{36} \sum V_{ijklmn}^{(3)} a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l$$

Normal-order w.r.t. some reference state  $\Phi$  (e.g., HF) :

$$H = E_{vac} + \sum f_i N(a_i^\dagger a_i) + \frac{1}{4} \sum \Gamma_{ijkl} N(a_i^\dagger a_j^\dagger a_l a_k) + \frac{1}{36} \sum W_{ijklmn} N(a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l)$$

$$E_{vac} = \langle \Phi | H | \Phi \rangle$$

$$f_i = t_{ii} + \sum_h \langle ih | V_2 | ih \rangle n_h + \frac{1}{2} \sum_{hh'} \langle ihh' | V_3 | ihh' \rangle n_h n_{h'}$$

$$\Gamma_{ijkl} = \langle ij | V_2 | kl \rangle + \sum_h \langle ijh | V_3 | klh \rangle n_h$$

$$W_{ijklmn} = \langle ijk | V_3 | lmn \rangle \quad \langle \Phi | N(\dots) | \Phi \rangle = 0$$

0-, 1-, 2-body terms contain some 3NF effects thru density dependence => Efficient truncation scheme for evolution of 3N?

## In-medium SRG for Infinite NM and closed-shell nuclei

- Normal order  $H$  w.r.t. fermi sea
- Choose SRG generator to eliminate “energy off-diagonal” pieces

$$\frac{dH(s)}{ds} = [\eta(s), H(s)] \quad \longrightarrow \quad \lim_{s \rightarrow \infty} \Gamma_{od}(s) = 0 \quad \lambda \equiv s^{-1/4}$$

$$\eta = [\hat{f}, \hat{\Gamma}] \quad \langle 12 | \Gamma_{od} | 34 \rangle = 0 \text{ if } f_{12} = f_{34}$$

- Truncate flow equations to 2-body normal-ordered operators
  - dominant parts of induced many-body forces included implicitly

$$H(\infty) = E_{vac}(\infty) + \sum f_i(\infty) N(a_i^\dagger a_i) + \frac{1}{4} \sum [\Gamma_d(\infty)]_{ijkl} N(a_i^\dagger a_j^\dagger a_l a_k)$$

$$E_{vac}(\infty) \rightarrow E_{gs}$$

$$f_k(\infty) \rightarrow \epsilon_k \text{ (fully dressed s.p.e.)}$$

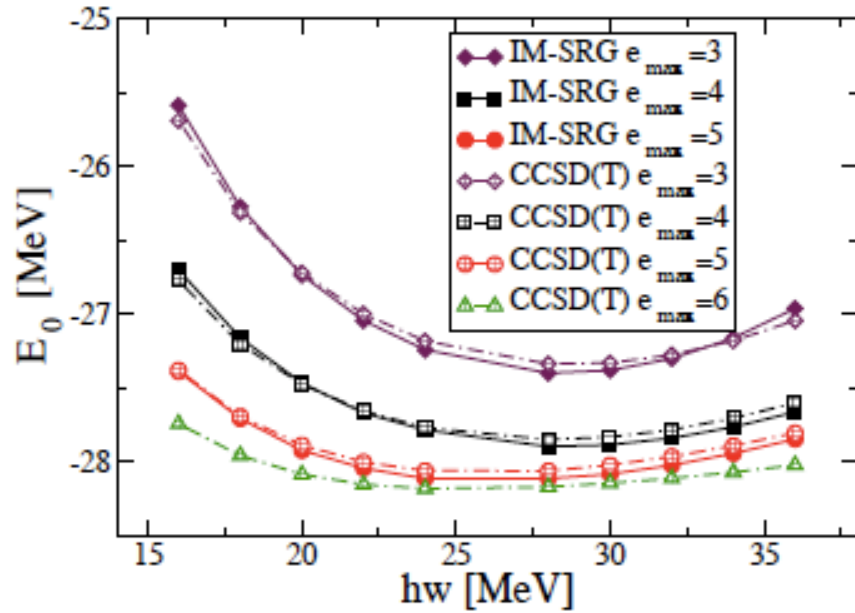
$$\Gamma_d(\infty) \rightarrow f(k', k) \text{ (Landau q.p. interaction)}$$

Microscopic realization of SM ideas: dominant MF + weak  $A$ -dependent  $NN_{\text{eff}}$

## Some observations

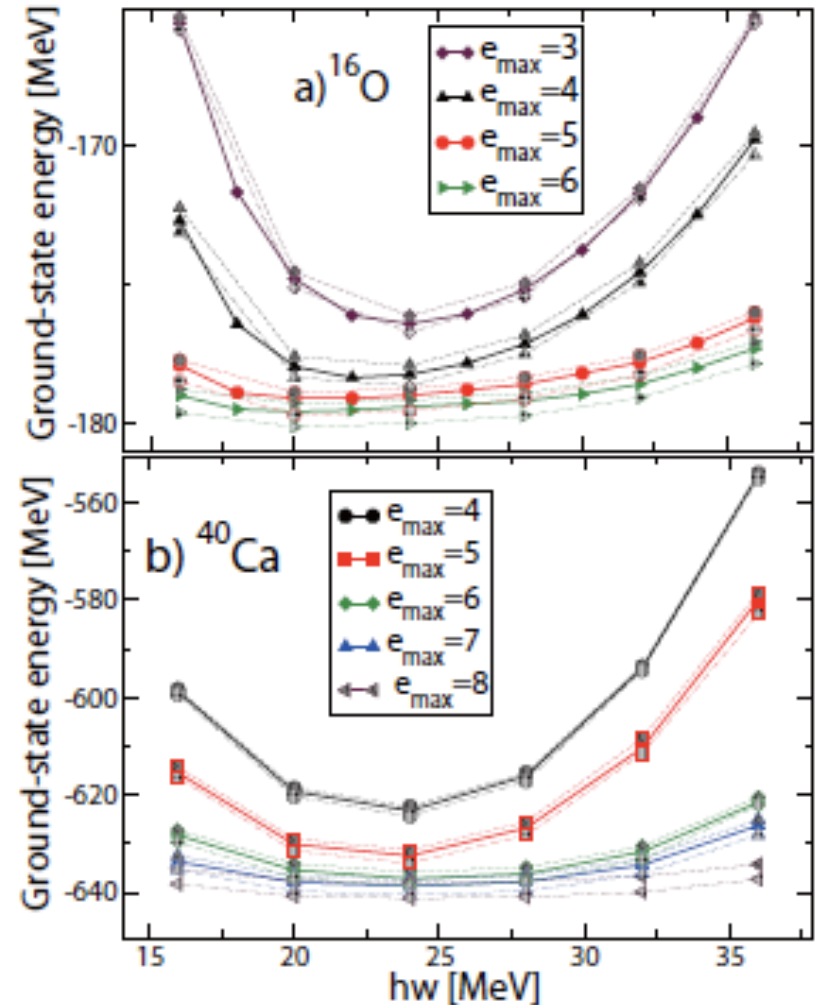
- 1)  $\frac{d}{ds}\langle H \rangle_0 \leq 0$  for monotonic  $f_k$       correlations weakened, HF picks up more binding with increasing  $s$ .
- 2) pp channel + 2 ph channels treated on equal footing
- 3) Intrinsically non-perturbative
- 4) no unlinked diagrams (size extensive, etc.)
- 5) "3rd-order exact" a-la CCSD
- 6) Extension to effective operators/Shell model possible

# In-medium SRG for nuclei Tsukiyama, SKB, Schwenk, in prep

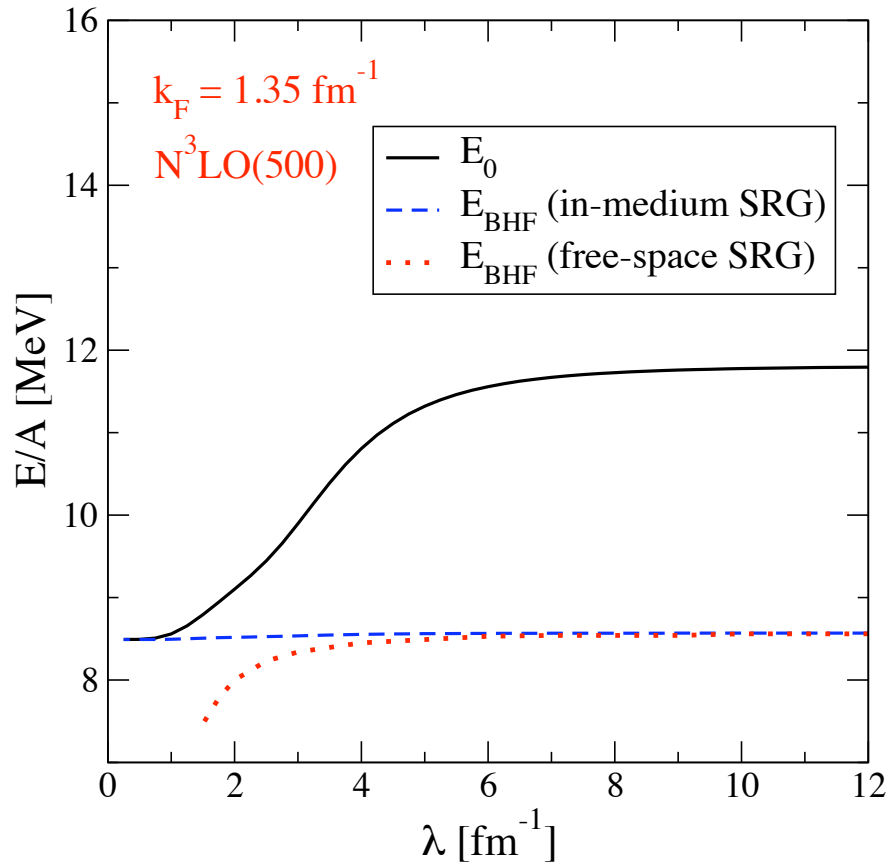


Comparable to CCSD(T) in closed shell nuclei

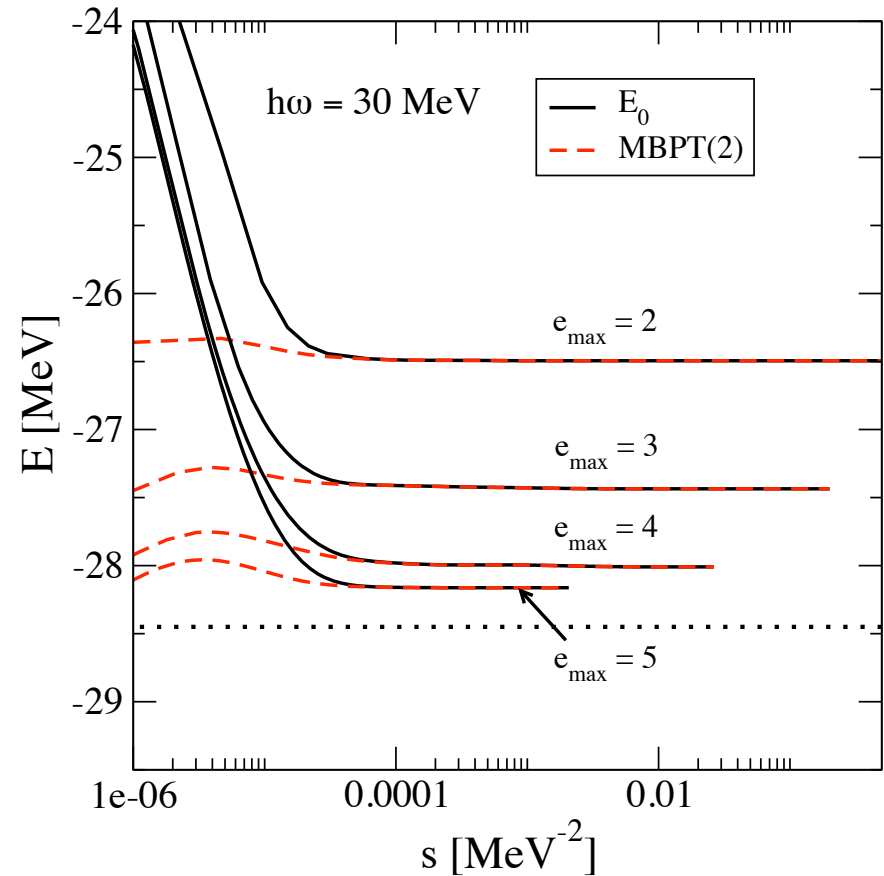
Promising method to calculate shell model valence  $H_{\text{eff}}/O_{\text{eff}}$



# Correlations “adiabatically” summed into $H(\lambda)$



INM

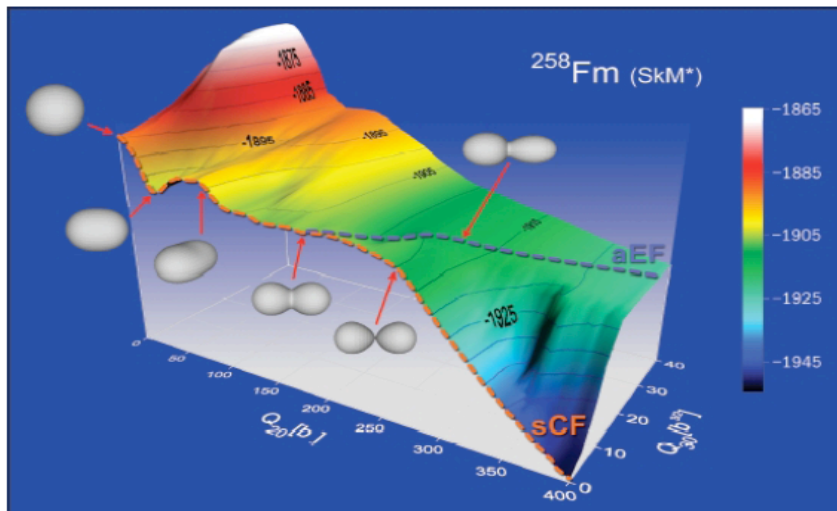
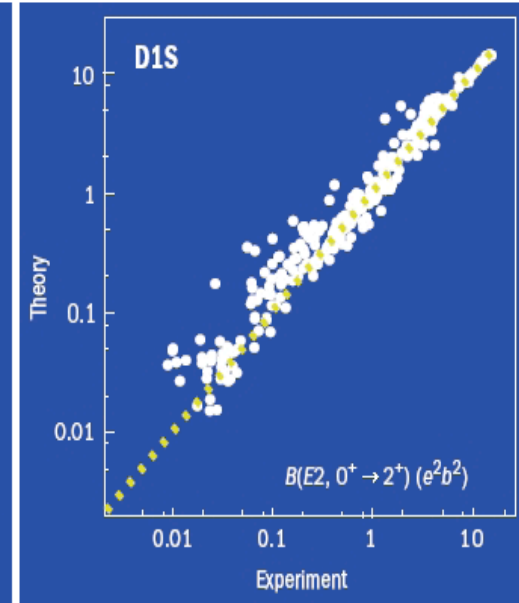
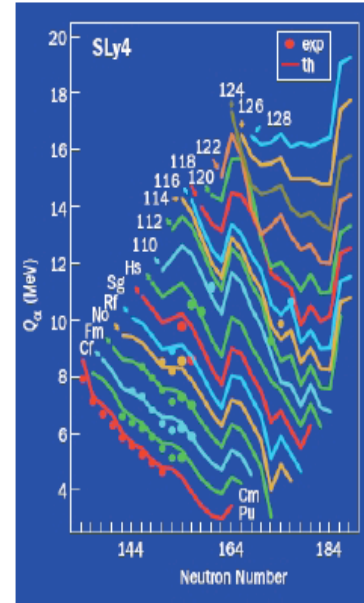
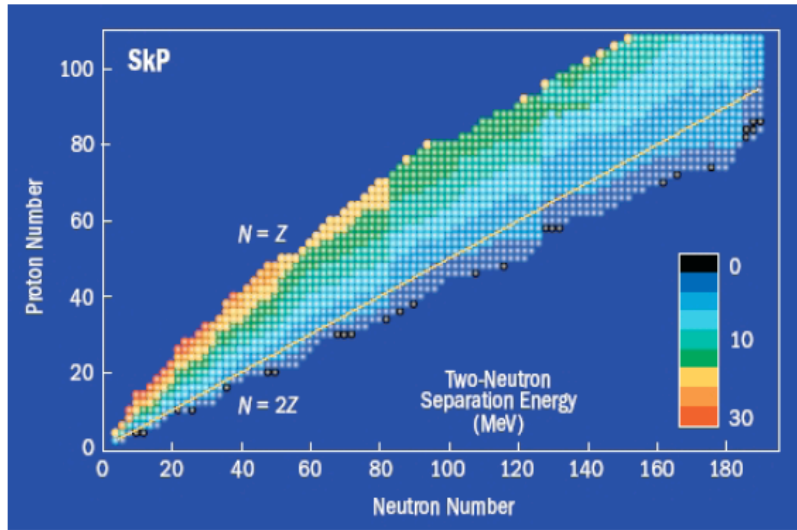


$s = 1/\lambda^4$

${}^4\text{He}$

Useful for ab initio DFT? Shell model?

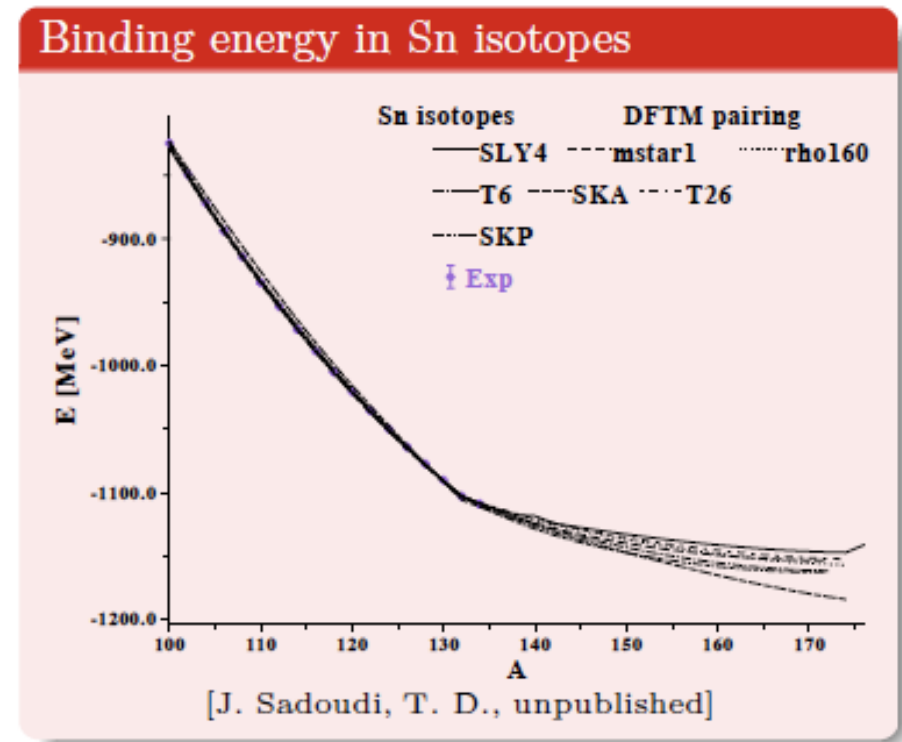
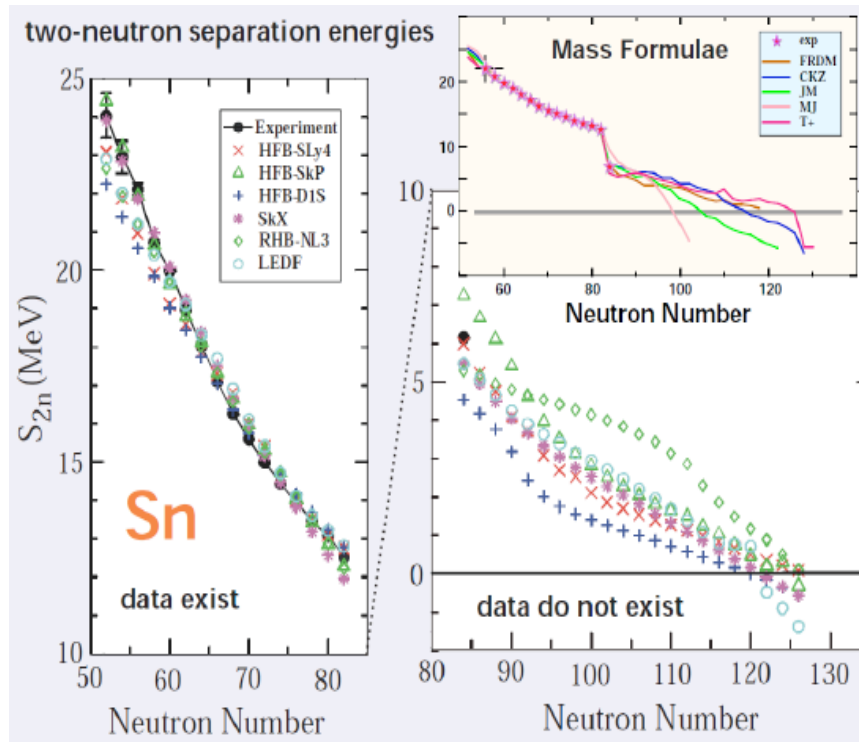
# Accomplishments of Phenomenological Energy Functionals



2N separation energies, Quadrupole and BE2 values, Fission energy surfaces, mass tables in a day, plus many other impressive feats

**BUT...**

# Limitations of Existing Energy Functionals (Predictability)



- Uncontrolled extrapolations away from known data
- Theoretical error-bars?



## What's missing in phenomenological EDFs ?

- Density dependencies too simplistic
- Isovector components not well constrained
- No way to estimate theoretical uncertainties
- What's the connection to many-body forces?

Turn to microscopic many body theory for guidance, aided by the simplifications enabled by RG-evolved interactions



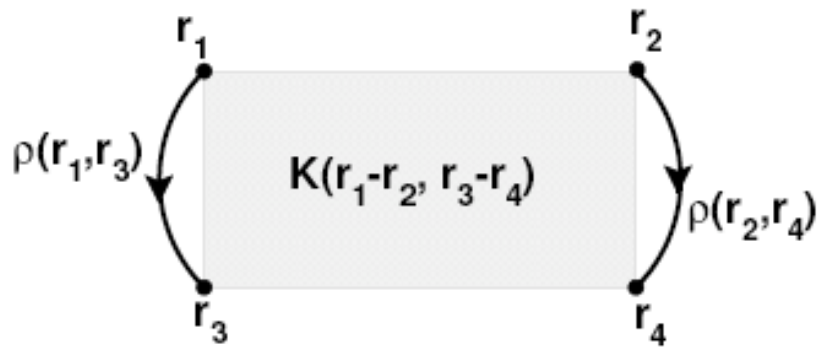
UNEDF SciDAC Collaboration  
Universal Nuclear Energy Density Functional

[www.unedf.org](http://www.unedf.org)

# Local Skyrme-like Functionals from RG-evolved Interactions

Dominant MBPT contributions to bulk properties take the form

$$\langle V \rangle \sim \text{Tr}_1 \text{Tr}_2 \int d\mathbf{R} d\mathbf{r}_{12} d\mathbf{r}_{34} \rho(\mathbf{r}_1, \mathbf{r}_3) K(\mathbf{r}_{12}, \mathbf{r}_{34}) \rho(\mathbf{r}_2, \mathbf{r}_4) + \text{NNN} \dots$$



$K$  is either free-space interaction (HF)  
or resummed in-medium vertex (BHF)

Written in terms on non-local quantities

density matrices and s.p. propagators  
finite range interaction vertex  $K$

Connection to  $E = E[\rho]$  is not obvious!

## Density Matrix Expansion Revisited (Negele and Vautherin)

Expand of DM in local operators w/factorized non-locality

$$\langle \Phi | \psi^\dagger(\mathbf{R} - \frac{1}{2}\mathbf{r}) \psi(\mathbf{R} + \frac{1}{2}\mathbf{r}) | \Phi \rangle = \sum_n \Pi_n(k_F r) \langle \mathcal{O}_n(\mathbf{R}) \rangle$$

$$\langle \mathcal{O}_n(\mathbf{R}) \rangle = [\rho(\mathbf{R}), \nabla^2 \rho(\mathbf{R}), \tau(\mathbf{R}), \mathbf{J}(\mathbf{R}), \dots]$$

Dependence on local densities/currents now manifest

$$\begin{aligned} \langle V_2 \rangle &\sim \sum_{n,m} \int d\mathbf{R} \mathcal{O}_n(\mathbf{R}) \mathcal{O}_m(\mathbf{R}) \int d\mathbf{r} \Pi_n(k_F r) \Pi_m(k_F r) V_2(r) \\ &\sim \sum_t \int d\mathbf{R} \left\{ C_t^{\rho\rho} \rho_t^2 + C_t^{\rho\tau} \rho_t \tau_t + C_t^{\rho\Delta\rho} \rho_t \Delta\rho_t + C_t^{JJ} \mathbf{J}_t^2 + C_t^{J\nabla\rho} \mathbf{J}_t \nabla\rho_t \dots \right\} \end{aligned}$$

Skyrme-like EDF with **density-dependent** couplings  
dominated by long-range pion-physics

# Prescriptions for $\Pi_n$ -functions

Phase space averaging (PSA-DME) (Gebremariam et al. arXiv:0910.4979)

$$\rho(\vec{r}_1, \vec{r}_2) = e^{i\vec{r}\cdot\vec{k}} e^{\frac{\vec{r}}{2}\cdot(\nabla_1 - \nabla_2) - i\vec{r}\cdot\vec{k}} \rho(\vec{r}_1, \vec{r}_2) \Big|_{\vec{r}_1 = \vec{r}_2 = \vec{R}}$$

Average the non-locality operator over local momentum distribution  $g(\mathbf{R}, \mathbf{k})$  and expand exponentiated gradients

$$\rho(\vec{r}_1, \vec{r}_2) \approx \int d^3\vec{k} g(\vec{R}, \vec{k}) e^{i\vec{k}\cdot\vec{r}} \sum_{n=0}^2 \frac{1}{n!} \left\{ \vec{r} \cdot \left( \frac{\nabla_1 - \nabla_2}{2} - i\vec{k} \right) \right\}^n \rho(\vec{r}_1, \vec{r}_2) \Big|_{\vec{r}_1 = \vec{r}_2 = \vec{R}}$$

Easy to build in physics associated with surface effects in finite fermi systems

Crucial to accurately describe spin-vector part of OBDM

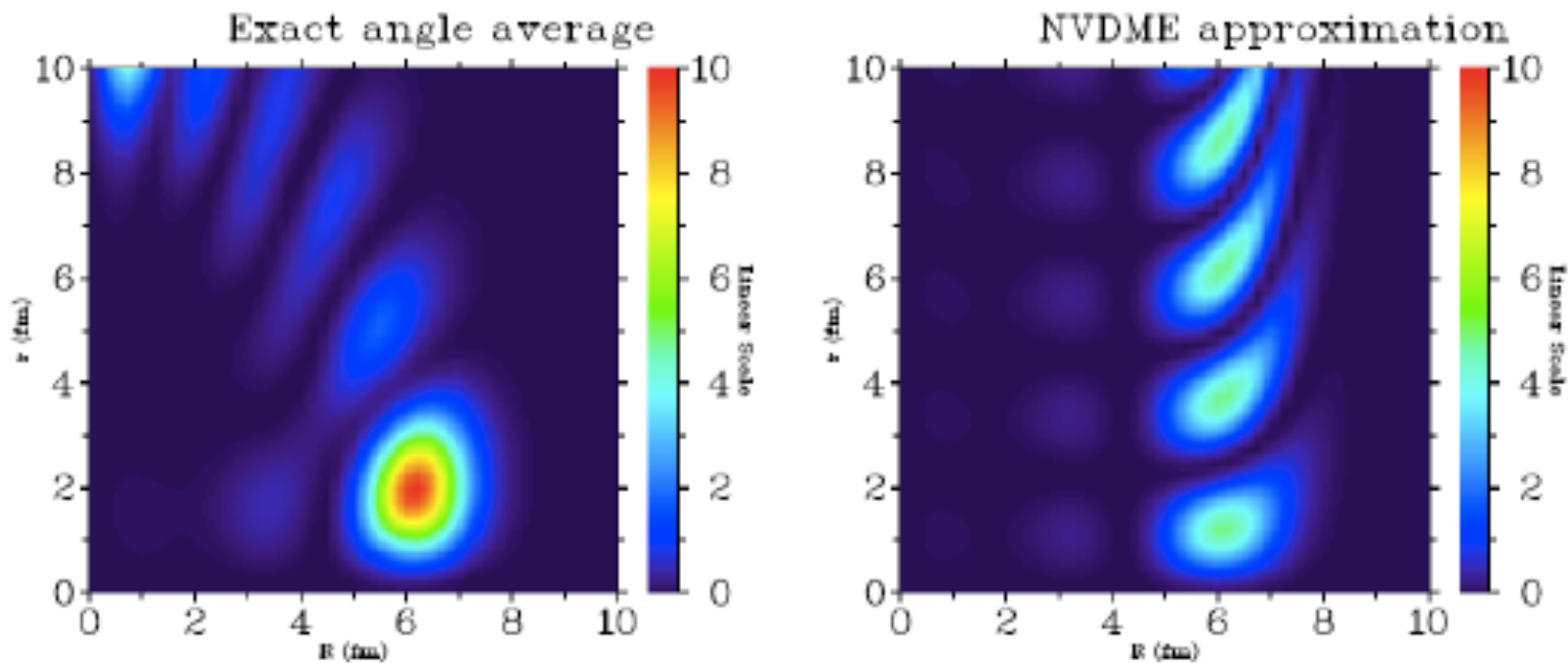
# Prescriptions for $\Pi_n$ -functions

Negele and Vautherin (NV-DME)

Truncated Bessel expansion of non-locality operator

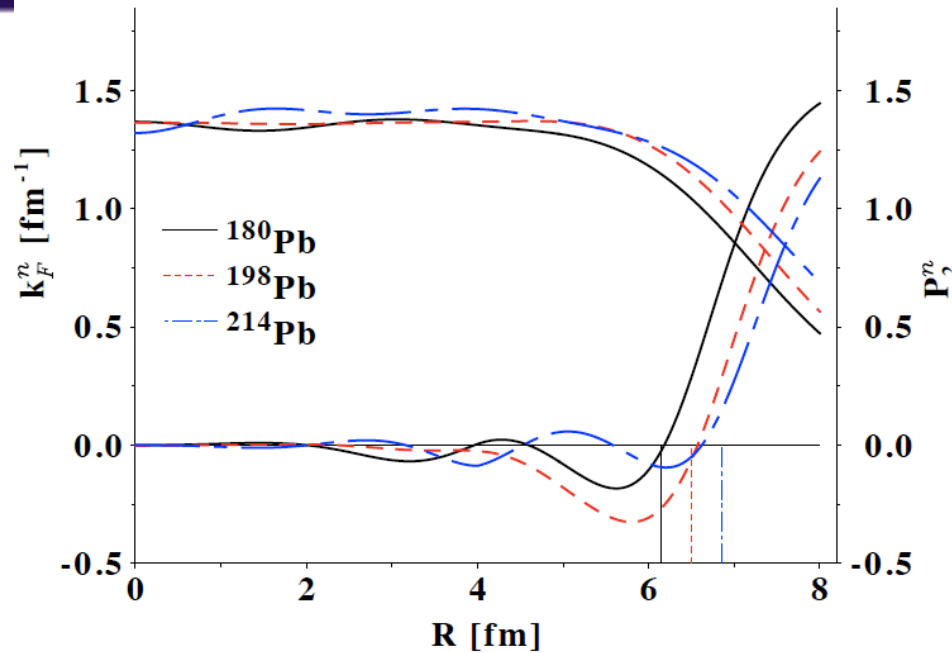
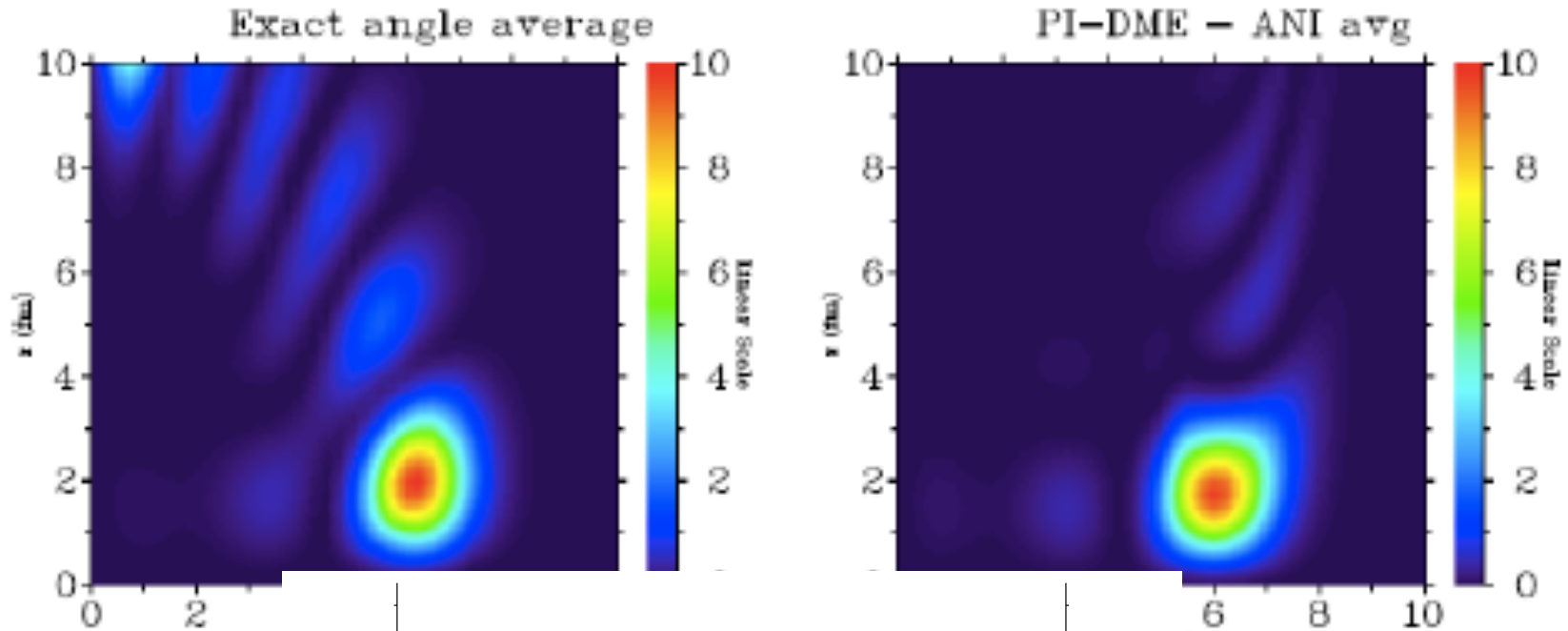
**Sufficient for spin-unsaturated nuclei only**

$$\int d\Omega_r \mathbf{s}_n(\mathbf{r}_1, \mathbf{r}_2) \cdot \mathbf{s}_n(\mathbf{r}_2, \mathbf{r}_1)$$



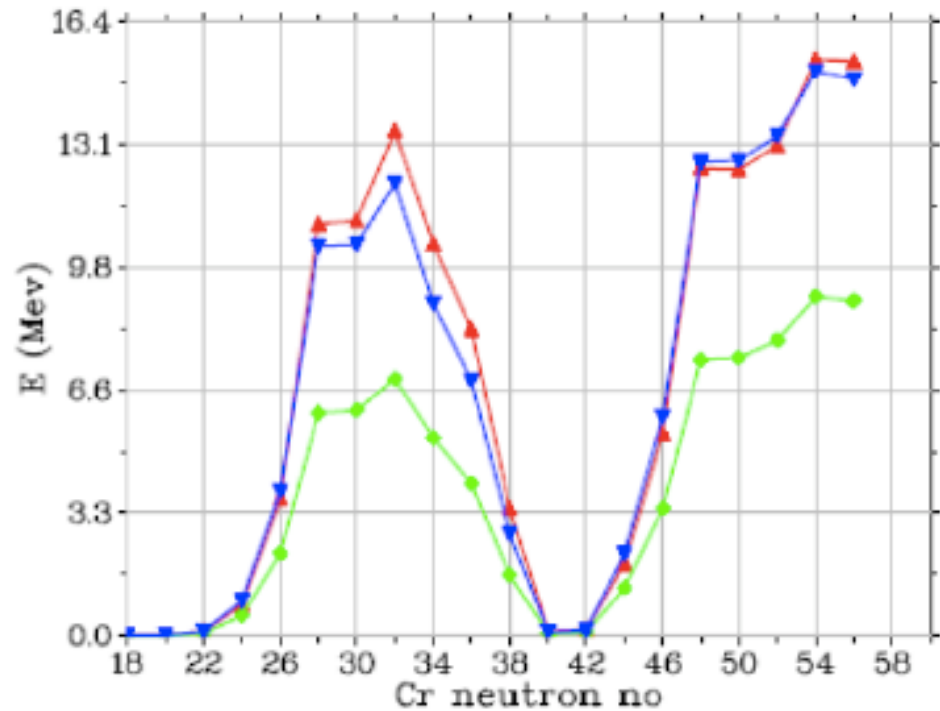
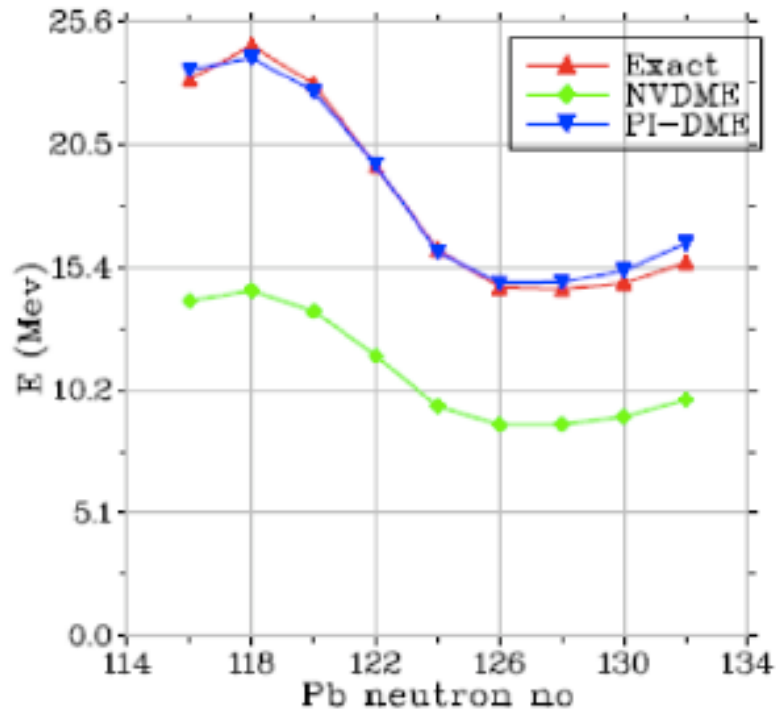
Why it fails: no phase space averaging done for spin-vector part

# Improved Vector PSA-DME



anisotropy of  $g(R,k)$  in the spatial surface (Bulgac et al.)

Look at  $\int dr dR V_{1\pi}(r) \mathbf{s}_n(\mathbf{r}_1, \mathbf{r}_2) \cdot \mathbf{s}_n(\mathbf{r}_2, \mathbf{r}_1)$ :



- Inclusion of finite fermi phase space effects crucial for **quantitative** agreement
- completely parameter-free

Can now apply modified DME with confidence to spin-unsaturated systems

# Including Long Range Chiral EFT in Skyrme-like EDFs

$$V_{EFT} = V_{ct}(\Lambda) + V_{1\pi} + V_{2\pi} + \dots$$

Each EDF coupling function splits into 2 terms

- 1)  $\Lambda$ -**dependent** Skyrme-like coupling **constants (short-distance)**
- 2)  $\Lambda$ -**independent** coupling **functions** from “universal” pion physics

$$C_t^{\rho\tau} \Rightarrow C_t^{\rho\tau}(\Lambda; V_{ct}) + C_t^{\rho\tau}[k_F(\mathbf{R}); V_\pi] \quad \text{Etc...}$$

From contact terms in  
EFT/RG V's

From pion exchanges

Suggests a microscopically-improved Skyrme phenomenology

Add pion-exchange couplings to existing Skyrmes and refit constants using guidance from EFT (naturalness, etc.)



# Gameplan - Include pion physics in Skyrme EDFs and refit

- Include DME coupling functions from finite-range NN and NNN chiral EFT thru N2LO
- Refit the Skyrme coupling constants (EFT constraints => naturalness)
- Look for improved observables and for sensitivities
- Can we “see” the pion as in NN phase shift analyses

Expect interesting spin-orbit consequences (NN vs NNN)

	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$			
N <sup>2</sup> LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			
N <sup>3</sup> LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			
	+ ...	+ ...	+ ...

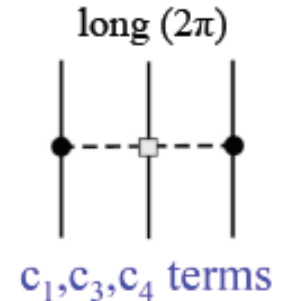
in progress w/ORNL group (Stoitsov et al.)

## New development: DME for chiral NNN force (N2LO)

- Expect interesting spin-orbit/tensor couplings from TPE

$$V_c(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) \sim \frac{\sigma_1 \cdot \mathbf{q}_1 \sigma_2 \cdot \mathbf{q}_2}{(q_1^2 + m_\pi^2)(q_2^2 + m_\pi^2)} F_{123}^{\alpha\beta} \tau_1^\alpha \tau_2^\beta + perms$$

$$F_{123}^{\alpha\beta} \equiv \delta_{\alpha\beta} \left[ -4 \frac{c_1 m_\pi^2}{f_\pi^2} + 2 \frac{c_3}{f_\pi^2} \mathbf{q}_1 \cdot \mathbf{q}_2 \right] + \frac{c_4}{f_\pi^2} \epsilon^{\alpha\beta\gamma} \tau_3^\gamma \sigma_3 \cdot (\mathbf{q}_1 \times \mathbf{q}_2)$$



Empirical EDFs (Skyrme, Gogny,...) spin-orbit coupling is density independent => appropriate for NN spin-orbit forces (short range)

This is a **mismatch** since microscopic NNN interactions are long-range (DME => strong density dependent  $\mathbf{J} \cdot \nabla \rho$  couplings)

Complexity explodes => Automated symbolic tools developed (Gebremariam et al) will be available at [www.unedf.org](http://www.unedf.org)

$$\begin{aligned}
\mathcal{E}^{CRA,2x} = \int d\vec{r} \left\{ & \mathcal{C}_7^{\rho_0^3} \rho_0^3(\vec{r}) + \mathcal{C}_7^{\rho_0 \rho_1^2} \rho_0(\vec{r}) \rho_1^2(\vec{r}) + \mathcal{C}_7^{\rho_0 \rho_1 \varsigma_1^1} \rho_0(\vec{r}) \rho_1(\vec{r}) \varsigma_1^1(\vec{r}) \right. \\
& + \mathcal{C}_7^{\rho_0^2 \Delta \rho_0} \rho_0^2(\vec{r}) \Delta \rho_0(\vec{r}) + \mathcal{C}_7^{\rho_0 \rho_1 \Delta \rho_1} \rho_0(\vec{r}) \rho_1(\vec{r}) \Delta \rho_1(\vec{r}) + \mathcal{C}_7^{\rho_0^2 \varsigma_0^2} \rho_0^2(\vec{r}) \varsigma_0^2(\vec{r}) \\
& + \mathcal{C}_7^{\rho_1^2 \varsigma_0^2} \rho_1^2(\vec{r}) \varsigma_0^2(\vec{r}) + \mathcal{C}_7^{\rho_0 \rho_1 \varsigma_1^2} \rho_0(\vec{r}) \rho_1(\vec{r}) \varsigma_1^2(\vec{r}) + \mathcal{C}_7^{\rho_0^2 \varsigma_0^1} \rho_0^2(\vec{r}) \varsigma_0^1(\vec{r}) \\
& + \mathcal{C}_7^{\rho_0 J_0^2} \rho_0(\vec{r}) \vec{J}_0(\vec{r}) \cdot \vec{J}_0(\vec{r}) + \mathcal{C}_7^{\rho_1 J_0 J_1} \rho_1(\vec{r}) \vec{J}_0(\vec{r}) \cdot \vec{J}_1(\vec{r}) + \mathcal{C}_7^{\rho_0 J_1^2} \rho_0(\vec{r}) \vec{J}_1(\vec{r}) \cdot \vec{J}_1(\vec{r}) \\
& + \mathcal{C}_7^{J_0^2 \nabla J_0} \vec{J}_0(\vec{r}) \cdot \vec{J}_0(\vec{r}) \vec{\nabla} \cdot \vec{J}_0(\vec{r}) + \mathcal{C}_7^{J_1^2 \nabla J_0} \vec{J}_1(\vec{r}) \cdot \vec{J}_1(\vec{r}) \vec{\nabla} \cdot \vec{J}_0(\vec{r}) + \mathcal{C}_7^{J_0 J_1 \nabla J_1} \vec{J}_0(\vec{r}) \vec{J}_1(\vec{r}) \vec{\nabla} \cdot \vec{J}_1(\vec{r}) \\
& + \mathcal{C}_7^{\Delta \rho_0 J_0^2} \Delta \rho_0(\vec{r}) \vec{J}_0(\vec{r}) \cdot \vec{J}_0(\vec{r}) + \mathcal{C}_7^{\varsigma_0^2 J_0^2} \varsigma_0^2(\vec{r}) \vec{J}_0(\vec{r}) \cdot \vec{J}_0(\vec{r}) + \mathcal{C}_7^{\varsigma_0^1 J_0^2} \varsigma_0^1(\vec{r}) \vec{J}_0(\vec{r}) \cdot \vec{J}_0(\vec{r}) \\
& + \mathcal{C}_7^{\nabla \rho_0 J_0 \nabla J_0} \vec{\nabla} \rho_0(\vec{r}) \vec{J}_0(\vec{r}) \vec{\nabla} \cdot \vec{J}_0(\vec{r}) + \mathcal{C}_7^{\rho_0 \nabla J_0 \nabla J_0} \rho_0(\vec{r}) [\vec{\nabla} \cdot \vec{J}_0(\vec{r})]^2 + \mathcal{C}_7^{\rho_0 J_0 \Delta J_0} \rho_0(\vec{r}) \vec{J}_0(\vec{r}) \cdot \Delta \vec{J}_0(\vec{r}) \\
& + \mathcal{C}_7^{\varsigma_1^1 J_0 J_1} \varsigma_1^1(\vec{r}) \vec{J}_0(\vec{r}) \cdot \vec{J}_1(\vec{r}) + \mathcal{C}_7^{\Delta \rho_1 J_0 J_1} \Delta \rho_1(\vec{r}) \vec{J}_0(\vec{r}) \cdot \vec{J}_1(\vec{r}) + \mathcal{C}_7^{\varsigma_1^2 J_0 J_1} \varsigma_1^2(\vec{r}) \vec{J}_0(\vec{r}) \cdot \vec{J}_1(\vec{r}) \\
& + \mathcal{C}_7^{\nabla \rho_1 J_1 \nabla J_0} \vec{\nabla} \rho_1(\vec{r}) \cdot \vec{J}_1(\vec{r}) \vec{\nabla} \cdot \vec{J}_0(\vec{r}) + \mathcal{C}_7^{\rho_1 J_1 \Delta J_0} \rho_1(\vec{r}) \vec{J}_1(\vec{r}) \cdot \Delta \vec{J}_0(\vec{r}) + \mathcal{C}_7^{\varsigma_0^2 J_1^2} \varsigma_0^2(\vec{r}) \vec{J}_1(\vec{r}) \cdot \vec{J}_1(\vec{r}) \\
& + \mathcal{C}_7^{\nabla \rho_1 J_0 \nabla J_1} \vec{\nabla} \rho_1(\vec{r}) \cdot \vec{J}_0(\vec{r}) \vec{\nabla} \cdot \vec{J}_1(\vec{r}) + \mathcal{C}_7^{\rho_1 \nabla J_0 \nabla J_1} \rho_1(\vec{r}) \vec{\nabla} \cdot \vec{J}_0(\vec{r}) \vec{\nabla} \cdot \vec{J}_1(\vec{r}) + \mathcal{C}_7^{\rho_0 \nabla J_1 \nabla J_1} \rho_0(\vec{r}) [\vec{\nabla} \cdot \vec{J}_1(\vec{r})]^2 \\
& \left. + \mathcal{C}_7^{\rho_1 J_0 \Delta J_1} \rho_1(\vec{r}) \vec{J}_0(\vec{r}) \cdot \Delta \vec{J}_1(\vec{r}) + \mathcal{C}_7^{\rho_0 J_1 \Delta J_1} \rho_0(\vec{r}) \vec{J}_1(\vec{r}) \cdot \Delta \vec{J}_1(\vec{r}) \right\}. \tag{91}
\end{aligned}$$

+ 4 other classes of similar terms

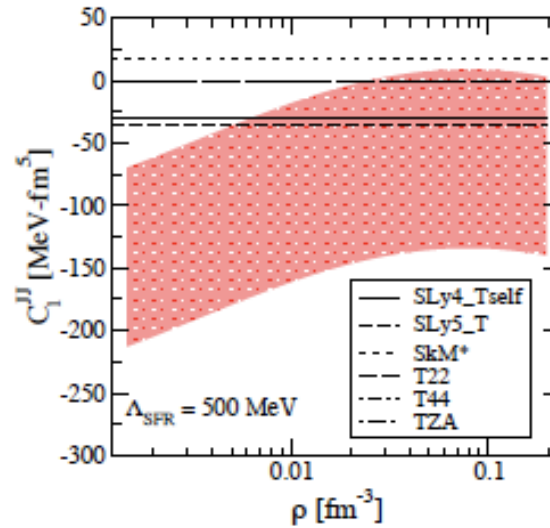
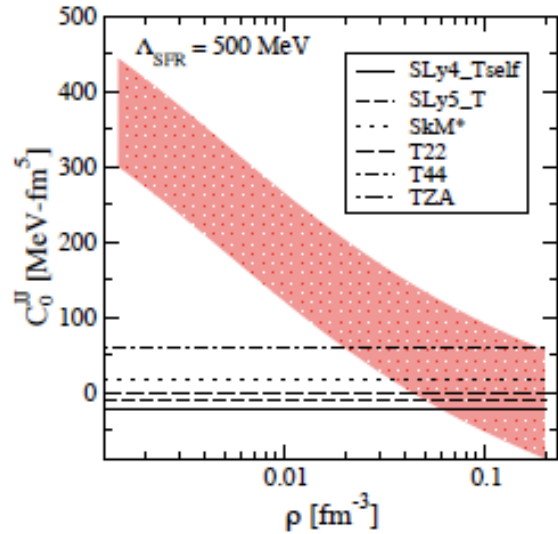
Looks ugly (or beautiful, depending on your view), but a regular structure emerges:

$$C^{ijk}[u] \xi_i \xi_j \xi_k, \quad u \equiv \frac{k_F(R)}{m_\pi} \quad (\text{note: } u \text{ is NOT small})$$

$$C^{ijk}[u] = C_1^{ijk}[u] + C_2^{ijk}[u] \ln(1 + 4u^2) + C_3^{ijk}[u] \arctan(2u),$$

$$C_\alpha^{ijk}[u] = \text{rational polynomial}$$

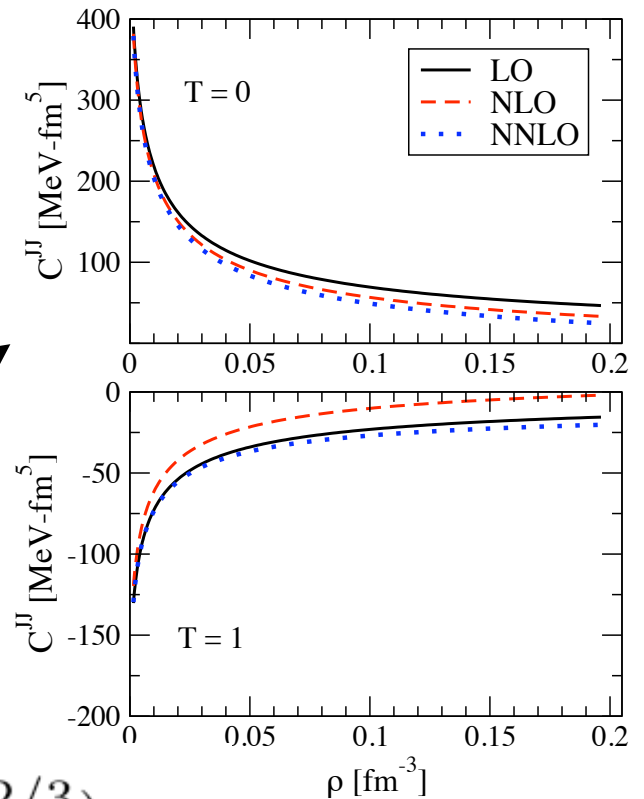
# Some examples (Gebremariam, SKB, Duguet 2010)



red “error bands” from EFT naturalness

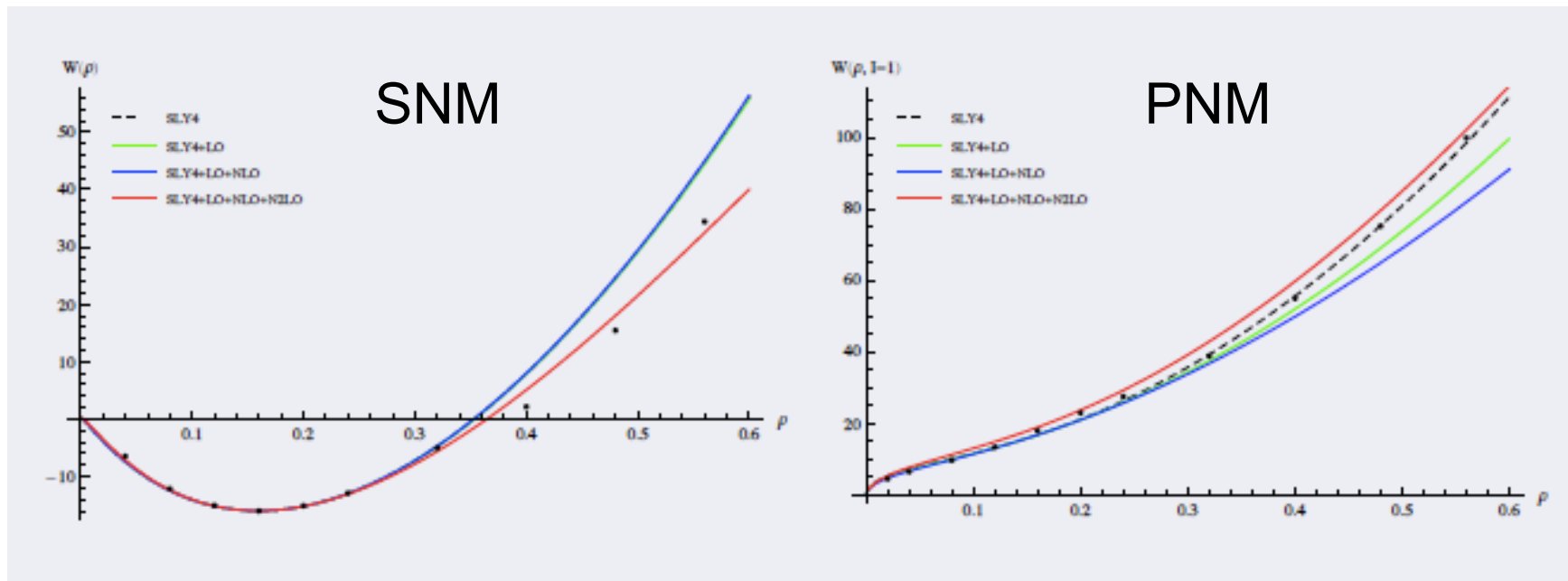
Comparison of EDF w/pions to Skyrme couplings w/tensor force

density dependence controlled by longest range component



$$\rho^{7/3}, \rho^{4/3}, \rho^{2/3}, \frac{1}{\rho^{2/3}} \log(1 + c\rho^{2/3}), \dots$$

Moral: Simple many-body theory + current understanding of underlying NN + NNN interactions tells us Skyrme is way too simple.



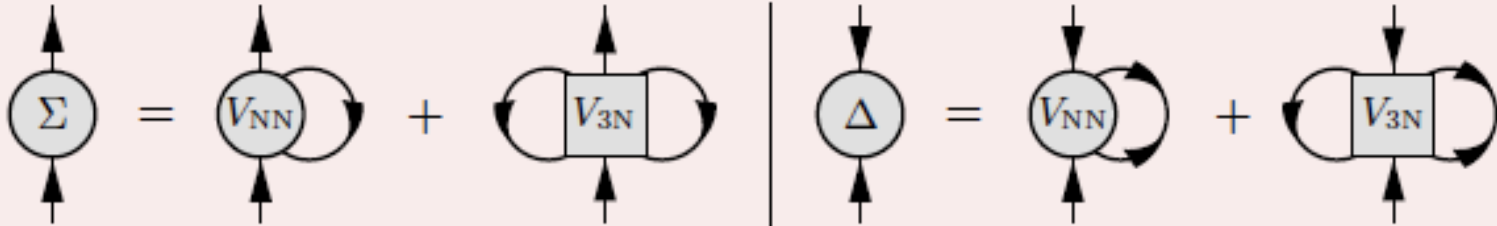
First exploratory calculations in progress w/M.Stoitsov et al. using the extended EDF (implemented in HFBRAD and HFBTHO)

Mathematica nb's with 2N/3N DME couplings available at [www.unedf.org](http://www.unedf.org)

# Other efforts developing non-empirical EDFs

## Non-empirical pairing functional (Duguet, Lesinski, Hebeler, Schwenk)

Build first  $\Sigma^q$  and  $\Delta^q$  at lowest-order in  $V_{NN}$  and  $V_{NNN}$  (RG-evolved)



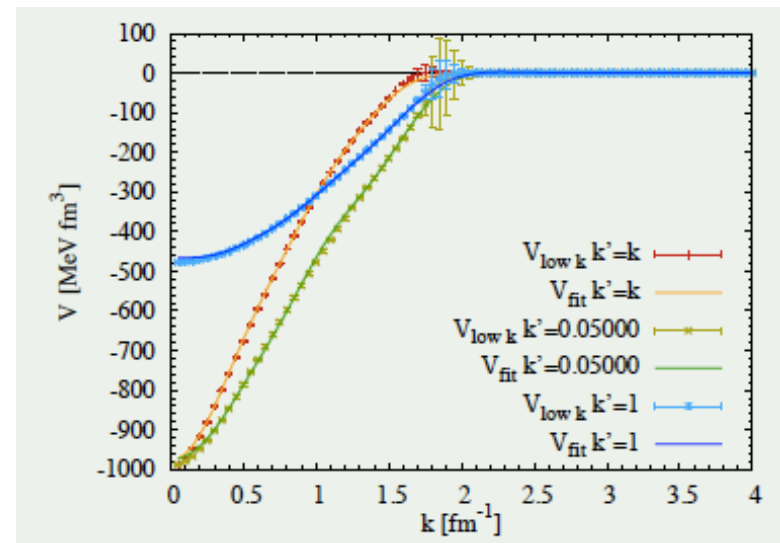
■  $v^{pp}$ : microscopically built from  $V_{NN}$  and  $V_{NNN}$

■  $v^{ph}$ : semi-empirical from constrained Skyrme EDF ( $m^* \approx 0.7 m$ )

$$V_{qq}^1 S_0(k, k') = \sum_{\alpha, \beta=1}^n g_{\alpha}(k) \lambda_{\alpha\beta} g_{\beta}(k')$$

Low-rank separable expansion good at low  $\Lambda$

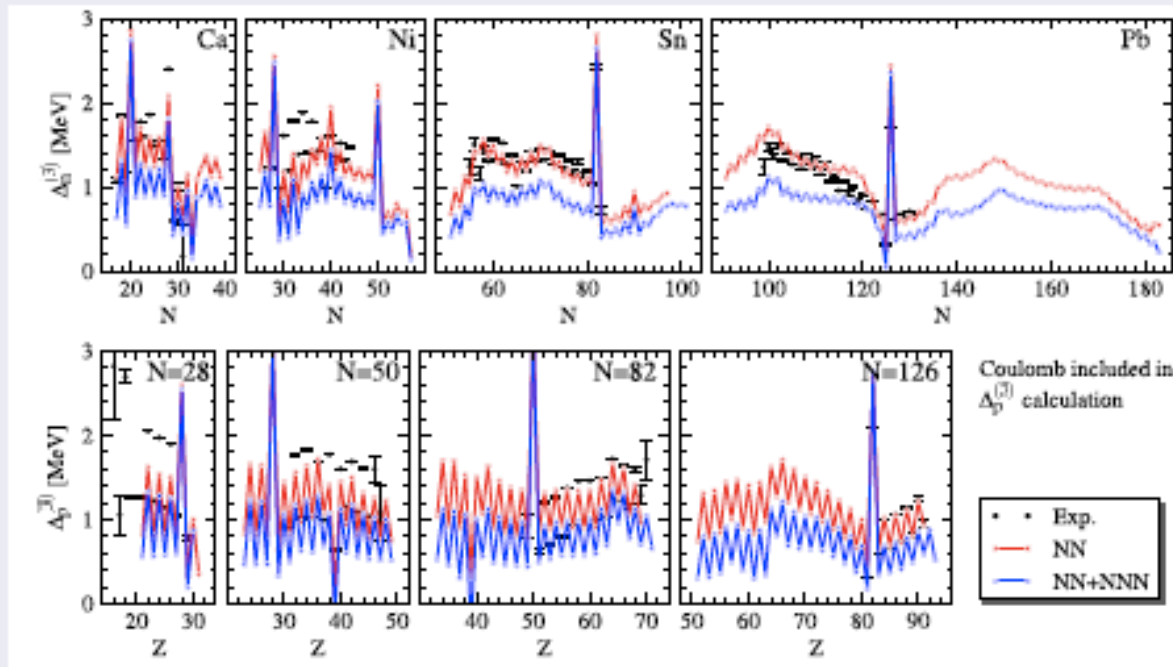
Almost as cheap as local pairing EDF calculations



With  $V_{\text{low } k} + V_{\text{coulomb}} + \text{approximate NNN}$

[T. Lesinski, T. D., K. Bennaceur, J. Meyer, in preparation]

$\Delta_{q, \text{exp}}^{(3)}$  versus  $\Delta_{q, \text{th}}^{(3)}$



- $\Delta_q^{(3)}$  decreased by 20% with slight isovector trend ( $|V_{qq\langle\bar{q}\rangle}^1 S_0| > |V_{qq\langle q\rangle}^1 S_0|$ )
- Leave  $\sim 20 - 30\%$  for coupling to (collective) fluctuations

Next: Beyond 1st order (Gorkov 2nd-order, In-medium SRG), local approx's

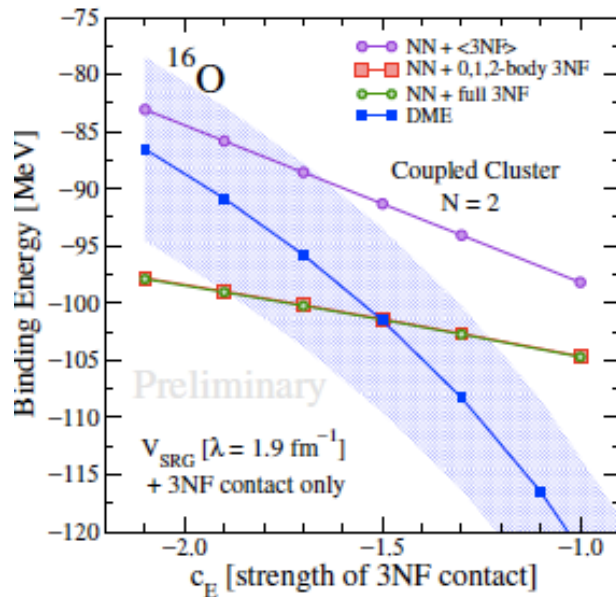
# Other efforts developing non-empirical EDFs

## DME functional vs. ab initio (SKB, Furnstahl, Platter)

Start from the **same** H and compare with no adjustments

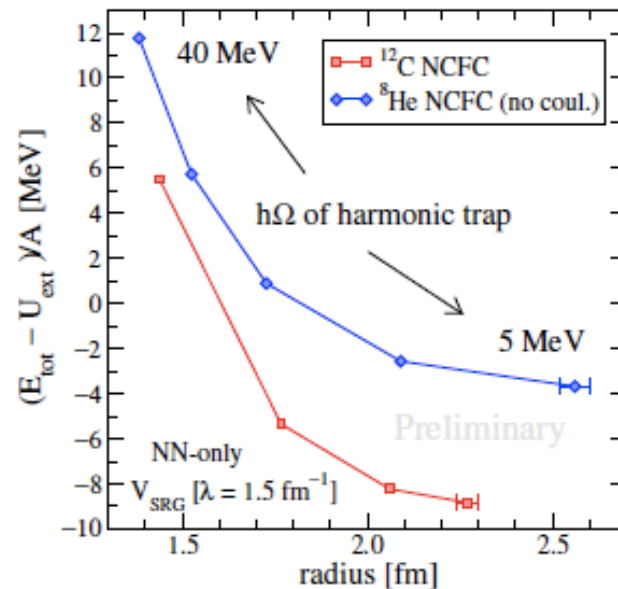
### “Coester Lines”

- Compare systematics, e.g., by varying 3NF coupling in Hamiltonian



### External Potentials

- DFT from response of energy to perturbation of densities  
⇒ Apply external fields

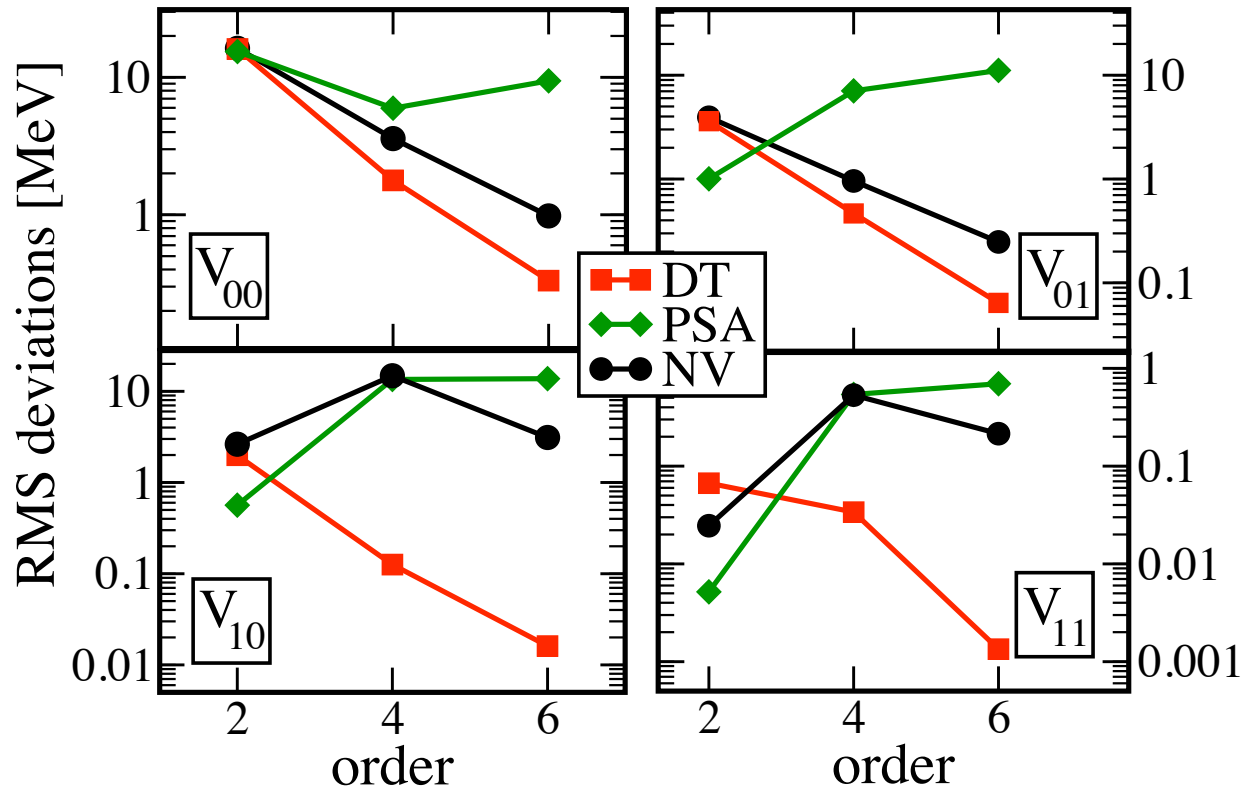


**This was pre- PSADME improvements and implementation of exact Hartree. Worth revisiting!**



# Other efforts developing non-empirical EDFs

DME beyond  $\nabla^2$  ( Carlsson, Dobaczewski, arXiv:1003.2543)



New “Damped Taylor” DME gives dramatic improvements with higher order gradients

Solves the problem of exploding # of parameters with higher  $\nabla$

# Summary

- RG methods simplify nuclear many-body calculations
  - faster convergence, more perturbative, low k “universality”
  - empirical NM saturation within theoretical errors
- In-medium SRG
  - Normal-ordering => simple way to evolve many-body operators
  - analogous to CC; diagonalize many-body problems
  - non-perturbative path for shell model and possible ab-initio DFT
- Microscopic connections to DFT now possible
  - explicit inclusion of long-distance chiral EFT physics via the density matrix expansion (microscopic guidance for density dependence, isovector and spin-orbit properties, etc.)