

Novel High Performance Computational Aspects of the Shell Model Approach for Medium Nuclei

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Overview

- New shell model ideas for the low-lying states
- Novel developments with shell model nuclear level densities
- Potential impact of the shell model nuclear level densities for nuclear astrophysics predictions

Nuclear Configuration Interaction

$0\hbar\omega$

$$H' = H + \beta(H_{CoM} - 3/2\hbar\omega)$$

Center-of-mass spurious states

$1\hbar\omega$

$$\Psi^{(J)} = [\Phi_{CoM}(NL)\Phi_{int}^{(J')}]^{(J)} \rightarrow \Phi_{CoM}(00)\Phi_{int}^{(J)}$$

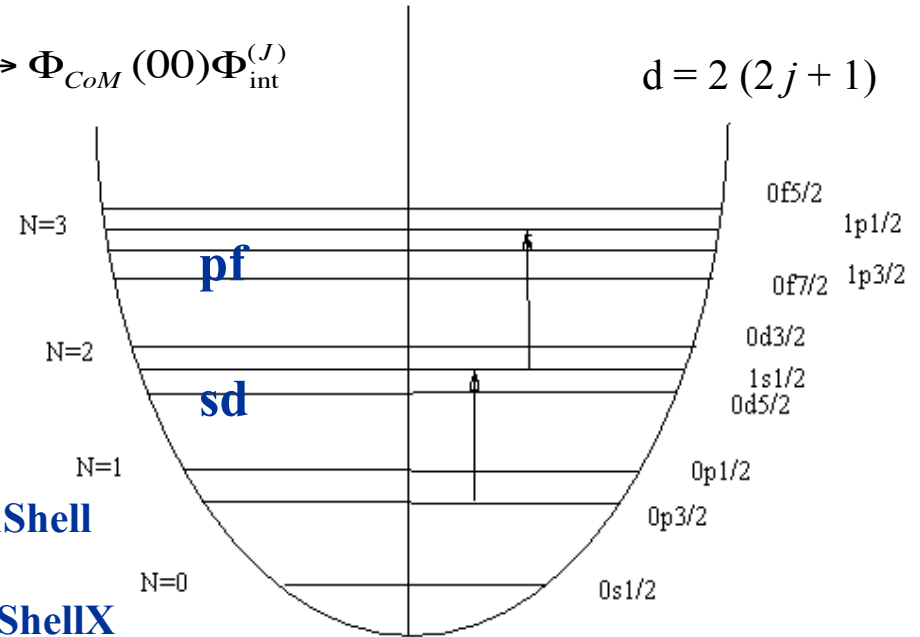
$$d = 2(2j + 1)$$

$(0 + 2)\hbar\omega$

$(1 + 3)\hbar\omega$

N_{max}

$$H = \sum_k \epsilon_K a_k^+ a_k + \frac{1}{2} \sum_{klmn} V_{kl;mn} a_k^+ a_l^+ a_n a_m + \dots$$



$$|\alpha\rangle = \sum_i C_i^\alpha |i(JT)\rangle \quad \text{JT-scheme: OXBASH, NuShell}$$

$$|\alpha\rangle = \sum_i C_i^\alpha |i(JT_z)\rangle \quad \text{J-scheme: NATHAN, NuShellX}$$

$$|\alpha\rangle = \sum_i C_i^\alpha |i(MT_z)\rangle \quad \text{M-scheme: Oslo-code, Antoine, MFDN, MSHELL, CMichSM, ...}$$

$$\sum_j \langle i | H | j \rangle C_j^\alpha = E_\alpha C_i^\alpha$$

Lanczos algorithm: provides few lower energies, especially the M-scheme codes.

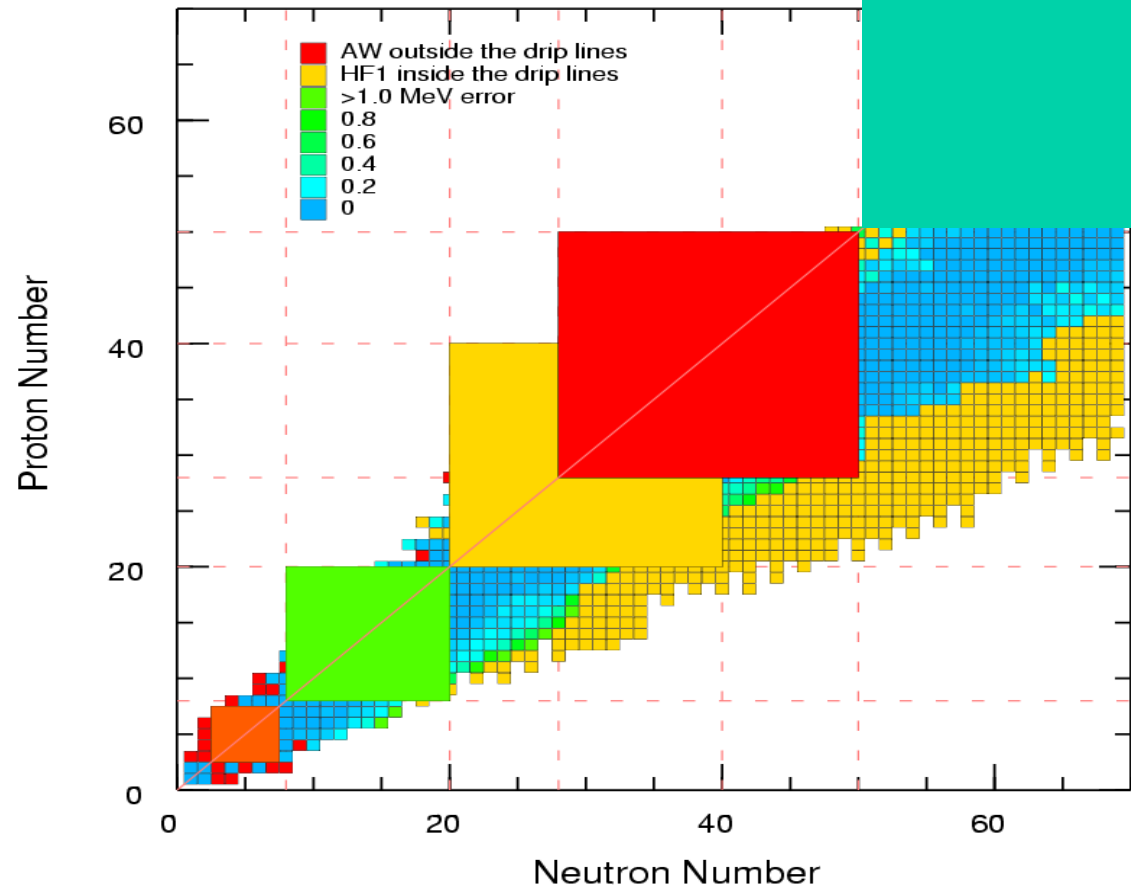
p - 10^2 - 1960's
 sd - 10^5 - 1980's
 pf - 10^9 - 1990's
 pf_{5/2}g_{9/2} - 10^{10} - 2006
 g_{7/2}sdh_{11/2} - 5×10^{10} - '08

Example: ⁷⁶Sr

pf_{5/2}g_{9/2} dimension

11 090 052 440

Nuclear drip lines

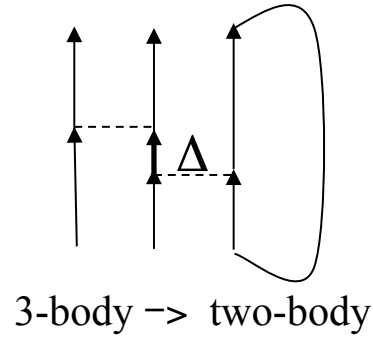
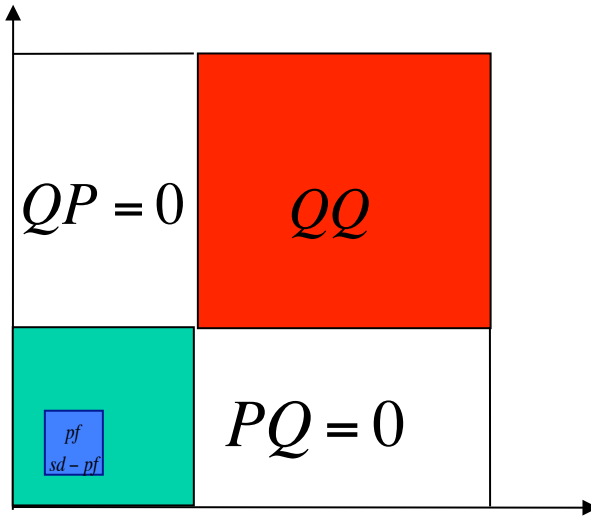


Current limit: 10^{10} - 10^{11} → 46-48 np valence s.p. states

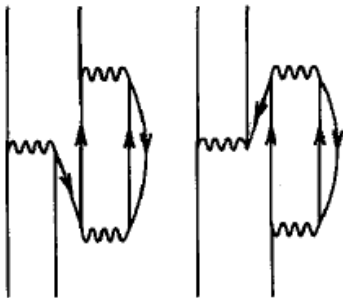
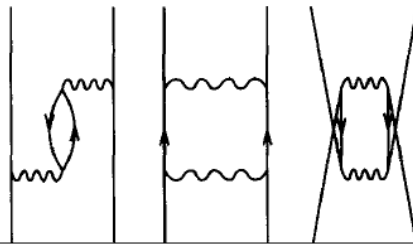
Extensions: Truncations, Exponential Convergence Method,

Coupled Clusters, Projected CI, ...

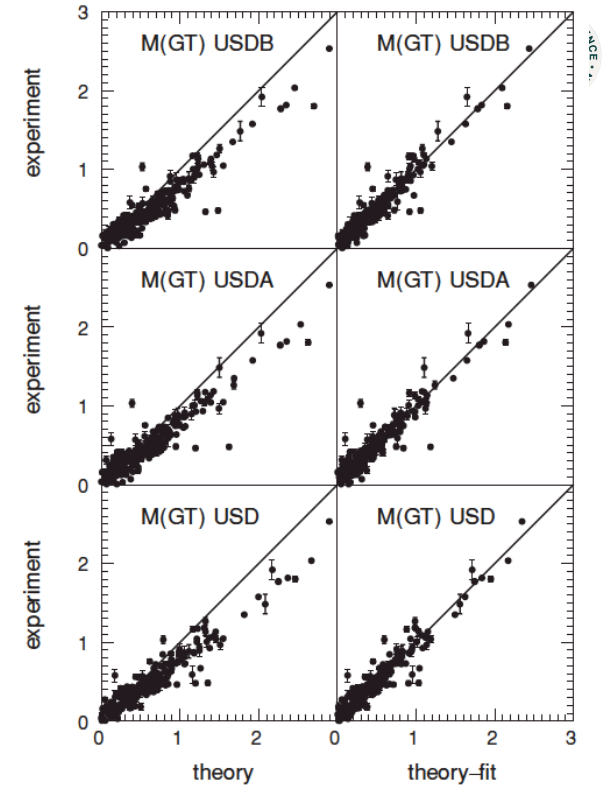
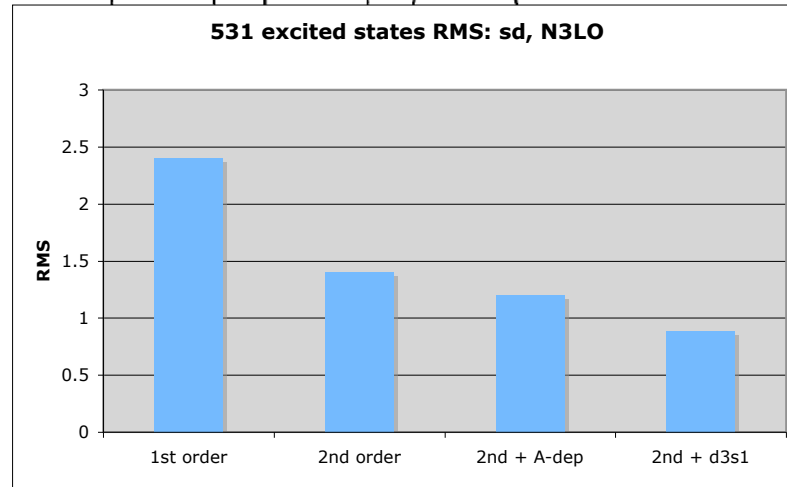
Effective Hamiltonians



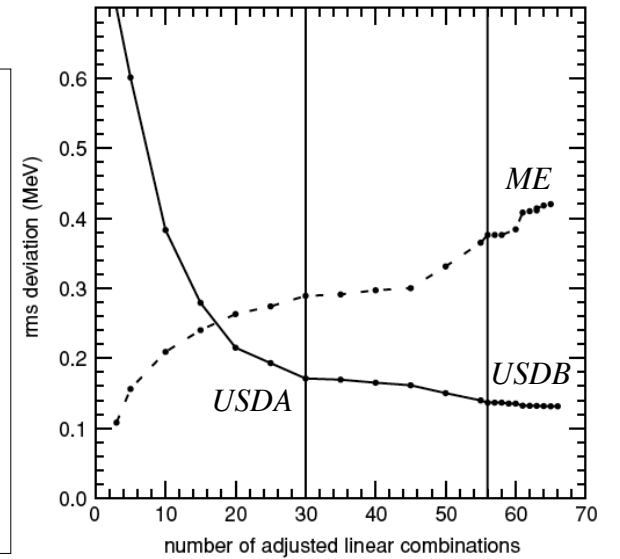
core polarization:
Phys.Rep. **261**,
125 (1995)

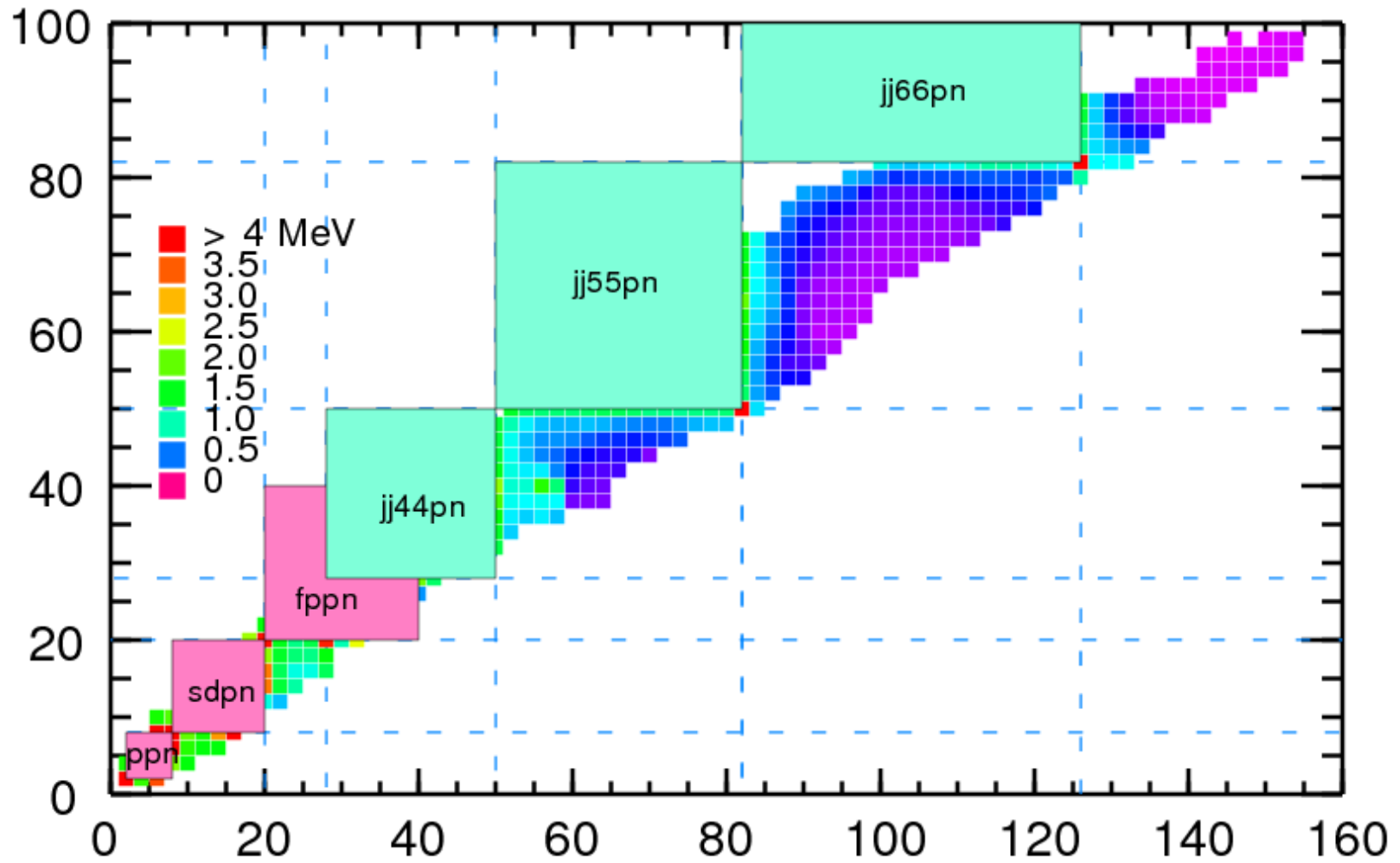


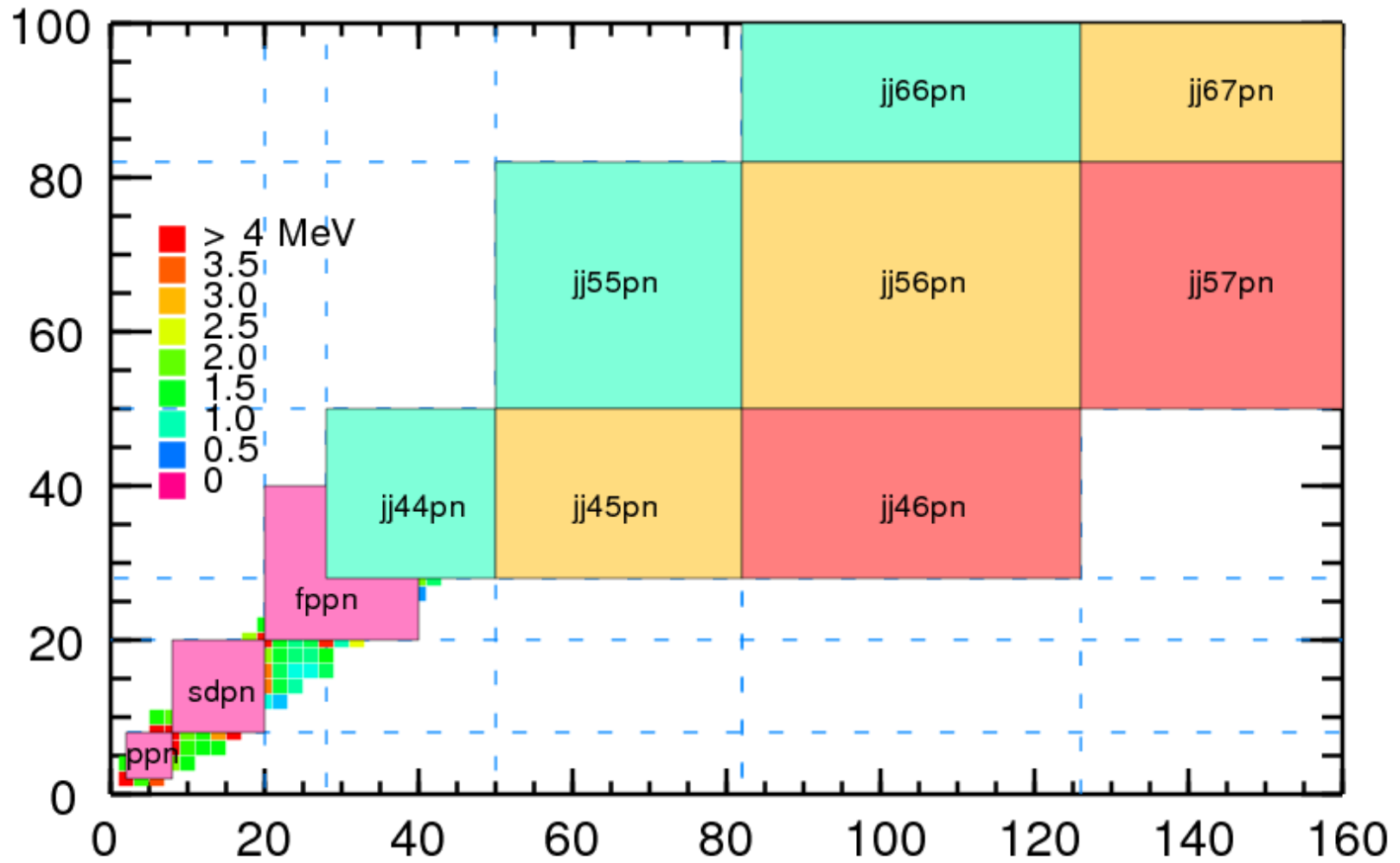
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PRC 74, 34315 (2006), 78, 064302 (2008)







GXPF1A Effective Interaction: $f_{7/2}p_{3/2}p_{1/2}f_{5/2}$

Renormalized G-matrix \rightarrow GXPF1

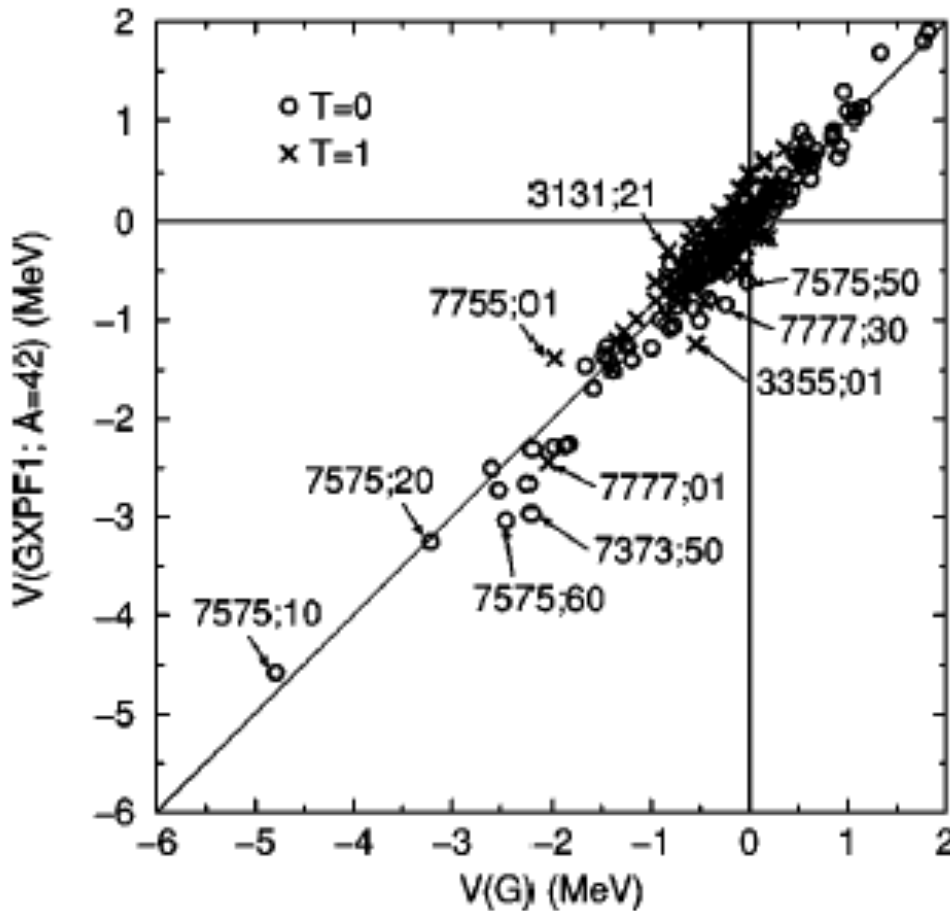
Phys.Rev. C **69**, 34355 (2004)

699 energies, 87 nuclei, **rms**=168 keV

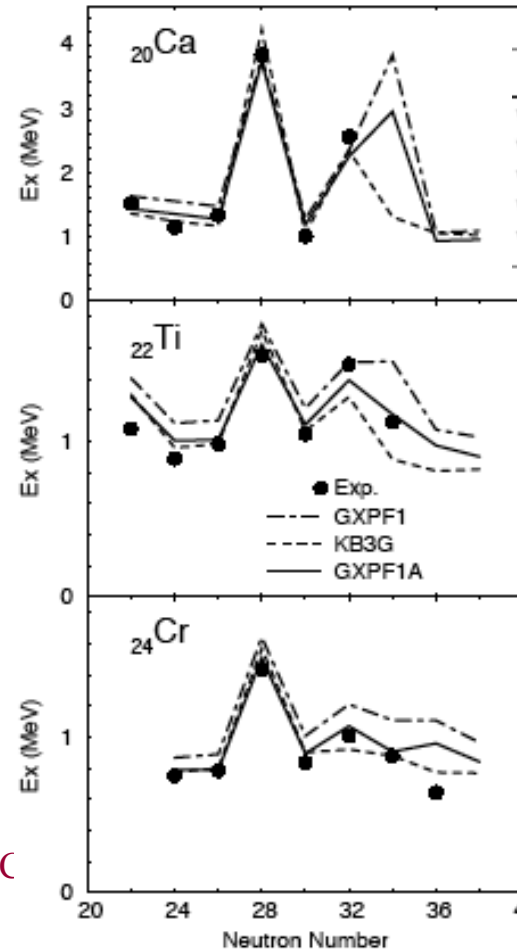
GXPF1 \rightarrow GXPF1A

M. Honma et al, ENAM04

5 matrix elements adjusted for N=34



1/11/2011 2:30, 2011



V	GXPF1	GXPF1A
V(7777; 01)	-2.439	-2.239
V(5511; 01)	-0.809	-0.309
V(1111; 01)	-0.447	+0.053
V(5151; 21)	-0.152	-0.502
V(5151; 31)	+0.238	+0.488

⁵⁶Ni states

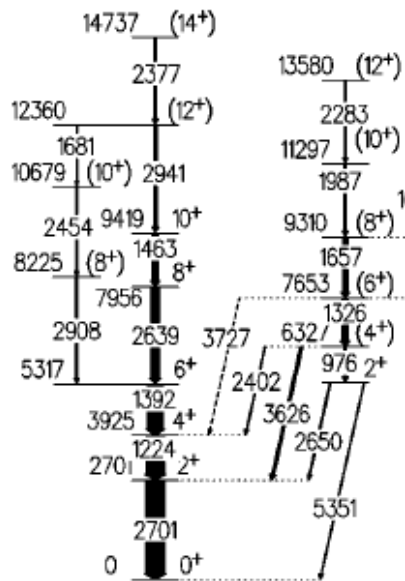
J	#
0	1
2	1
4	1
0	2
0	3
6	1
2	2
4	2
4	3
8	1
10	1

Koeln Low Spin States in ^{56}Ni : complete spectroscopy

$^{56}\text{Ni}_{28}$

(sph)

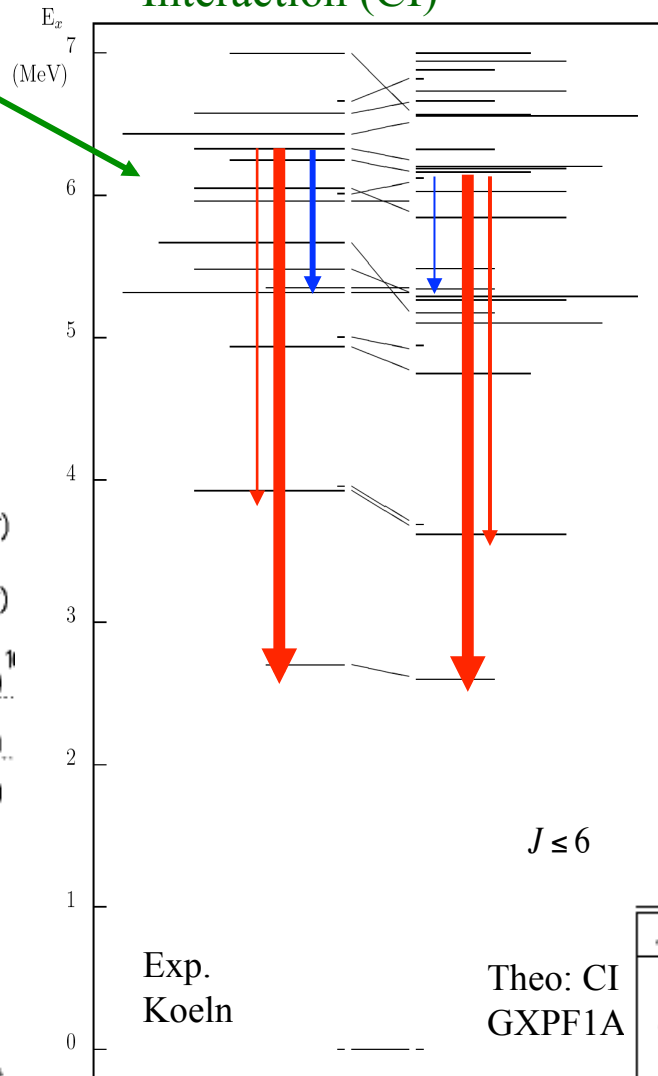
①



LBNL high spin experiment
D.Rudolf et al., PRL 88, 1999

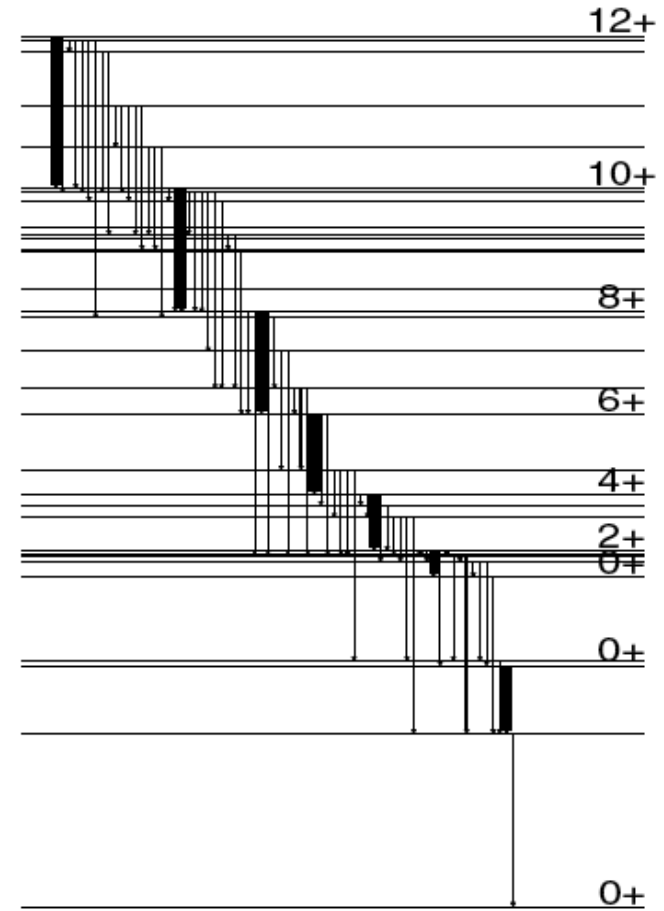
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Configuration Interaction (CI)



M. Horoi et al., in preparation
M. Horoi CM

M. Horoi et al., Phys.Rev. C 73, 061305(R) (2006)



J_n	E_x (MeV)	$\langle Q \rangle_{J0}$	$(Q_0)_{sp}$	$B(E2; J \rightarrow J-2)$	$ Q_0 _{tr}$
2 ₃	5.342	-41.6	+145.3	413.2	144.1
4 ₅	6.027	-55.2	+151.9	598.0	143.8
6 ₃	7.556	-56.2	+140.6	609.3	139.5
8 ₄	9.300	-47.2	+112.2	558.4	130.5
10 ₆	10.782	-63.9	+147.0	591.1	132.5
12 ₅	13.071	-62.7	+141.1	612.3	133.7

$e_{eff}^p = 1.5 \quad e_{eff}^n = 0.5$

Angular Momentum Projected CI (PCI)

Deformed Slater Determinant

Deformed Nilsson Single Particle operator

Spherical H.O.

$$|\Phi_{\kappa}\rangle = b_{i_1}^{\dagger} b_{i_2}^{\dagger} \dots b_{i_n}^{\dagger} |(\epsilon_2, \epsilon_4)\rangle$$

$$b_k = \sum_i W_{ki} c_i$$

Nuclear State Wave-function:
$$|\Psi_{IM}^{\sigma}\rangle = \sum_{K\kappa} f_{IK\kappa}^{\sigma} P_{MK}^I |\Phi_{\kappa}\rangle$$

$$\sum_{K'\kappa'} (H_{K\kappa, K'\kappa'}^I - E_I^{\sigma} N_{K\kappa, K'\kappa'}^I) f_{IK\kappa'}^{\sigma} = 0$$

Angular Momentum Projection

Shell Model Hamiltonian

$$H_{K\kappa, K'\kappa'}^I = \langle \Phi_{\kappa} | H P_{KK'}^I | \Phi_{\kappa'} \rangle$$

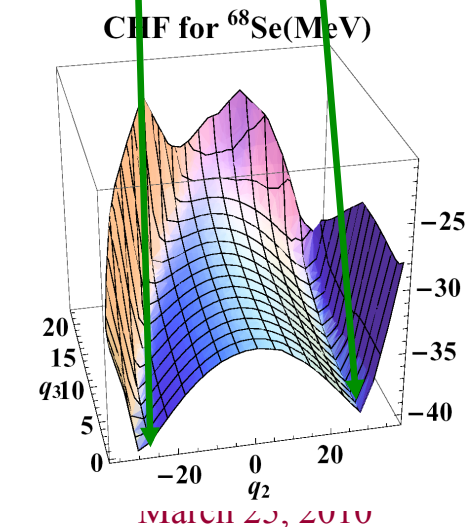
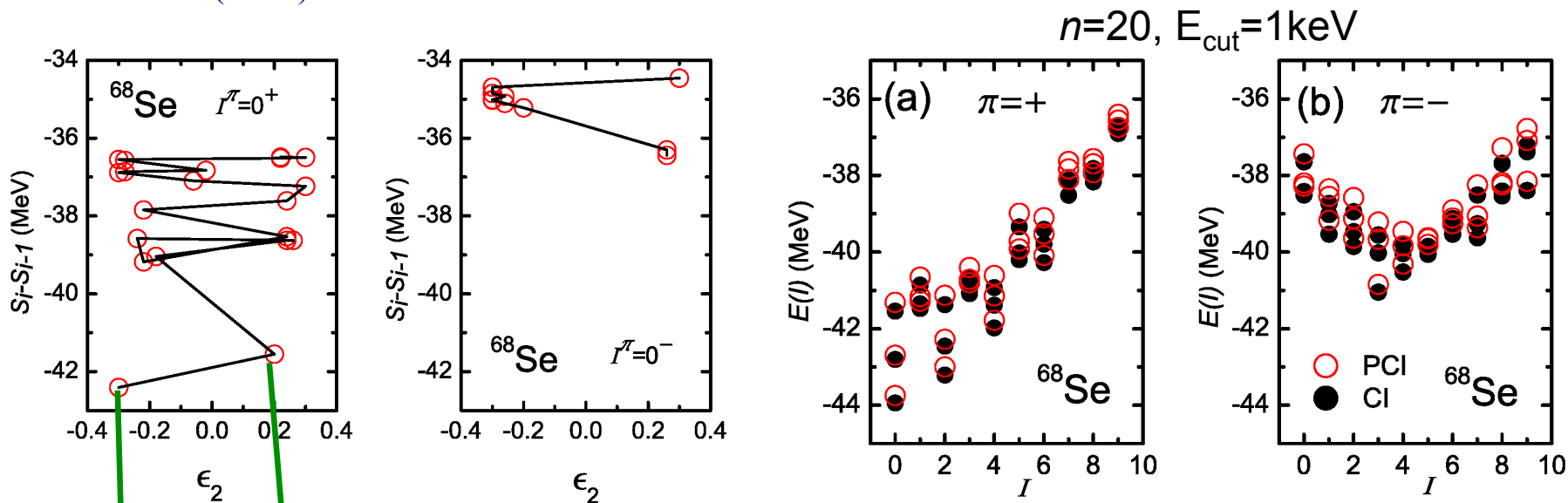
$$N_{K\kappa, K'\kappa'}^I = \langle \Phi_{\kappa} | P_{KK'}^I | \Phi_{\kappa'} \rangle$$

. Horoi CM

Extension of PCI: Gao, Horoi
 PRC **79**, 014311 (2009); PRC **80**,
 034325 (2009).

Interaction taken from:

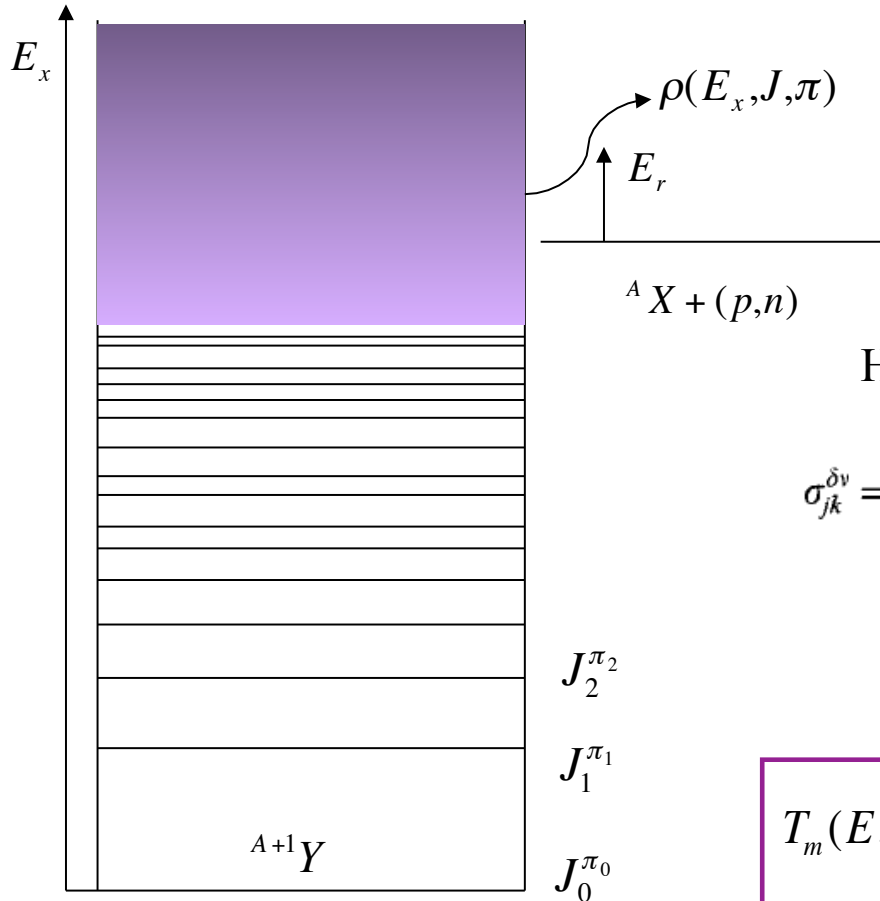
K. Kaneko, M. Hasegawa, and T. Mizusaki,
 Phys. Rev. C **70**, 051301(R) (2004).



Conclusions:

- 1, The physics of shape coexistence in ^{68}Se can be clearly seen from the PCI basis.
- 2, With new method of basis selection the PCI energies are very close to those of full CI for both positive parity and negative parity states.

Nuclear Level Densities (NLD)



$(n, \gamma), (n, xn), (n, n'), (n, p), (n, f), \dots$

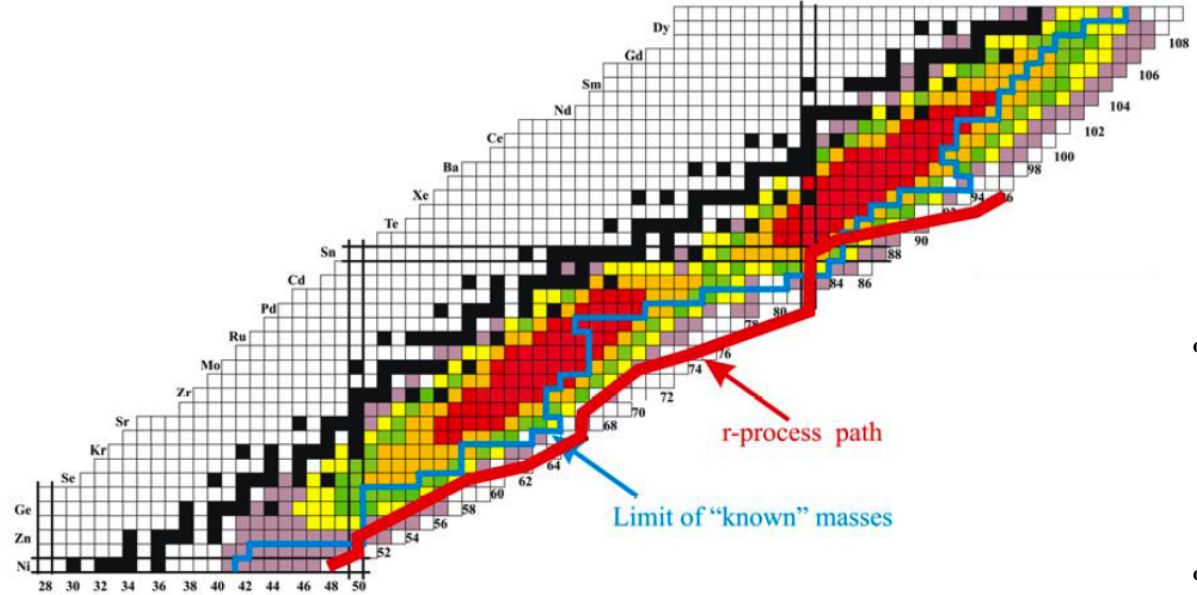
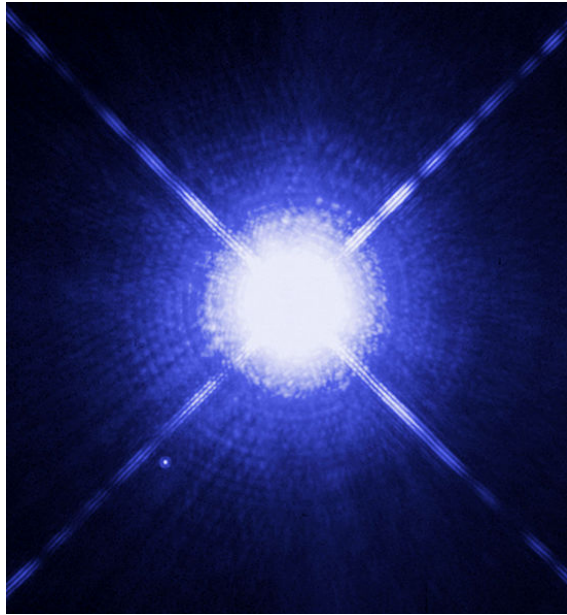
Hauser and Feshbach, Phys. Rev **87**, 366 (1952)

$$\sigma_{jk}^{\delta\nu} = \frac{\pi \hbar^2}{2\mu_{ij} E_{ij}} \frac{1}{(2J_i^\delta + 1)(2J_j + 1)} \times \sum_{J, \pi} (2J + 1) \frac{T_j^\delta(E, J, \pi, E_j^\delta, J_j^\delta, \pi_j^\delta) T_k^\nu(E, J, \pi, E_k^\nu, J_k^\nu, \pi_k^\nu)}{\sum_m T_m(E, J, \pi)}$$

$$T_m(E, J, \pi) = \int_{E_{\min}}^{E_{\max}} T(E, J, \pi; E_x, J_x, \pi_x) \rho(E_x, J_x, \pi_x) dE_x$$

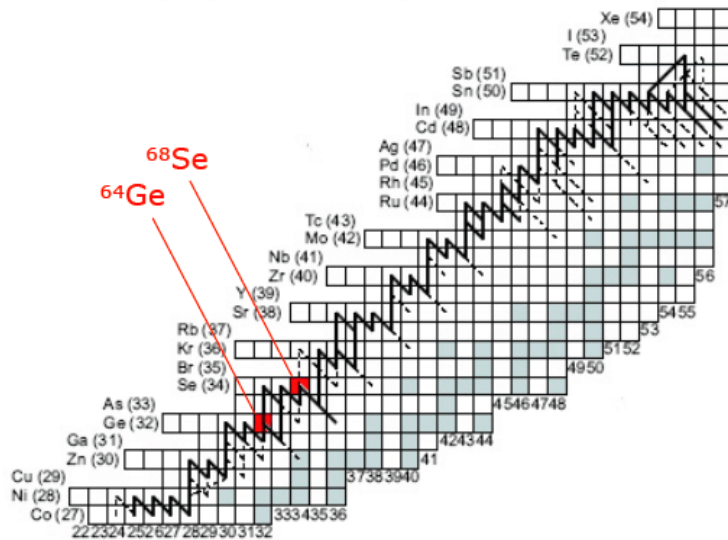
Where are NLD Needed: Nuclear Astrophysics

Binary stars XRB: Sirius



SN 1987 A

the rp-process path



Schatz et al. Phys. Rep 294 (1998) 167-298

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The Back-Shifted Fermi Gas Model for Nuclear Level Density

Rauscher et al, Phys. Rev. C 56, 1613 (1997)

$$\rho(U, J, \pi) = \frac{1}{2} \mathcal{F}(U, J) \rho(U), \quad (8)$$

with

$$\rho(U) = \frac{1}{\sqrt{2\pi\sigma}} \frac{\sqrt{\pi}}{12a^{1/4}} \frac{\exp(2\sqrt{aU})}{U^{5/4}},$$

$$\mathcal{F}(U, J) = \frac{2J+1}{2\sigma^2} \exp\left(\frac{-J(J+1)}{2\sigma^2}\right), \quad (9)$$

$$\sigma^2 = \frac{\Theta_{\text{rigid}}}{\hbar^2} \sqrt{\frac{U}{a}}, \quad \Theta_{\text{rigid}} = \frac{2}{5} m_u A R^2, \quad U = E - \delta.$$

Y. Alhassid, G. F. Bertsch, S. Liu, and H. Nakada, Phys. Rev. Lett. **84**, 4313 (2000).

D. Mocolj, T. Rauscher, G. Martinez-Pinedo, and Y. Alhassid, Nucl. Phys. **A718**, 750c (2003).

$$\frac{a}{A} = c_0 + c_1 S(N, Z), \quad (10)$$

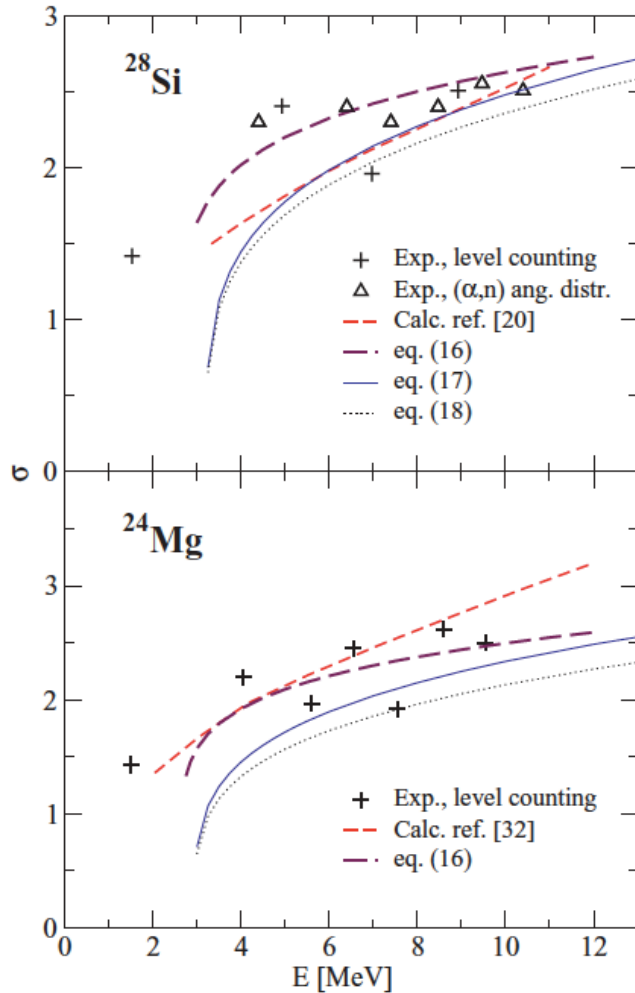
$$\delta = \Delta(Z, N)$$

$$\Delta_{\text{even-even}} = \frac{12}{\sqrt{A}},$$

$$\Delta_{\text{odd}} = 0, \quad (12)$$

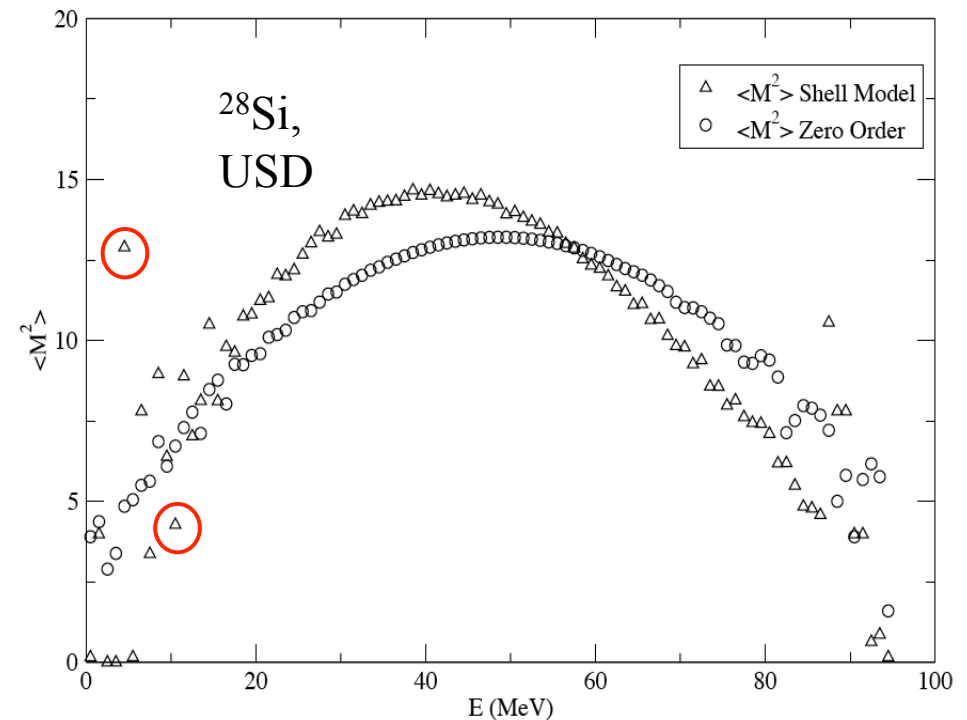
$$\Delta_{\text{odd-odd}} = -\frac{12}{\sqrt{A}}.$$

The Spin Cutoff Parameter



$$\langle M^2 \rangle (E) = \frac{1}{\rho(E)} \sum_{\bar{m}} d_{\bar{m}}(FRG)_{\bar{m}}(E) \langle M^2 \rangle_{\bar{m}}$$

$$\sigma^2(E) = \langle M^2 \rangle (E)$$



T. von Egidy & D. Bucurescu,
Phys.Rev. C 80, 054310 (2009)

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Horoi, Ghita, Zelevinsky,
Nucl. Phys. A 758, 142 (2005)

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Other Models of Nuclear Level Densities

$$\rho(E_x, J, \pi) = (1/2) \mathcal{F}(U, J) \rho_{FG}(U)$$

$$U = E_x - \Delta$$

$$\text{HF+BCS} \rightarrow \rho_{\text{HF+BCS}}(U)$$

- Goriely Nucl. Phys. A605, (1996) 28.
- Demetriou and Goriely, Nucl. Phys. A695 (2001) 95.
- <http://www-astro.ulb.ac.be/Html/nld.html>

	Z=10	Z=20	Z=30	Z=40	Z=50	Z=60	Z=70	Z=80	Z=90	Z=100
	Z=11	Z=21	Z=31	Z=41	Z=51	Z=61	Z=71	Z=81	Z=91	Z=101
	Z=12	Z=22	Z=32	Z=42	Z=52	Z=62	Z=72	Z=82	Z=92	Z=102
	Z=13	Z=23	Z=33	Z=43	Z=53	Z=63	Z=73	Z=83	Z=93	Z=103
	Z=14	Z=24	Z=34	Z=44	Z=54	Z=64	Z=74	Z=84	Z=94	Z=104
	Z=15	Z=25	Z=35	Z=45	Z=55	Z=65	Z=75	Z=85	Z=95	Z=105
	Z=16	Z=26	Z=36	Z=46	Z=56	Z=66	Z=76	Z=86	Z=96	Z=106
	Z=17	Z=27	Z=37	Z=47	Z=57	Z=67	Z=77	Z=87	Z=97	Z=107
Z=8	Z=18	Z=28	Z=38	Z=48	Z=58	Z=68	Z=78	Z=88	Z=98	Z=108
Z=9	Z=19	Z=29	Z=39	Z=49	Z=59	Z=69	Z=79	Z=89	Z=99	Z=109

$$\text{HFB+Combinatorial: } \rho(E_x, J, \pi)$$

- S. Hilaire, J.P. Delaroche and A.J. Koning, Nucl. Phys. A632, 417 (1998).
- S. Hilaire, J.P. Delaroche and M. Girod, Eur. Phys. J. A12 (2001) 169.
- S. Hilaire and S. Goriely, Nucl. Phys. A779 (2006) 63
- http://www-astro.ulb.ac.be/Html/nld_comb.html

Experimental Data

<http://ocl.uio.no/compilation/>

<http://inpp.ohiou.edu/~voinov/>

^{50}V , ^{51}V

^{44}Sc

^{56}Fe , ^{57}Fe

^{47}Ti

^{93}Mo , ^{94}Mo , ^{95}Mo , ^{96}Mo ,
 ^{97}Mo , ^{98}Mo

^{56}Fe , ^{57}Fe

^{60}Ni ,

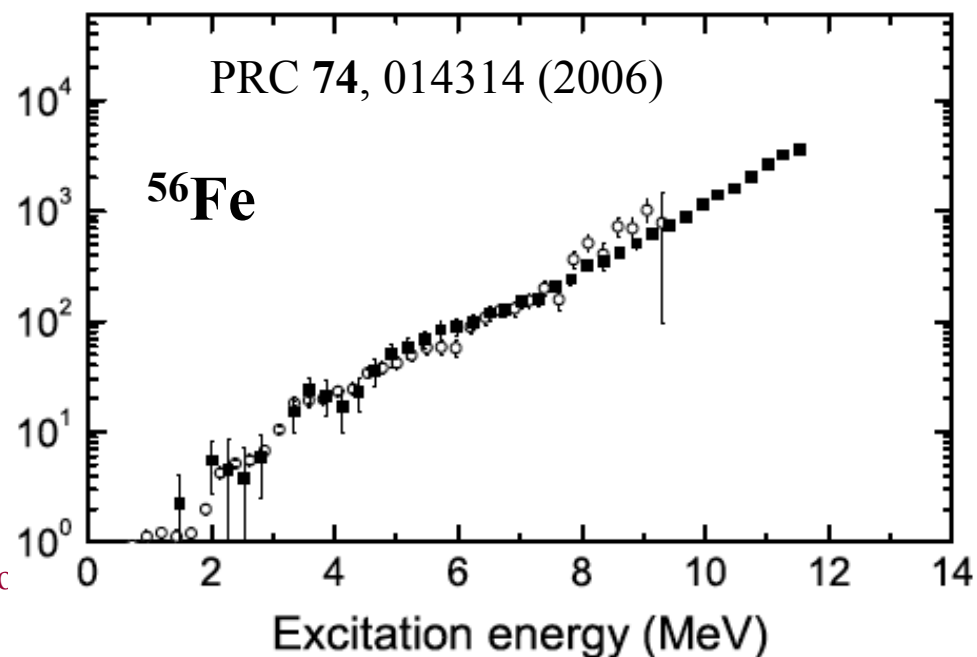
^{148}Sm , ^{149}Sm

^{60}Co

^{160}Dy , ^{161}Dy , ^{162}Dy

^{167}Er , ^{168}Er

^{170}Yb , ^{171}Yb , ^{172}Yb



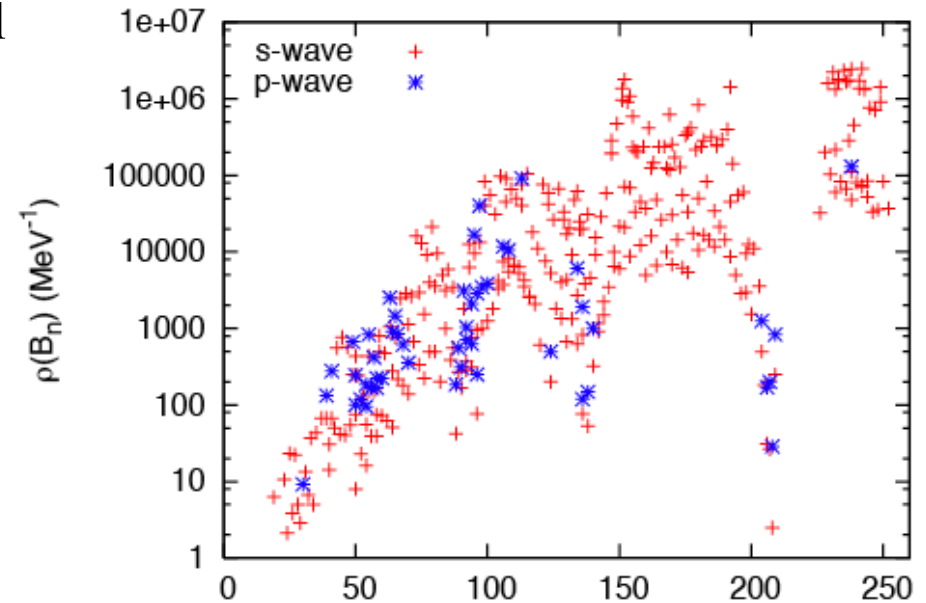
Experimental Data: Neutron Resonances

<http://www-nds.iaea.org/ripl-2/densities.html>

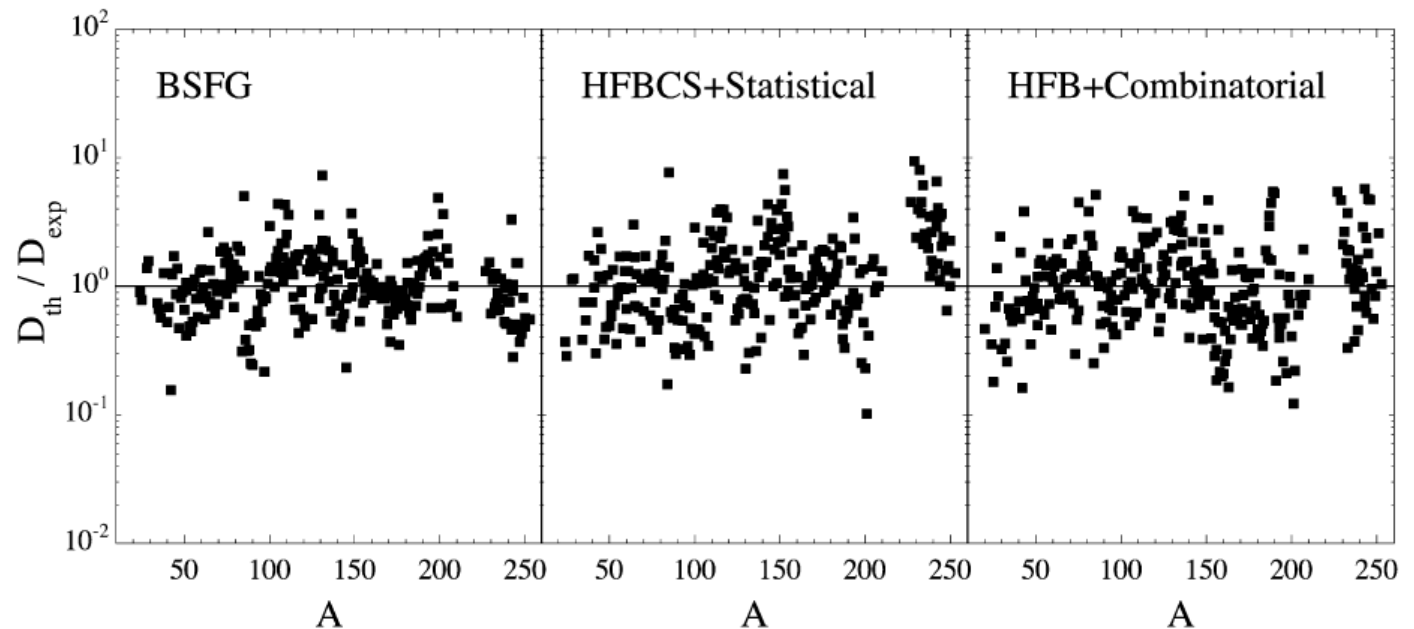
$$\rho(B_n) = \frac{1000}{D_0}$$

$$f_{rms} = \exp \left[\frac{1}{N_e} \sum_{i=1}^{N_e} \ln^2 \frac{D_{th}^i}{D_{exp}^i} \right]^{1/2}$$

$$f_{rms} \sim 2$$



Hilaire and Goriely,
NPA 799, 63 (2006)



Accurate Nuclear Level Densities

Comparison of:

1. CI,
2. HF+BCS

www-astro.ulb.ac.be/Html/nld.html

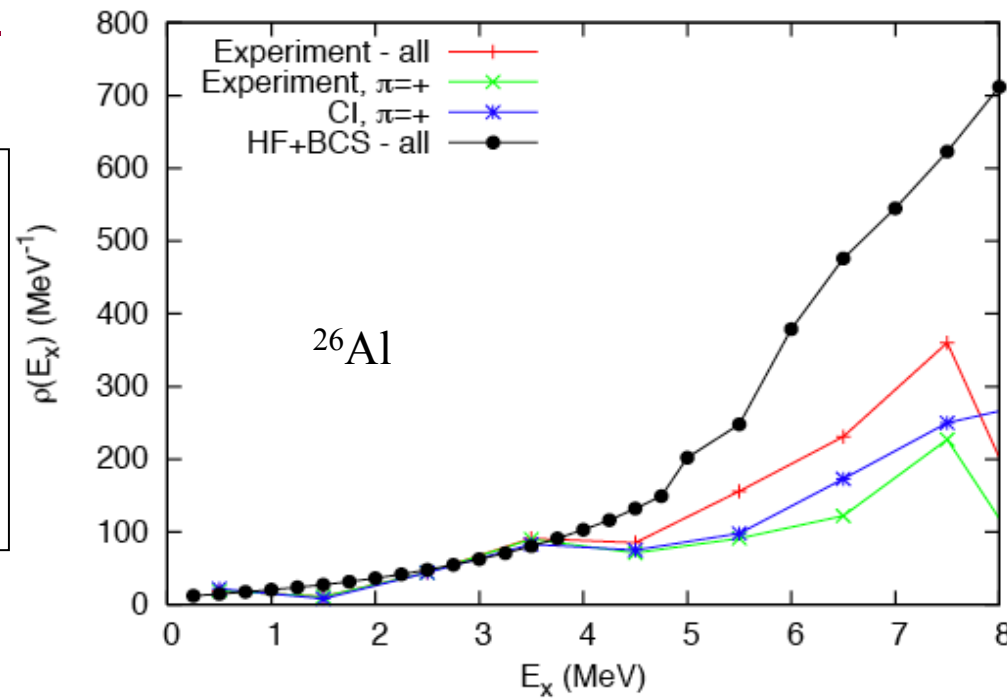
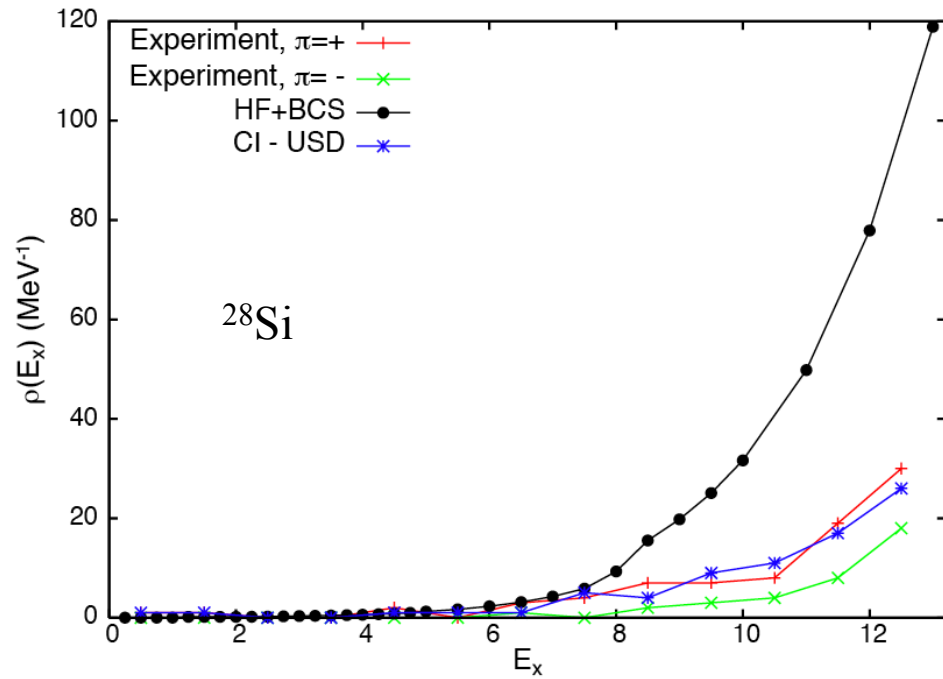
3. experimental data

Complete spectroscopy: sd-
shell nuclei

Conclusions:

- HF+BCS overestimates the data
- CI accurately describes the data

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NLD and Statistical Spectroscopy

M. Horoi et al. :

PRC **67**, 054309 (2003),

PRC **69**, 041307(R) (2004),

NPA **785**, 142 (2005).

PRL **98**, 265503 (2007)

Configurations: e.g. 4 particles in sd

d3 d5 s1

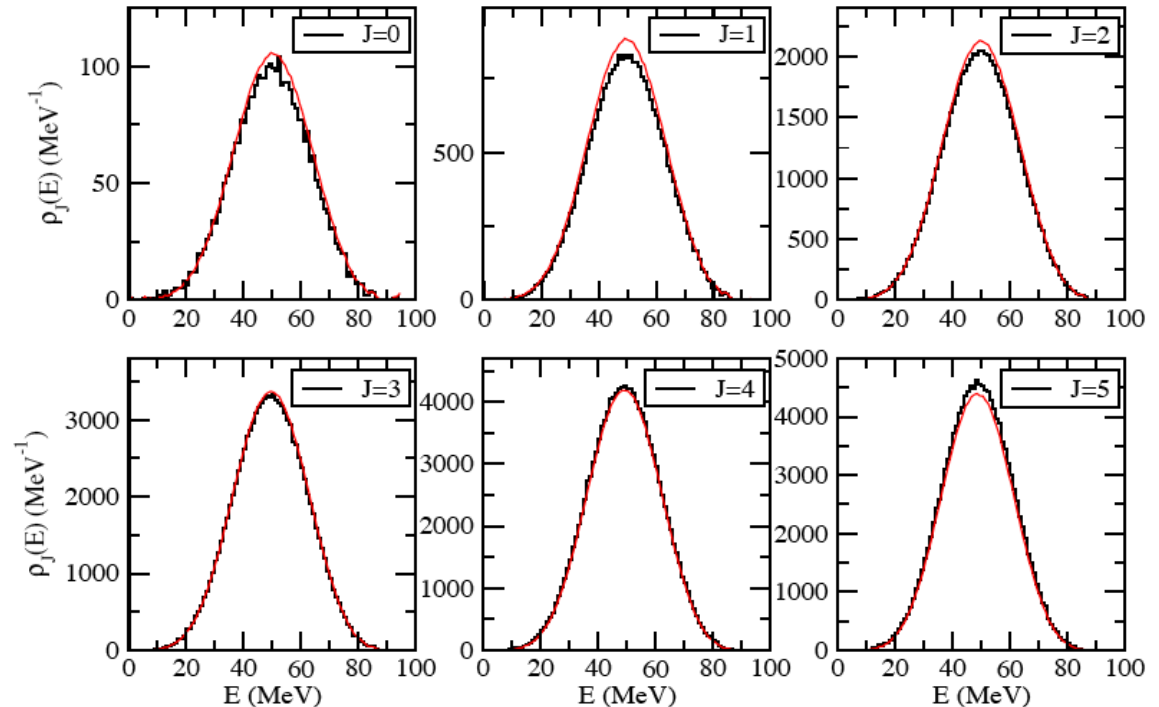
4 0 0

3 1 0

3 0 1 ...

preserve rotational invariance and parity

²⁸Si π = + staircase: CI, USD



$$\rho(E_x, J, \pi) = \sum_{c \in \text{conf}} D_c(J, \pi) G_{FR}(E, E_c(J), \sigma_c(J))$$

$$E_c(J), \sigma_c(J) \leftarrow \text{Tr}_{SD_c} \langle M | H^q | M \rangle_{SD_c}$$

$$E_x = E - E_{g.s.}$$

$E_c(J), \sigma_c(J)$: computational intensive

E_{g.s.} from CI, PCI, Exponential Convergence Method (PRL **82**, 2064 (1999)), CC, etc.

Configurations can be calculated in parallel

Fixed spin and parity nuclear level density for restricted shell model configurations

Mihai Horoi,¹ Monica Ghita,¹ and Vladimir Zelevinsky^{2,3}

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²National Superconducting Cyclotron Laboratory, East Lansing, Michigan 48824, USA

³Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA

(Received 14 January 2004; published 30 April 2004)

Fixed J Configura tion Centroids and Widths

$$\langle\langle H \rangle\rangle_{\vec{m}nMT_z} = \sum_i \epsilon_i D^i(\vec{m}nMT_z) + \sum_{i<j} V_{ijij} D^{ij}(\vec{m}nMT_z)$$

$$\begin{aligned} \langle\langle H^2 \rangle\rangle_{\vec{m}nMT_z} = & \sum_i \epsilon_i^2 D^i(\vec{m}nMT_z) + \\ & \sum_{i<j} \left[2\epsilon_i\epsilon_j + 2(\epsilon_i + \epsilon_j)V_{ijij} + \sum_{k<l} V_{ijkl}^2 \right] D^{ij}(\vec{m}nMT_z) + \\ & \sum_{(i<j)\neq l} \left[\sum_k (2V_{liik}V_{ljjk} - V_{ijkl}^2) + 2\epsilon_l V_{ijij} \right] D^{ijl}(\vec{m}nMT_z) + \\ & \sum_{(i<j)\neq(k<l)} [V_{ijkl}^2 + V_{ijij}V_{klkl} - 4V_{kiil}V_{kjjl}] D^{ijkl}(\vec{m}nMT_z) \end{aligned}$$

$$E_{\vec{m}}(J) = \langle H \rangle_{\vec{m}nJT_z} = (\langle\langle H \rangle\rangle_{nM=JT_z} - \langle\langle H \rangle\rangle_{\vec{m}nM=(J+1)T_z}) / D(\vec{m}nJT_z)$$

$$\langle H^2 \rangle_{\vec{m}nJT_z} = (\langle\langle H^2 \rangle\rangle_{\vec{m}nM=JT_z} - \langle\langle H^2 \rangle\rangle_{\vec{m}nM=(J+1)T_z}) / D(\vec{m}nJT_z)$$

$$\sigma_{\vec{m}nJT_z} = \sqrt{\langle\langle H^2 \rangle\rangle_{\vec{m}nJT_z} - \langle H \rangle_{\vec{m}nJT_z}^2}$$

C. Jacquemin,
Z. Phys. A
303, 135
(1981)

n - is the number of particles

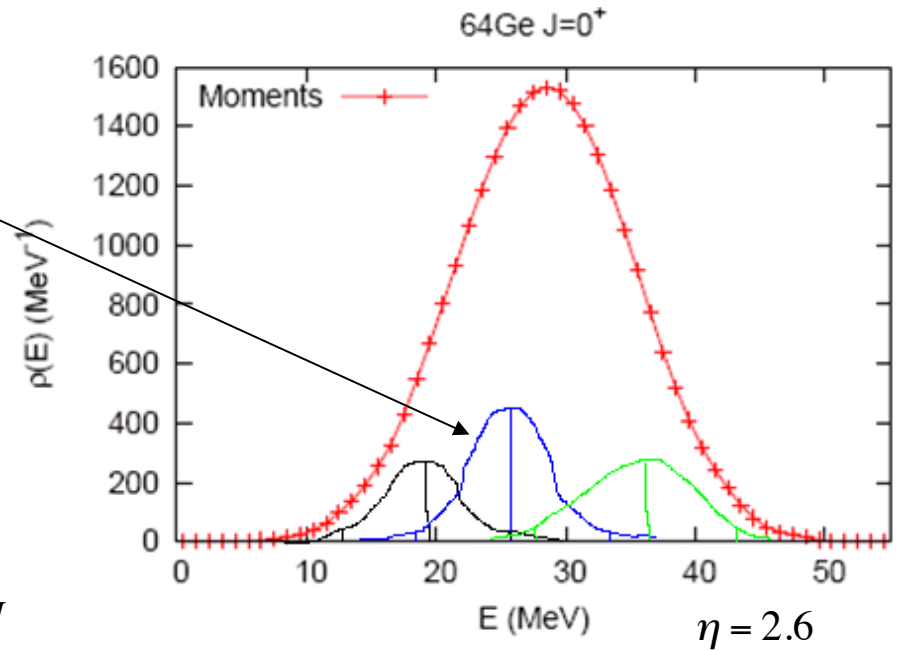
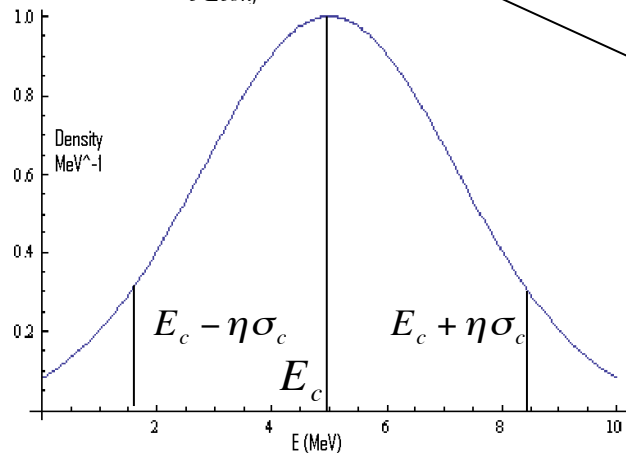
$D^i(\vec{m}nMT_z)$ is the number of determinants in a partition \vec{m} with nMT_z and state i occupied,

$$D^{\vec{k}}(n\vec{m}M) = \sum_{k \leq t \leq n} (-1)^{t-k} \sum_{t_1 + \dots + t_k = t} D((n-t)(\vec{m} - t_1 U_1 - \dots - t_k U_k)(M - t_1 m_{k_1} - \dots - t_k m_{k_k}))$$

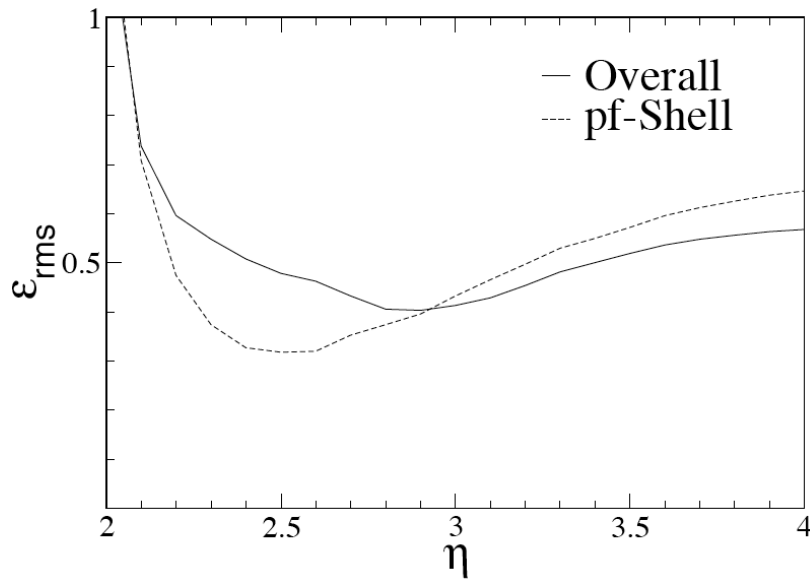
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Study of Errors

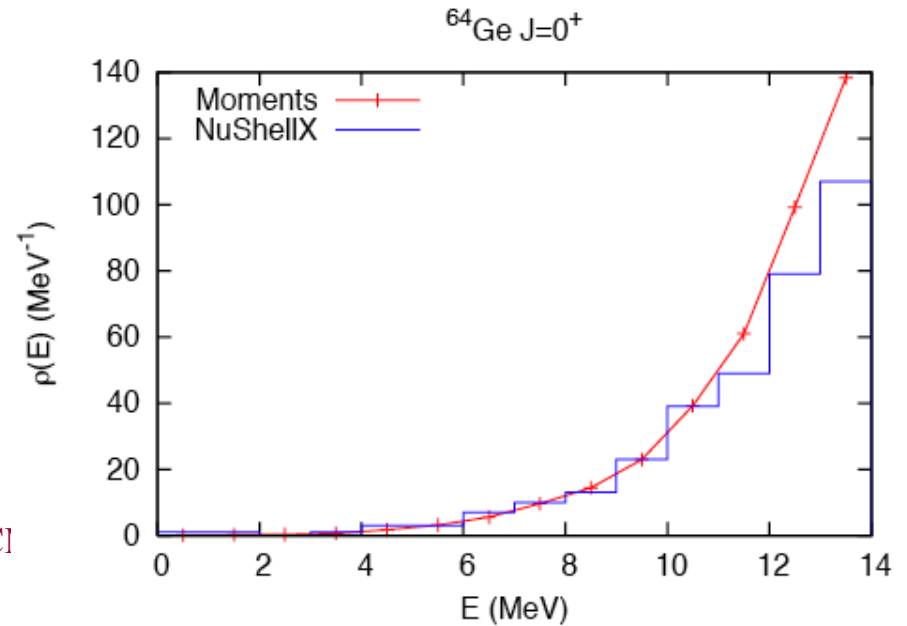
$$\rho(E_x, J, \pi) = \sum_{c \in \text{conf}} D_c(J, \pi) G_{FR}(E, E_c(J), \sigma_c(J))$$



$$\epsilon_{rms} = \exp \left[\sqrt{\frac{1}{N} \sum_i \left(\ln \frac{\rho_i^{MOM}}{\rho_i^{CI}} \right)^2} \right] - 1 \quad \text{error relative to CI}$$

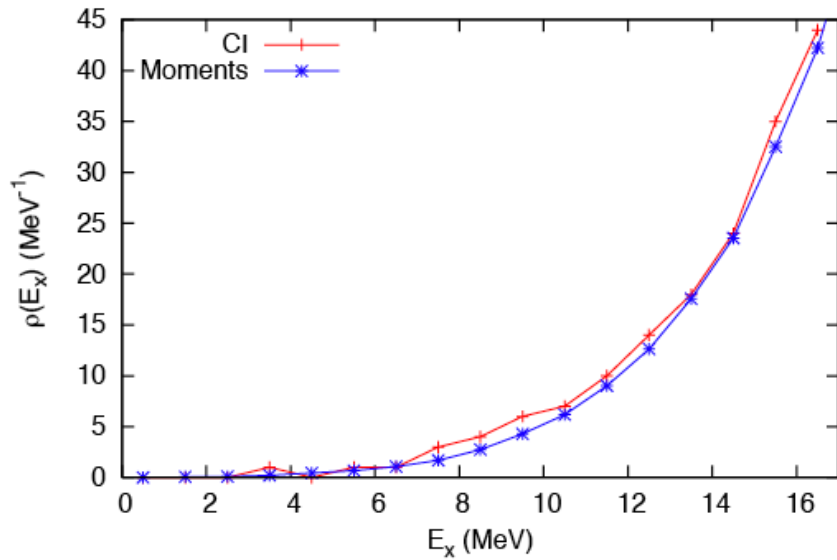


M. Horoi CI

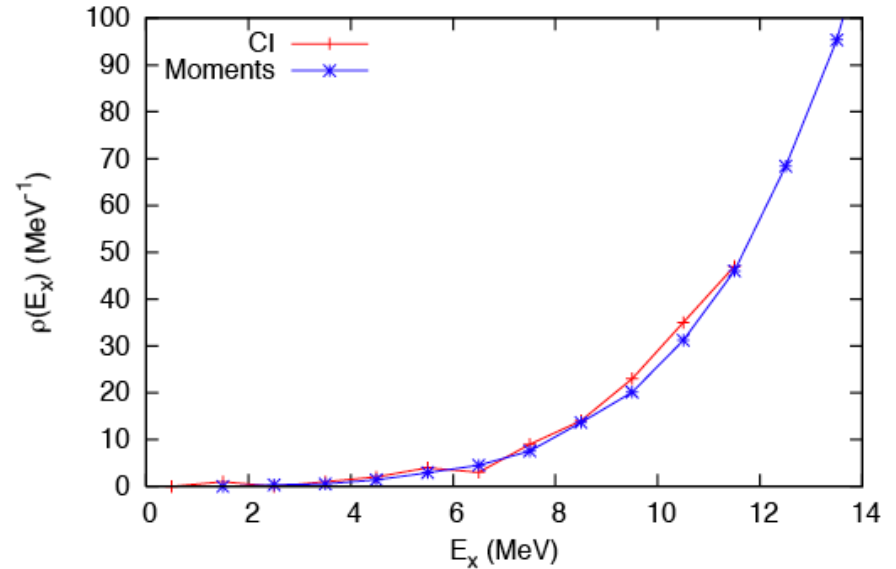


NLD Comparison: CI, Moments, HF+BCS

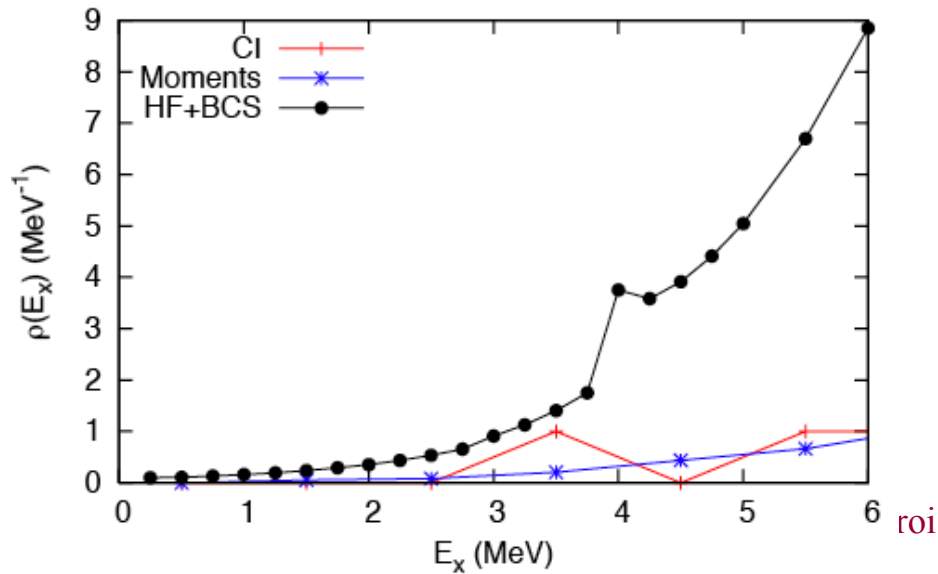
^{48}Cr , $J=0$, parity=+



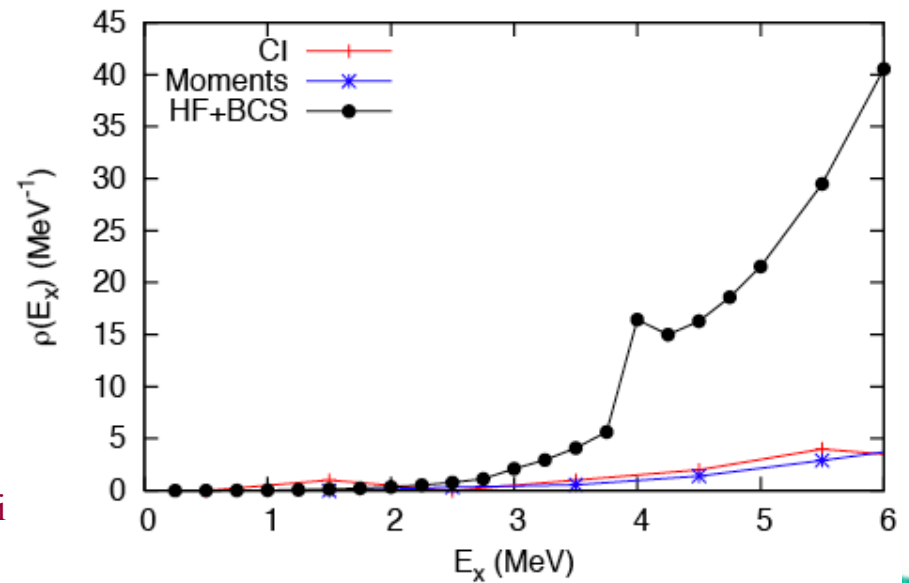
^{48}Cr , $J=4$, parity=+



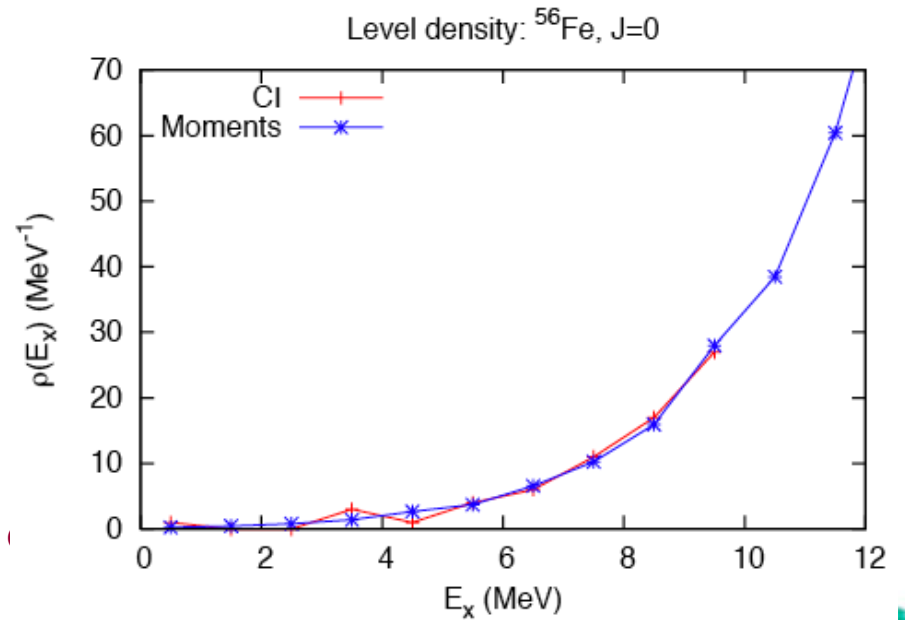
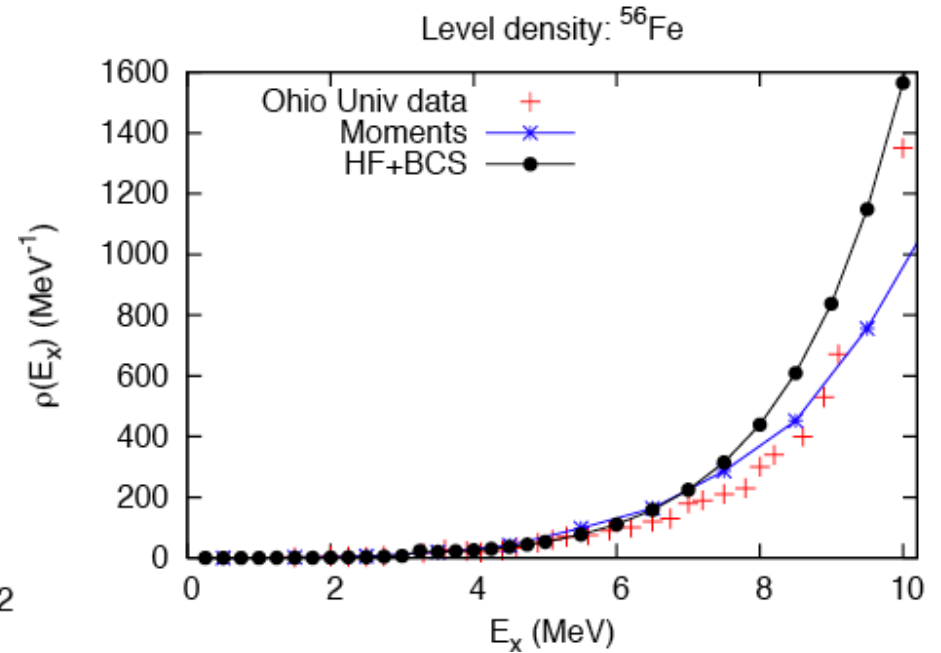
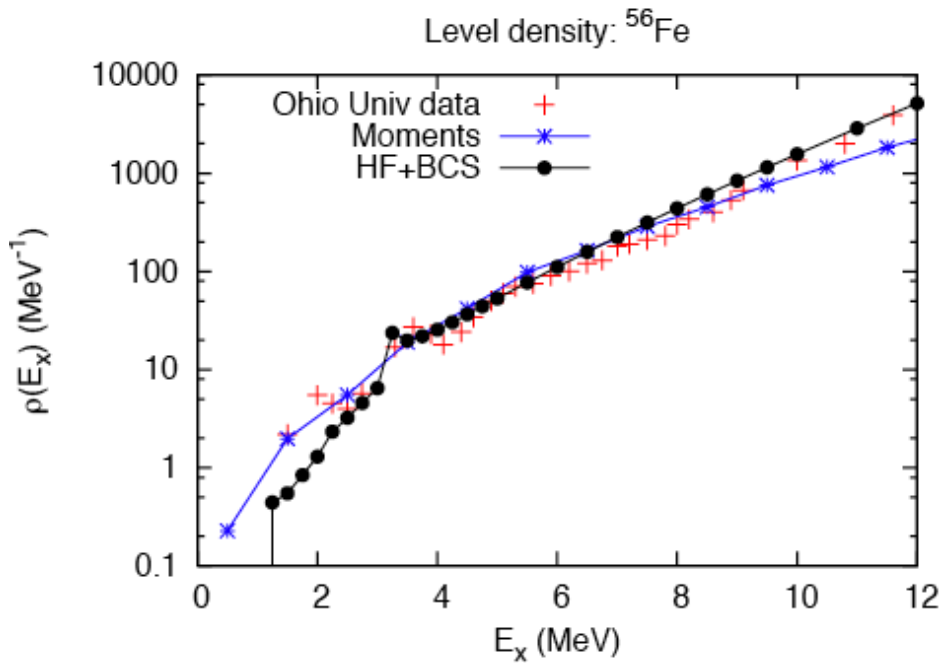
^{48}Cr , $J=0$, parity=+



^{48}Cr , $J=4$, parity=+



NLD of ^{56}Fe : CI, Moments, HF+BCS



Ohio data: PRC 74, 014314 (2006)

Theory: *pf* model space,
GXPF1A interaction

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Ratio of unnatural to natural NLD of different parities at low energies

$$\rho(E_x, J, \pi) = (1/2) \mathcal{F}(U, J) \rho_{FG}(U)$$

$$U = E_x - \Delta$$

Equal contribution to both parities

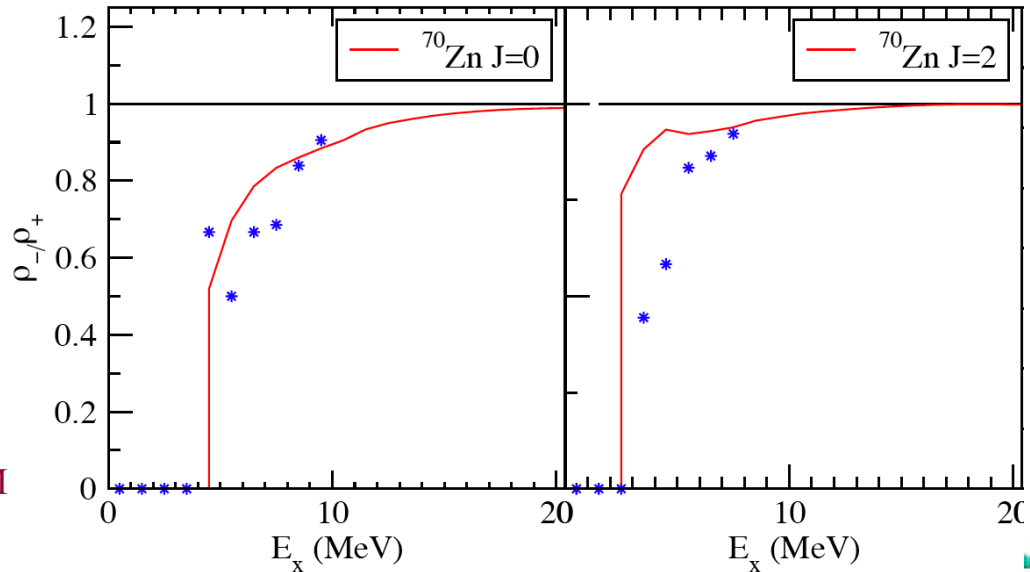
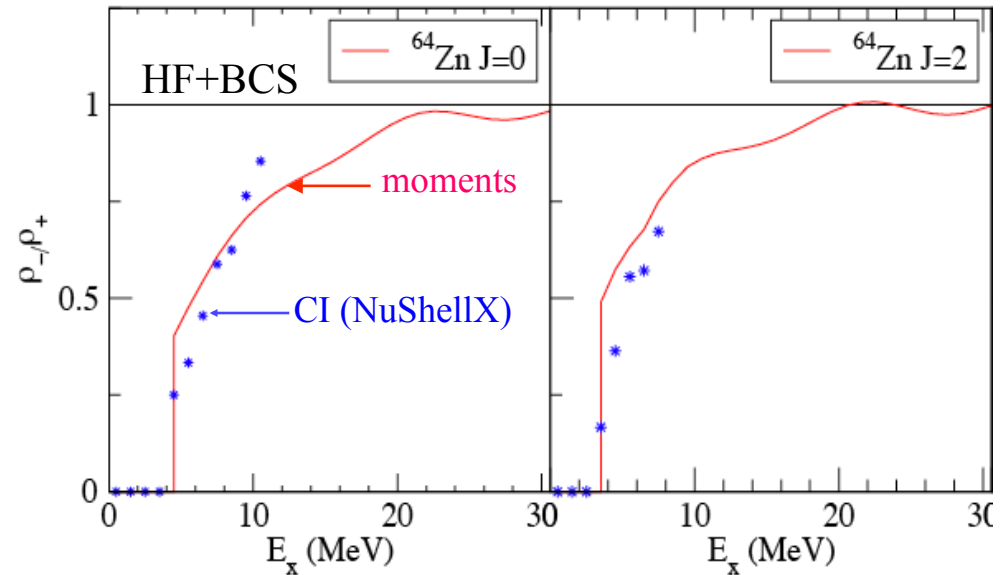
Remedy by Alhassid, Bertsch, Liu, Nakada, PRL 84, 4313 (2000) + Basel group (Rauscher)

Configurations: e.g. 4 particles in fpg

f5	p3	p1	g9	π
4	0	0	0	+
3	1	0	0	+
3	0	1	0	+
3	0	0	1	-

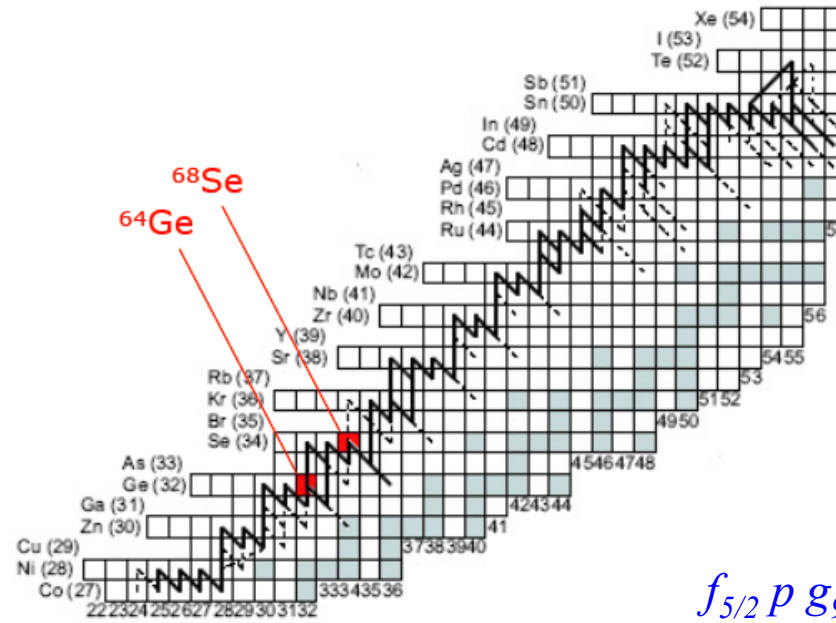
...

preserve rotational invariance and parity



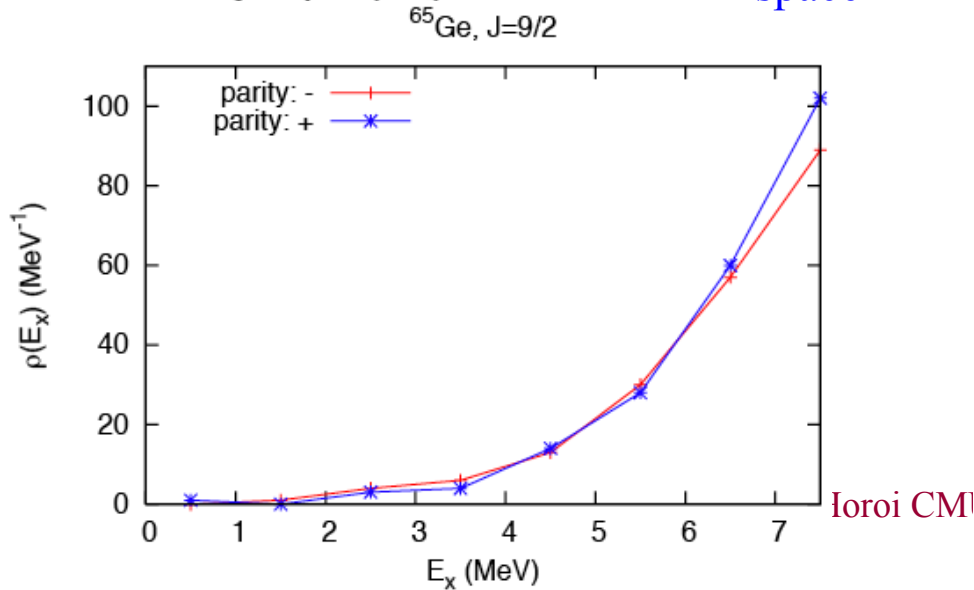
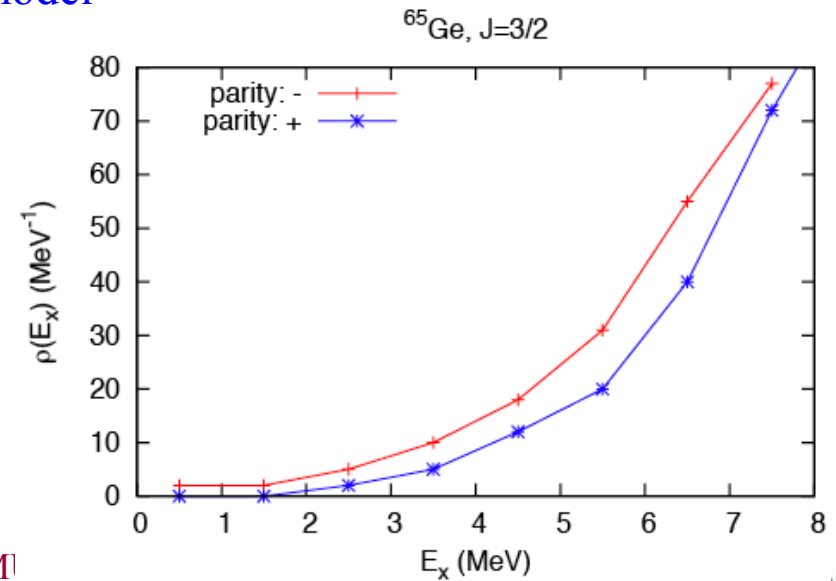
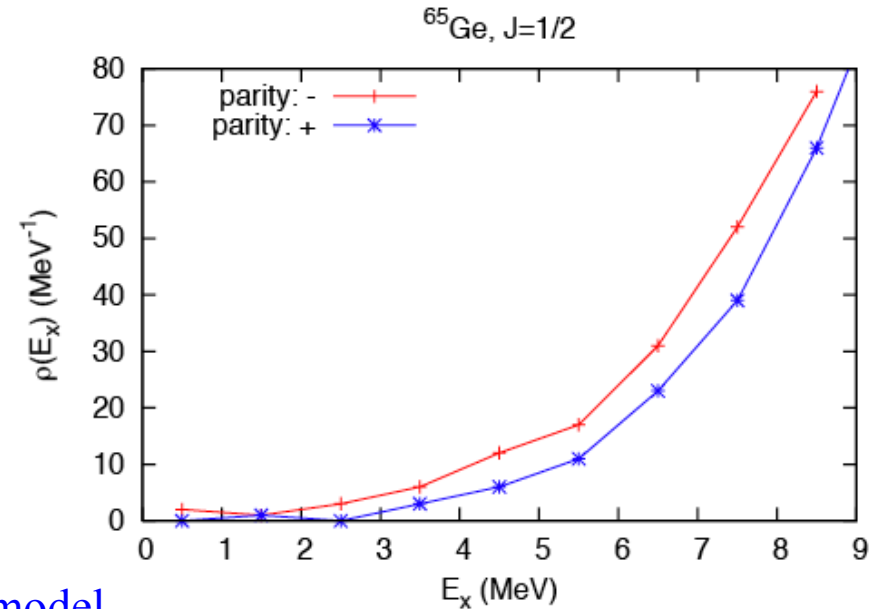
NLD for the rp-process

the rp-process path



Schatz et al. Phys. Rep 294 (1998) 167-298

$f_{5/2} p g_{9/2}$ model space



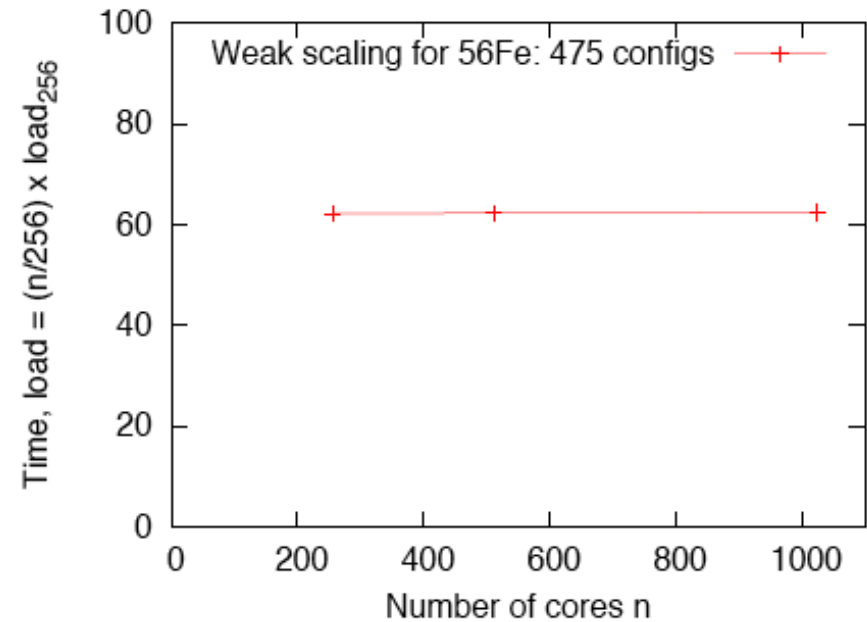
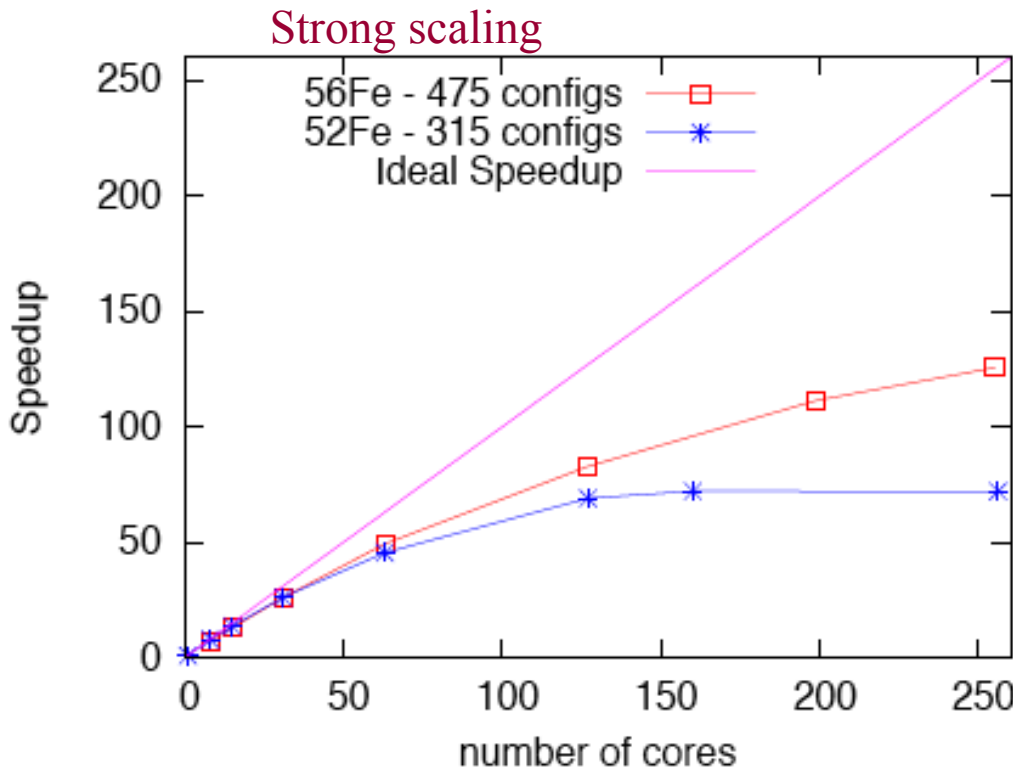
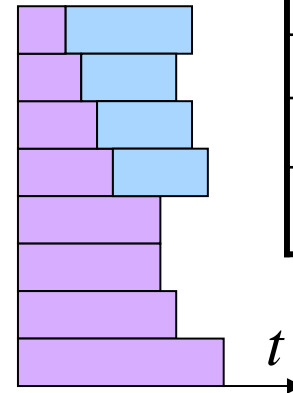
Scaling of the MPI JMOMENTS Code

Domain decomposition: many-body configurations

Algorithm: Dynamical Load Balancing

Machine: Franklin/NERSC

Nucleus / model space	Isospin configs	PN - configs
52Fe/pf	315	22028
56Fe/pf	475	51174
64Ge/pfg9/2	3749	510544



M. Horoi et al. :

PRC **67**, 054309 (2003),

PRC **69**, 041307(R) (2004),

NPA **785**, 142 (2005).

PRL **98**, 265503 (2007)

$$\rho(E_x, J, \pi) = \sum_{c \in \text{conf}} D_c(J, \pi) G_{FR}(E, E_c(J), \sigma_c(J))$$

$$E_c(J), \sigma_c(J) \leftarrow \text{Tr}_{SD_c} \langle M | H^q | M \rangle_{SD_c}$$

$$E_x = E - E_{g.s.}$$

$E_c(J), \sigma_c(J)$: computational intensive

Many more configurations, which can be more efficiently be calculated in parallel

Configurations: e.g. 4P and 4N in *pf* model space

	Pf7	Pp3	Pf5	Pp1	Nf7	Np3	Nf5	Np1
4	0	0	0	4	0	0	0	0
3	1	0	0	4	0	0	0	0
4	0	0	0	3	1	0	0	0

...

preserve rotational invariance and parity

E_{g.s.} from CI, PCI, Exponential Convergence Method (PRL **82**, 2064 (1999)), CC, etc, **or determined**.

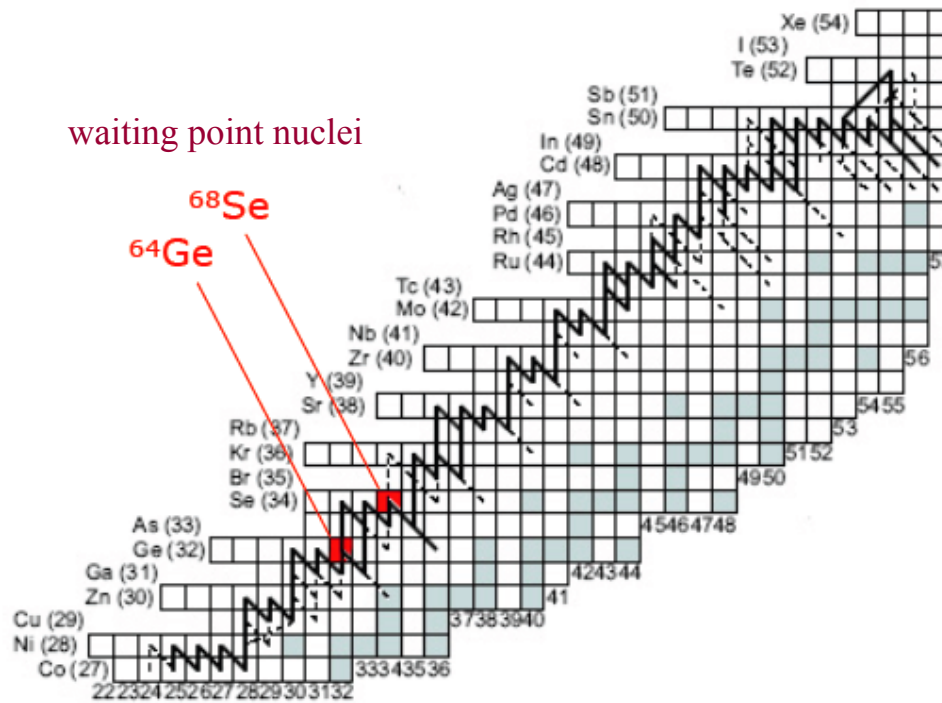
$$D^{\vec{k}}(PN\vec{m}M) = \sum_{M_P + M_N = M} D^{\vec{k}_P}(P\vec{m}M_P) D^{\vec{k}_N}(N\vec{m}M_N)$$

Domain decomposition: many-body configurations

Algorithm: Dynamical Load Balancing

Machine: Franklin/NERSC

the rp-process path



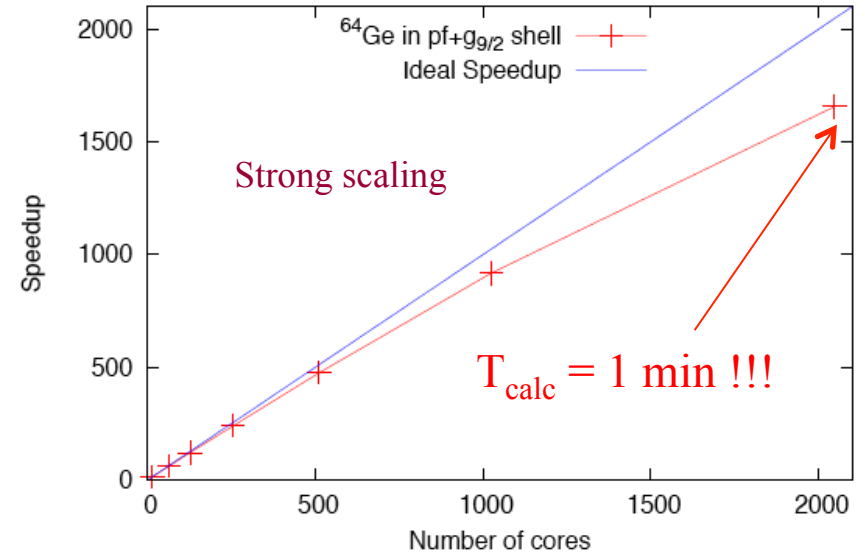
Schatz et al. Phys. Rep 294 (1998) 167-298

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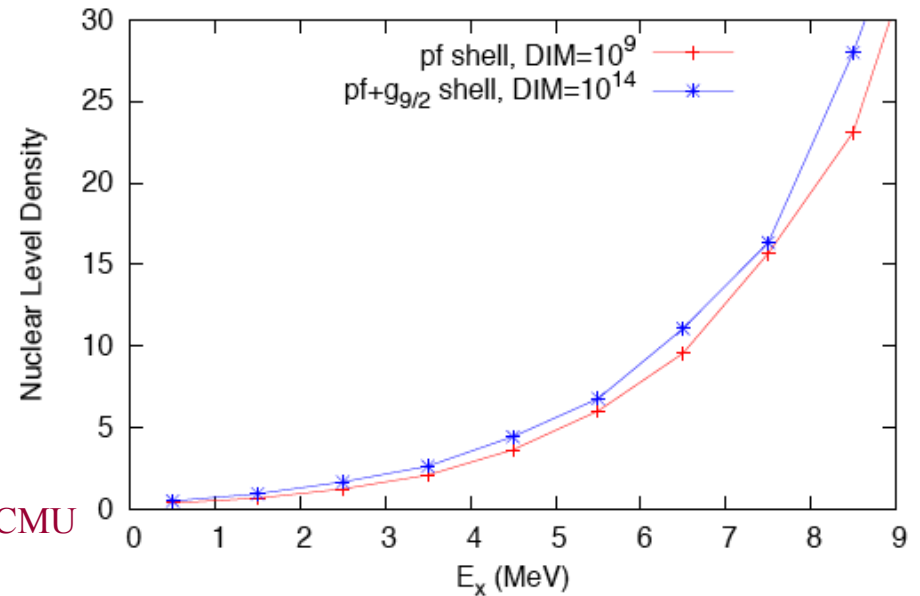
March 25, 2010

M. Horoi CMU

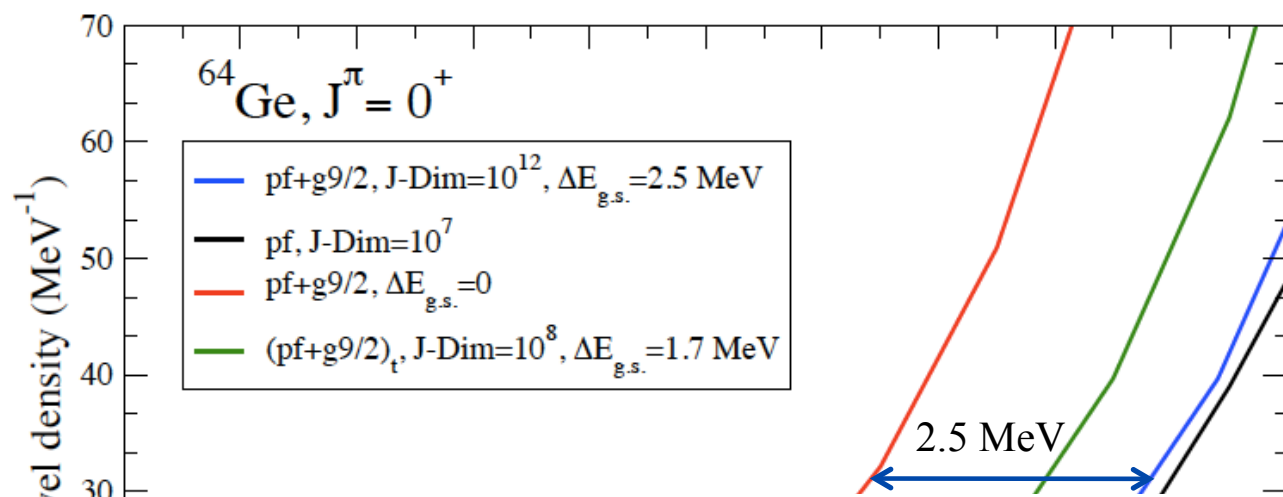
Nuclear Level Density: JMOMENTS code



^{64}Ge : comparison for two model spaces



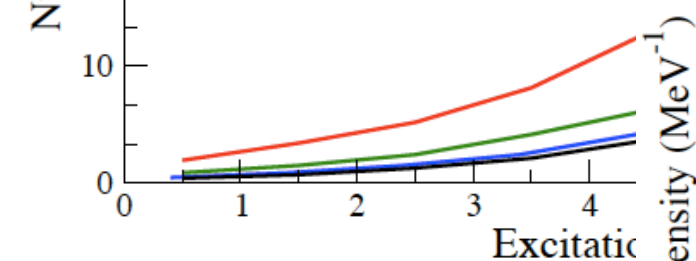
Results



$$E_{\text{gs}}(\text{pf}) = -304.25 \text{ MeV}$$

$$E_{\text{gs}}(\text{pf}+\text{g}9/2)_t = -305.95 \text{ MeV}$$

$$E_{\text{gs}}(\text{pf}+\text{g}9/2)_s = -306.75 \text{ MeV}$$

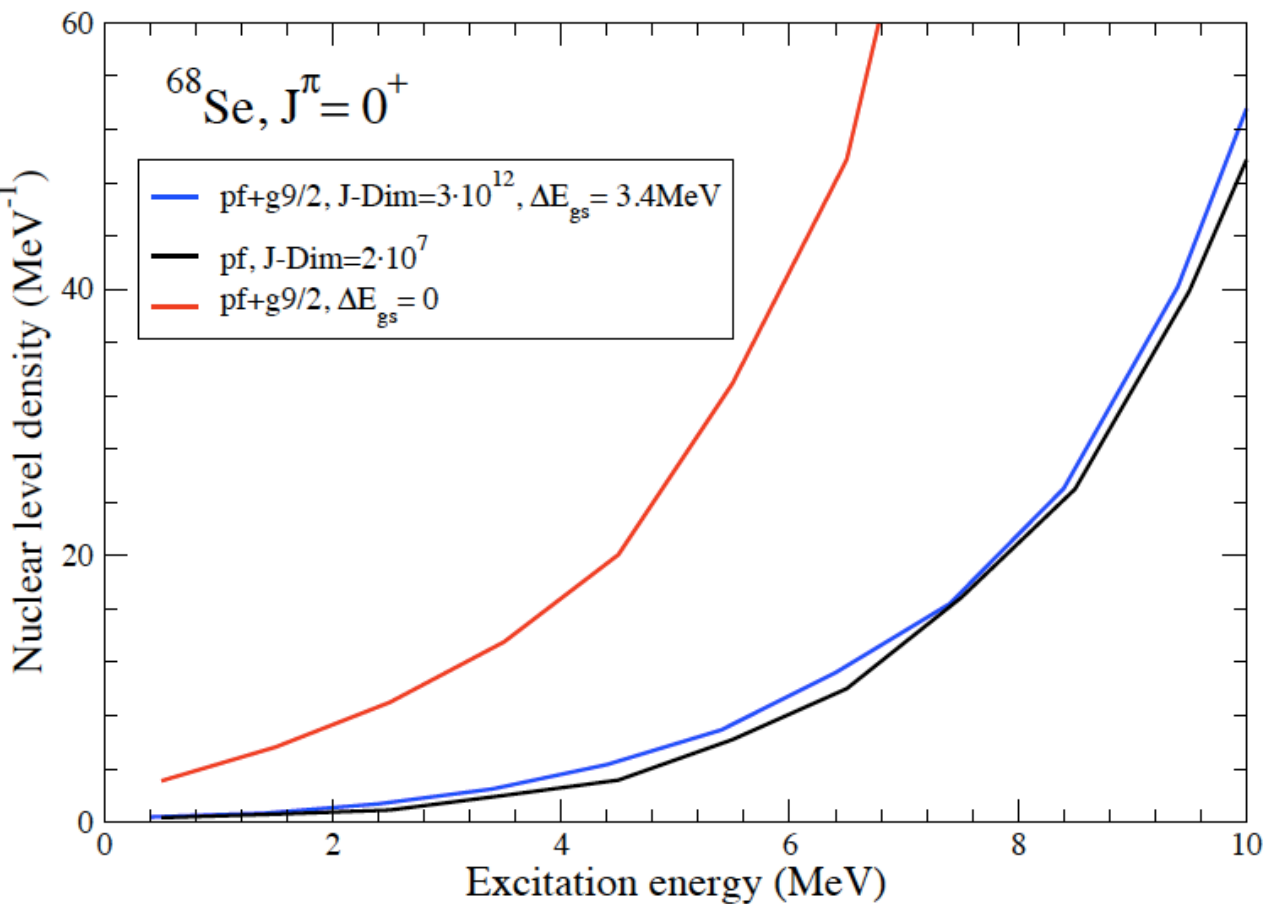


$$E_{\text{gs}}(\text{pf}) = -353.1 \text{ MeV}$$

$$E_{\text{gs}}(\text{pf}+\text{g}9/2)_s = -356.6 \text{ MeV}$$

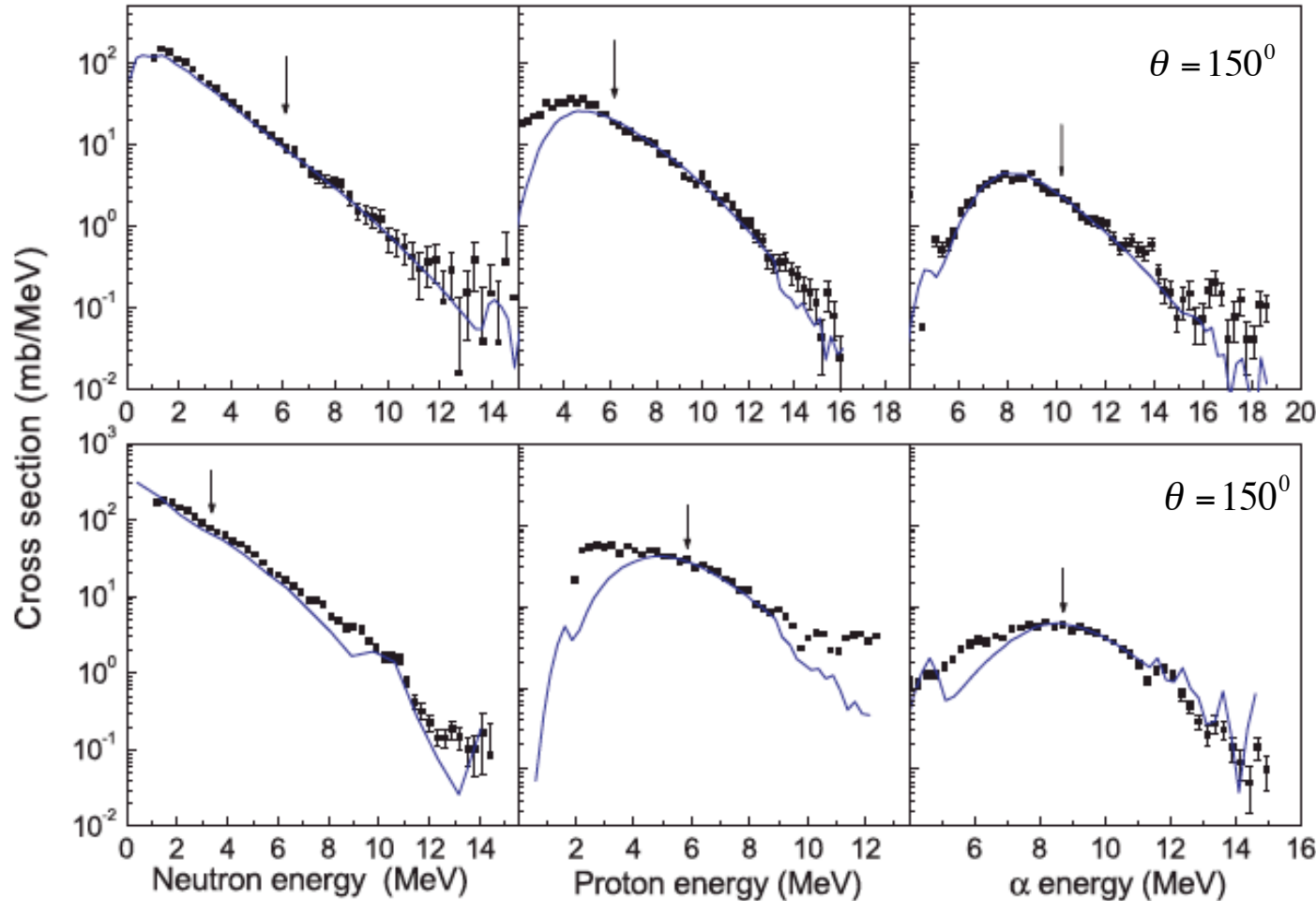
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NLD and Hauser-Feshbach

From A. Voinov et al., PRC **76**, 044602 (2007)



$\leftarrow {}^3\text{He} + {}^{58}\text{Fe}$
 $E_{{}^3\text{He}} = 10 \text{ MeV}$

${}^{61}\text{Ni}^*$ compound

$\leftarrow d + {}^{59}\text{Co}$

$E_d = 7.5 \text{ MeV}$

NLD and Hauser-Feshbach

talys 1.2 : www.talys.eu

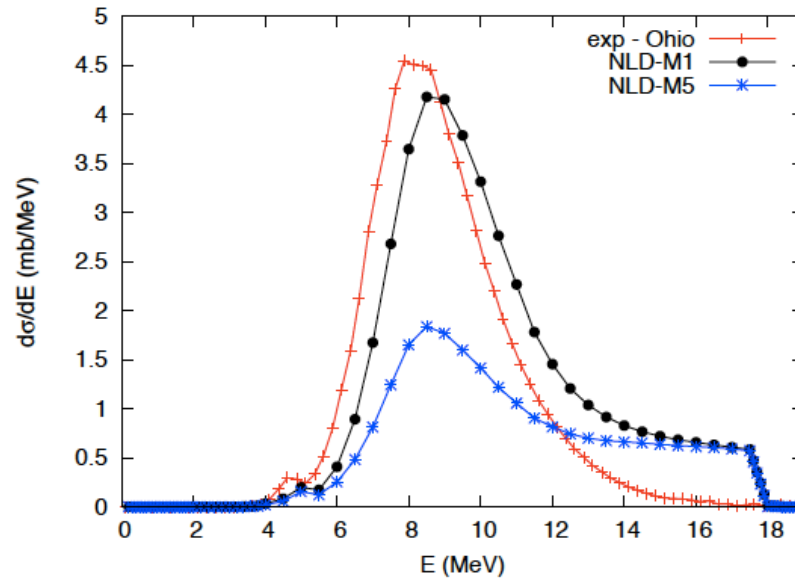
ldmodel 1: Constant temperature + Fermi gas model

ldmodel 2: Back-shifted Fermi gas model

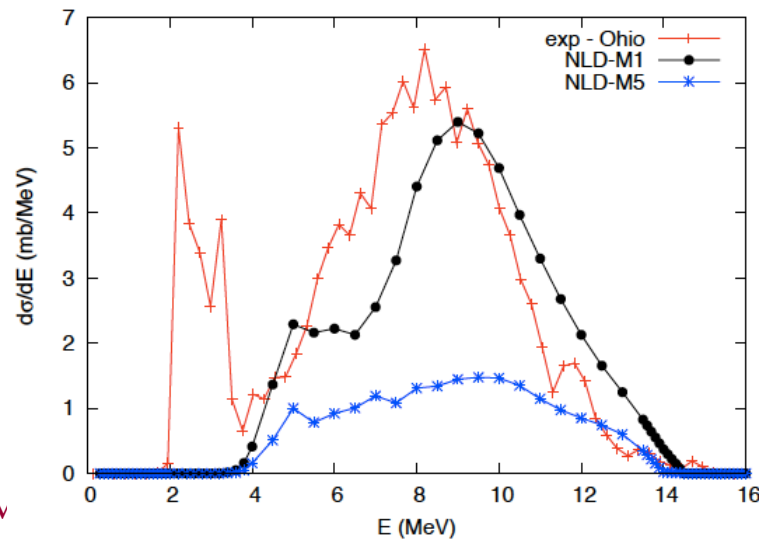
ldmodel 3: Generalised superfluid model

ldmodel 4: Microscopic level densities from Goriely's table

ldmodel 5: Microscopic level densities from Hilaire's table



← $^{58}\text{Fe}(^3\text{He}, \alpha)$
 $\theta = 150^\circ$



← $^{59}\text{Co}(d, \alpha)$
 $\theta = 150^\circ$

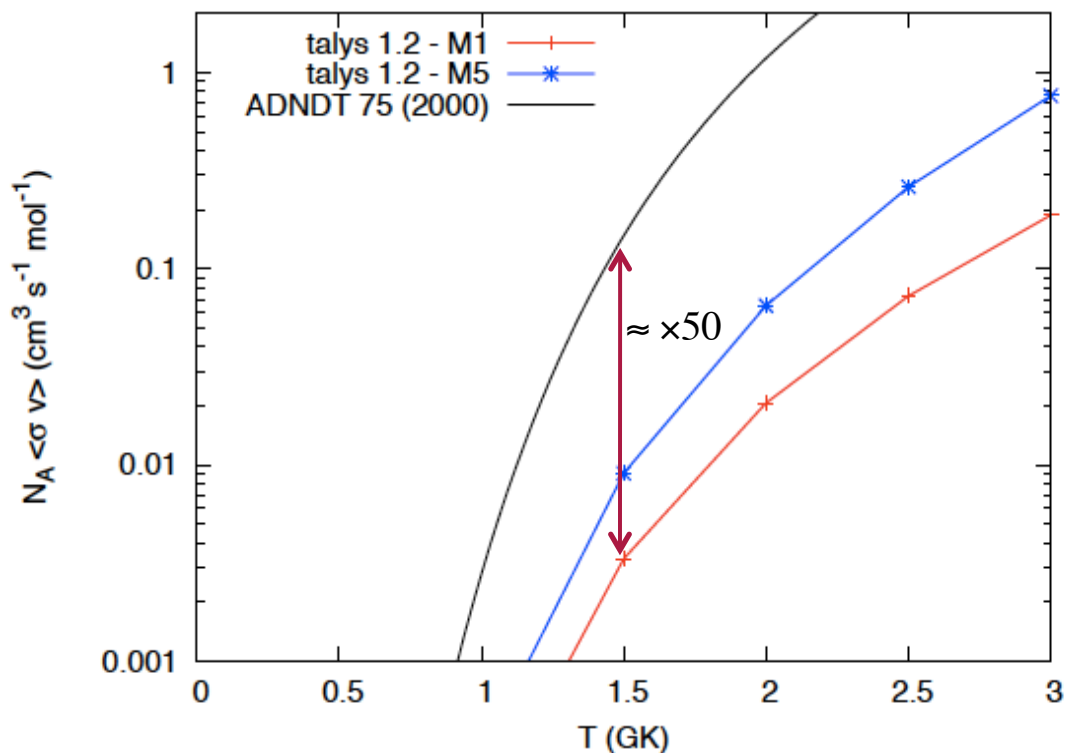
NLD: reaction rates

talys 1.2 : www.talys.eu

Rauscher & Thielemann ADNDT 75, 1 (2000)

$$N_A \langle \sigma v \rangle_{\alpha\alpha'}^*(T) = \left(\frac{8}{\pi m}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} G(T) \int_0^\infty \sum_\mu \frac{(2I^\mu + 1)}{(2I^0 + 1)} \times \sigma_{\alpha\alpha'}^\mu(E) E \exp\left(-\frac{E + E_x^\mu}{kT}\right) dE,$$

$$G(T) = \sum_\mu (2I^\mu + 1)/(2I^0 + 1) e^{-E_x^\mu/kT} \rightarrow \sum_{I,\pi} \int (2I^\pi + 1)/(2I^0 + 1) \rho(E_x, I, \pi) e^{-E_x/kT}$$



NLD: rp-process

See also PRC 75, 032801(R) (2007)

From P. Shury et al., PRC 75, 055801 (2007)

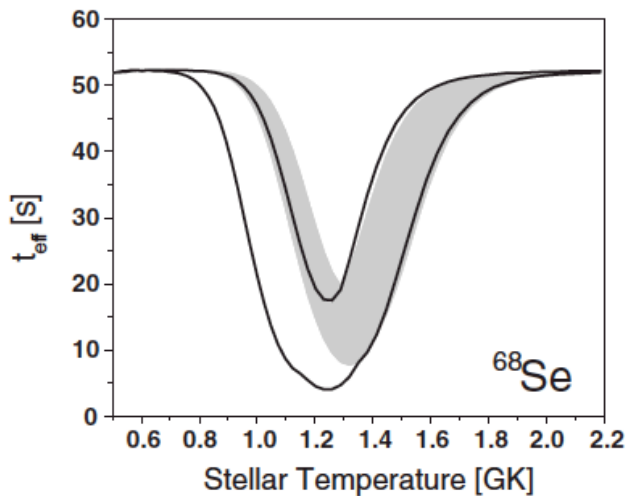
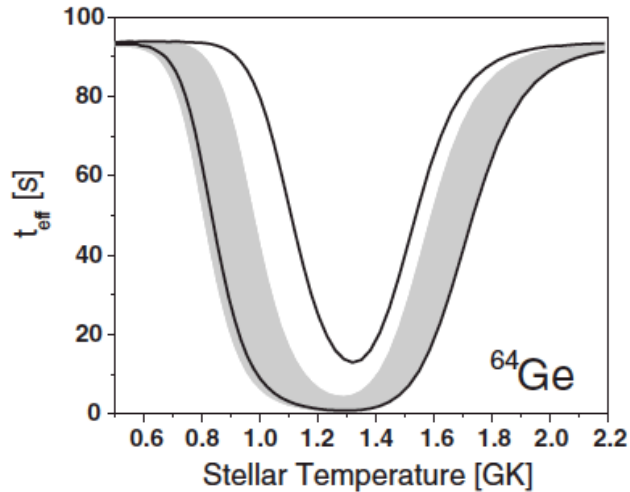
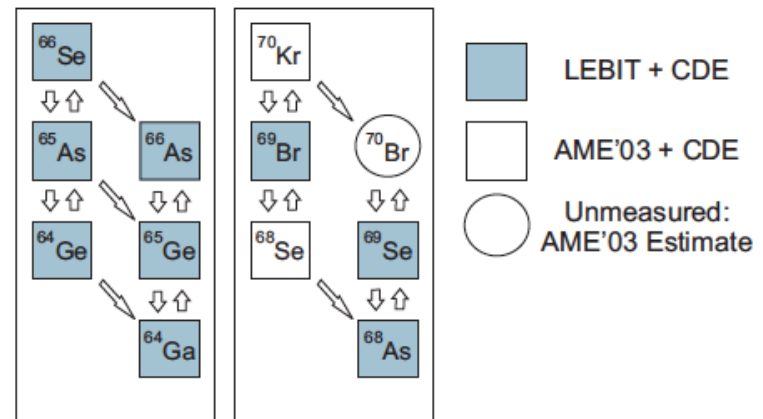


TABLE VI. Reaction Q-values derived from mass measurements and Coulomb displacement energies reported herein and derived from AME'03 [31]. All values in keV.

Reaction	This work	AME'03
$^{64}\text{Ge}(p,\gamma)^{65}\text{As}$	-255(104)	-354(172) ^a
$^{65}\text{As}(p,\gamma)^{66}\text{Se}$	2350(200)	2433(246)
$^{68}\text{Se}(p,\gamma)^{69}\text{Br}$	-679(119) ^b	-463(129) ^b
$^{69}\text{Br}(p,\gamma)^{70}\text{Kr}$	2450(216)	2234(227)



Rauscher & Thielemann ADNDT 75, 1 (2000)

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Removal of Spurious Center-of-Mass Excitations

$\rho(E, J, 0+2)$: total density in a model space including all $0+2$ h.o. excitations

$\rho_{nsp}(E, J, 0+2)$: center-of-mass excitations removed

$$\rho_{nsp}(E, J = 2, 0 + 2) =$$

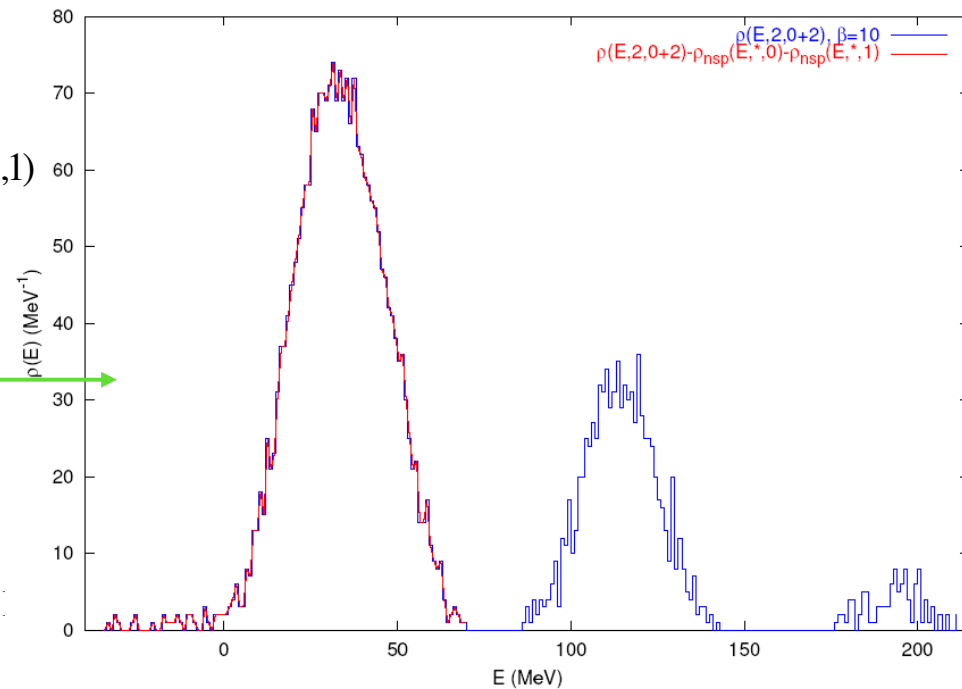
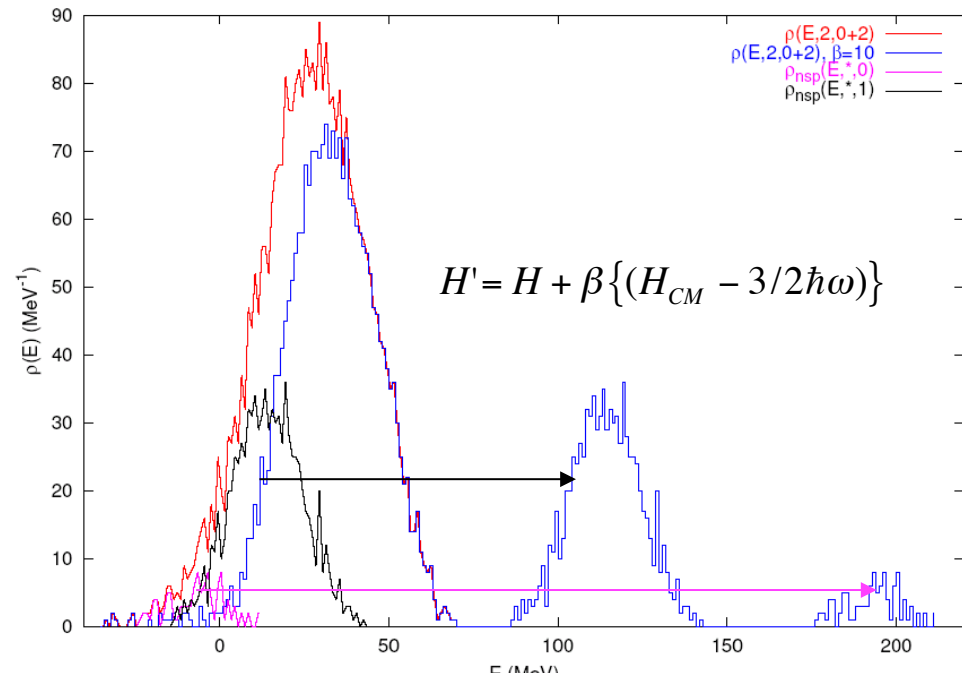
$$\rho(E, 2, 0 + 2) - \sum_{\substack{J_K=0 \\ \text{step 2}}}^2 \sum_{J' = |2 - J_K|}^{2 + J_K} \rho_{nsp}(E, J', 0) - \sum_{J'=1}^3 \rho_{nsp}(E, J', 1)$$

^{10}B : 10 particles in s - p - sd - pf shell model space

Horoi and Zelevinsky, PRL **98**, 265503 (2007)

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Fixed spin and parity nuclear level density for restricted shell model configurations

Fixed J
Restricted
Configuration
Widths

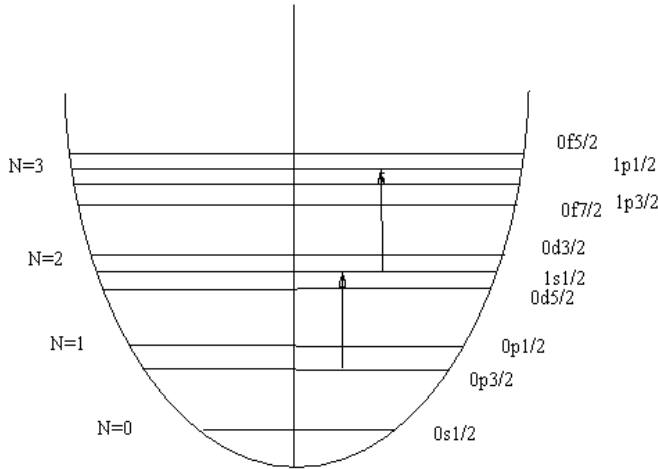
Mihai Horoi,¹ Monica Ghita,¹ and Vladimir Zelevinsky^{2,3}

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²National Superconducting Cyclotron Laboratory, East Lansing, Michigan 48824, USA

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(Received 14 January 2004; published 30 April 2004)



$$\begin{aligned} \langle\langle H^2 \rangle\rangle_{\vec{m}nMT_z} (m' = m) &\equiv \langle\langle H | m \rangle\langle m | H \rangle\rangle \\ &= \sum_i \epsilon_i^2 D^i(\vec{m}nMT_z) + \\ &\quad \sum_{i < j} [2\epsilon_i \epsilon_j + 2(\epsilon_i + \epsilon_j) V_{ijij} + V_{ijij}^2] D^{ij}(\vec{m}nMT_z) + \\ &\quad \sum_{(i < j) \neq l} [2V_{liil} V_{ljjl} + 2\epsilon_l V_{ijij}] D^{ijl}(\vec{m}nMT_z) + \\ &\quad \sum_{(i < j) \neq (k < l)} [V_{ijij} V_{klkl}] D^{ijkl}(\vec{m}nMT_z) \end{aligned}$$

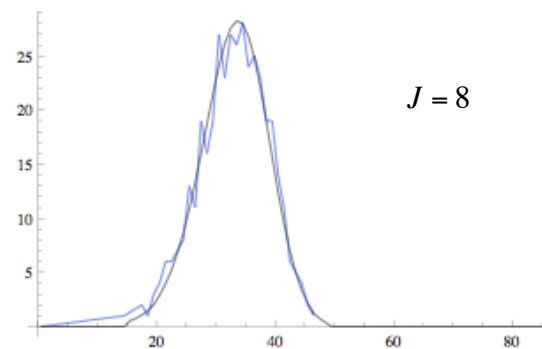
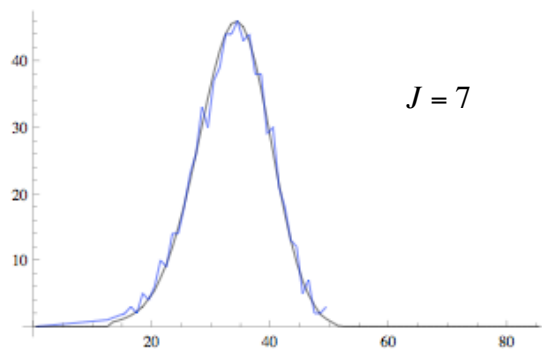
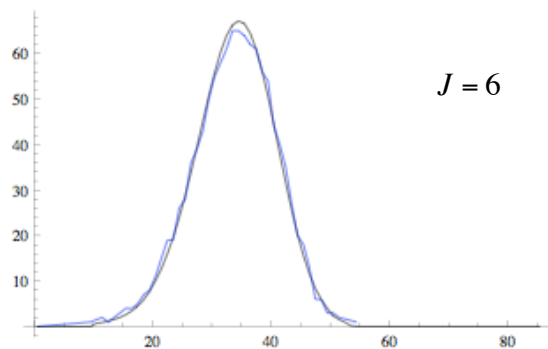
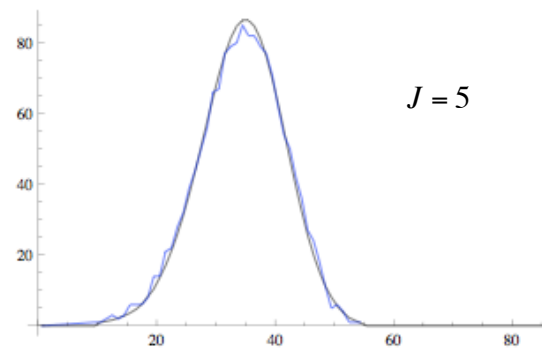
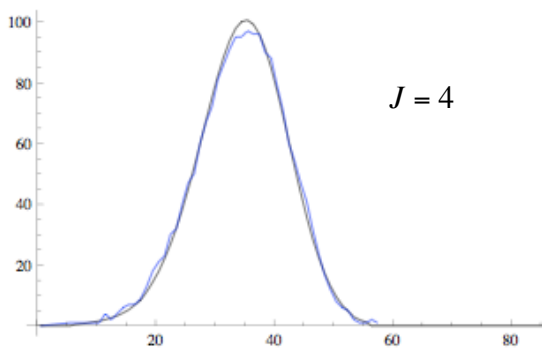
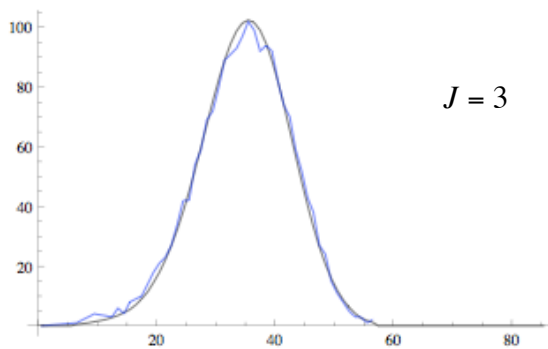
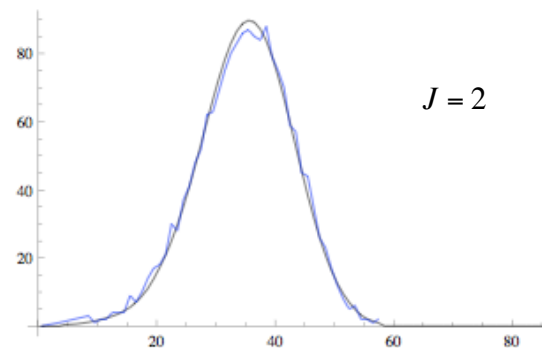
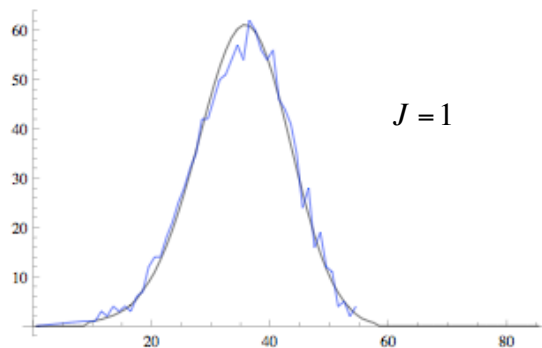
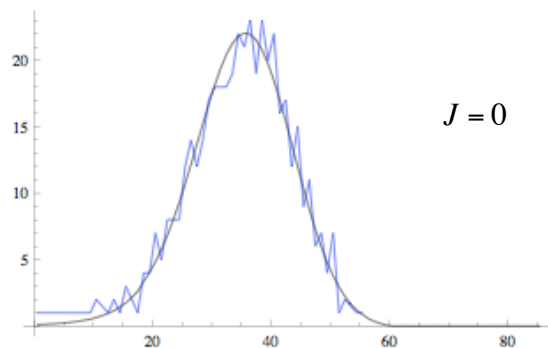
$$\begin{aligned} H^2 \rangle\rangle_{\vec{m}nMT_z} (\vec{m}' = \vec{m} : r^{-1} s^{+1}) &= \sum_i V_{riis}^2 D^{ir}(\vec{m}nMT_z) - \\ &\quad \sum_i V_{riis}^2 D^{irs}(\vec{m}nMT_z) + \\ &\quad \sum_{i \neq j} V_{riis} V_{rjjs} D^{ijs}(\vec{m}nMT_z) - \\ &\quad \sum_{i \neq j} V_{riis} V_{rjjs} D^{ijrs}(\vec{m}nMT_z) \end{aligned}$$

$$\begin{aligned} \langle\langle H^2 \rangle\rangle_{\vec{m}nMT_z} (\vec{m}' = \vec{m} : r^{-1} s^{-1} t^{+1} u^{+1}) &= V_{rstu}^2 D^{rs}(\vec{m}nMT_z) - \\ &\quad V_{rstu}^2 D^{rst}(\vec{m}nMT_z) - \\ &\quad V_{rstu}^2 D^{rsu}(\vec{m}nMT_z) + \\ &\quad V_{rstu}^2 D^{rstu}(\vec{m}nMT_z) \end{aligned}$$

$$D^{ij}, D^{ijr}, D^{ijrs}$$

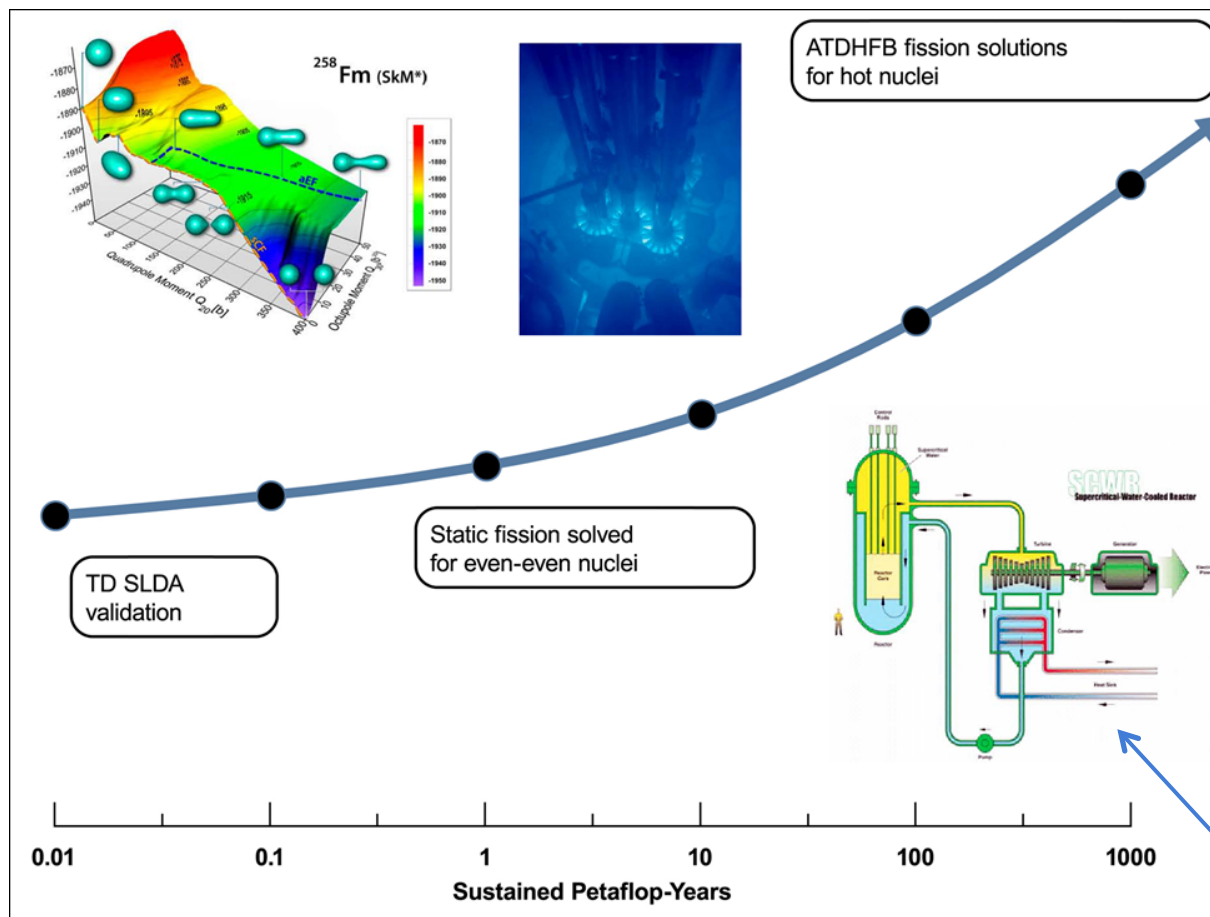
Moments method with restricted configurations

Si in sd model space: only 4p-4p configurations included



Extreme Computing Goals

<http://extremecomputing.labworks.org/nuclearphysics/report.stm>



Reaction DB TENDL-2009: www.talys.eu → [MCNPX: mcnpx.lanl.gov](http://mcnpx.lanl.gov)

Summary and Outlook

- ✓ Shell model techniques describe and predict a large number of data in medium and heavy nuclei.
- ✓ BSFG and mean-field models describe the existing data within a factor of 10 (average 2), but they can calculate NLD for all nuclei.
- ✓ CI J-dependent NLD seem to be more accurate, but can only be obtained for a limited number of nuclei.
- ✓ J-dependent **moments** method reproduces very well the CI J-dependent NLD and could provide accurate NLD for a much larger class of nuclei.
- ✓ Work to integrated SM NLD with reaction codes is in progress.