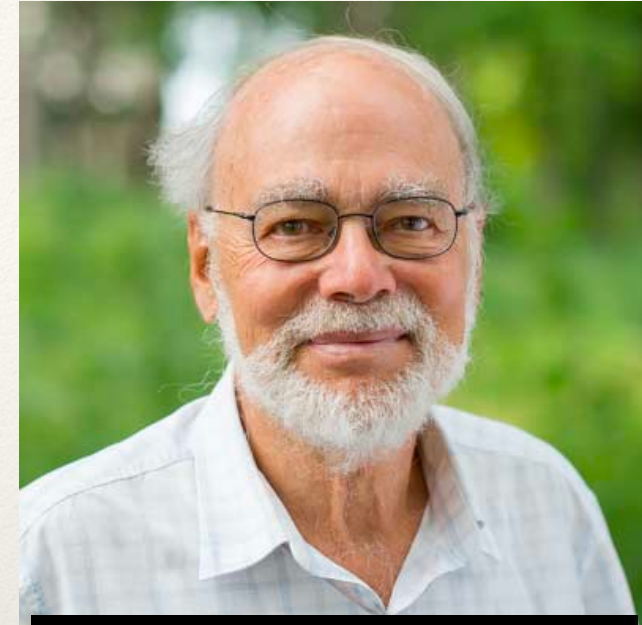


# Quantum Monte Carlo and Nuclear Physics: Steven C. Pieper

- Early days of Nuclear QMC
- Ground States
- EM form factors and transitions
- Excited States and Low Energy Scattering
- Neutron Matter and Drops
- Electron and Neutrino Scattering
- Outlook



AV18 celebration



Workshop on Nuclear and Dense Matter, May 3-6, 1977, Urbana, Illinois





## *Find in the picture*

- **Nobel Prize Winner**
- **Director of INT**
- **Former Secretary of DOE**
- **Professors from:**
  - Cornell, Illinois, MIT, Minnesota,**
  - Pisa, Stony Brook, ...**
- **Several other past, current  
leading scientists in NP**



## My First and last papers w / Steve

### Quantum Monte Carlo calculations of nuclei with $A \leq 7$

BS Pudliner, VR Pandharipande, J Carlson, SC Pieper, RB Wiringa  
Physical Review C 56 (4), 1720, 1997.

### Quantum Monte Carlo calculation of neutral-current $\nu$ - $^{12}\text{C}$ inclusive quasielastic scattering

A Lovato, S Gandolfi, J Carlson, E Lusk,  
SC Pieper, R Schiavilla, Physical Review C 97 (2), 022502, 2018



# ~26 Papers w/ S.C. Pieper

Nuclear Spectra with  $A \leq 7$

Pion exchange and three-nucleon forces

$A=8$  nuclei

Benchmark of  $A=4$

Excited states in  $A=6,8$

RMP 2015. (QMC methods for Nuclear Physics)

Tensor Forces and the Ground State of Nuclei

Neutron-alpha scattering

Nucleon and Pair-momentum distributions

Neutron Drops and Skyrme density Functionals

Cold neutrons trapped in external fields

Charge Form Factor and Sum Rules in  $^{12}\text{C}$

Neutral Weak currents in inclusive scattering from  $^{12}\text{C}$

EM and neutral-weak response functions of  $^{12}\text{C}$

Isovector spin-longitudinal and transverse response of nuclei

EM response of  $^{12}\text{C}$

Neutron Matter: A superfluid Gas



I was Vijay's Graduate Student at Urbana  
(VMC of  $A=3,4$ ; quark models)  
visited Bob at ANL to work with advanced computers  
Mal gave QMC lectures at Urbana

...

Bob and Vijay recruited Steve to QMC work  
I went to Courant (NYU) to work with Mal Kalos and Kevin Schmidt



Excellent combination of ingredients:

Good physics problem: interacting nucleons to nuclei

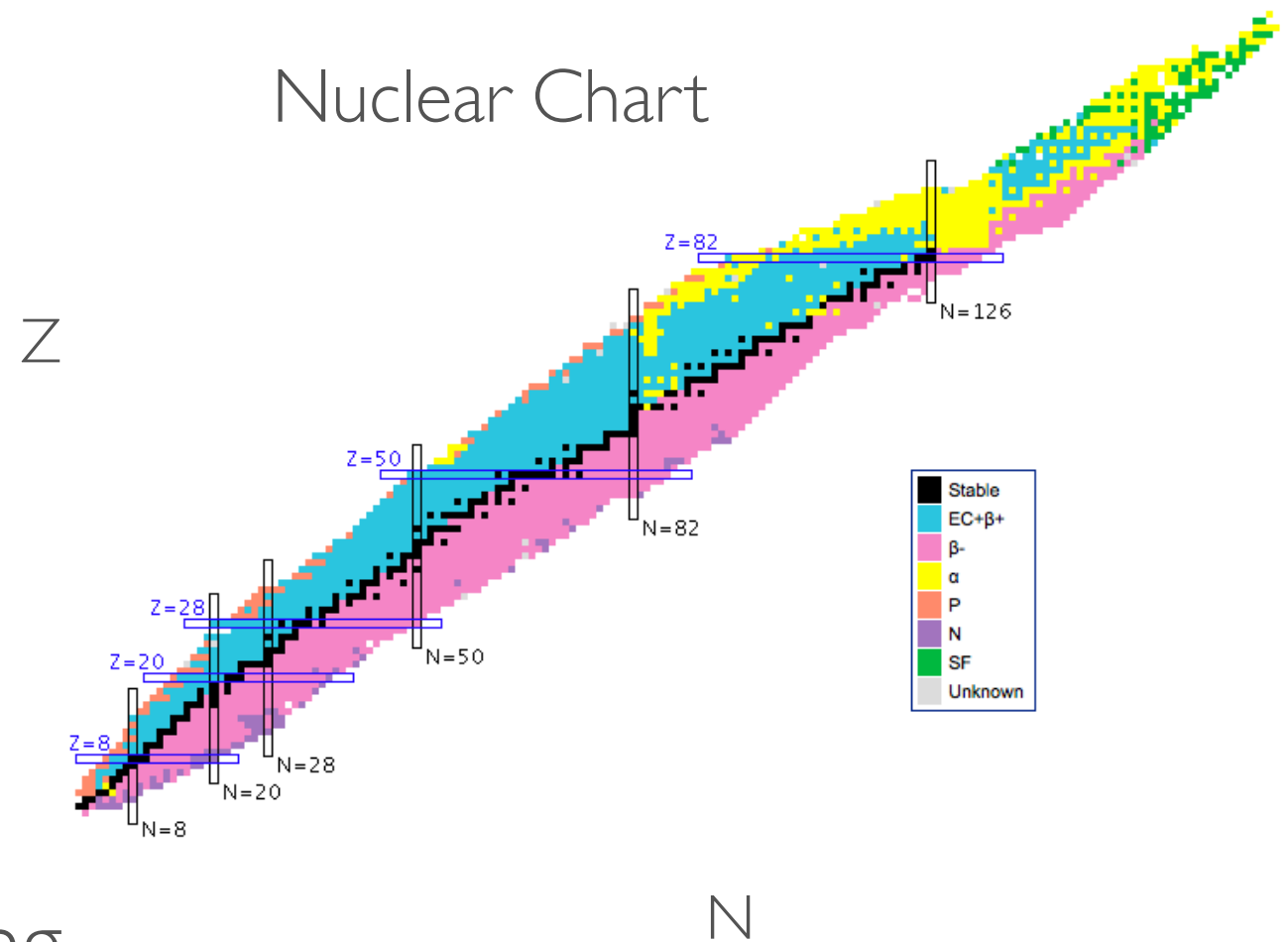
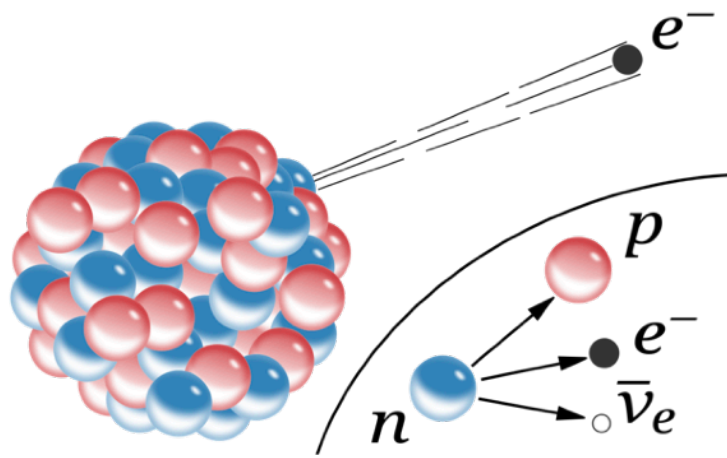
Rapidly advancing tools: from very primitive to very advanced computers

Excellent Collaborators: Steve Pieper



# Nuclei are a strongly correlated many-body systems

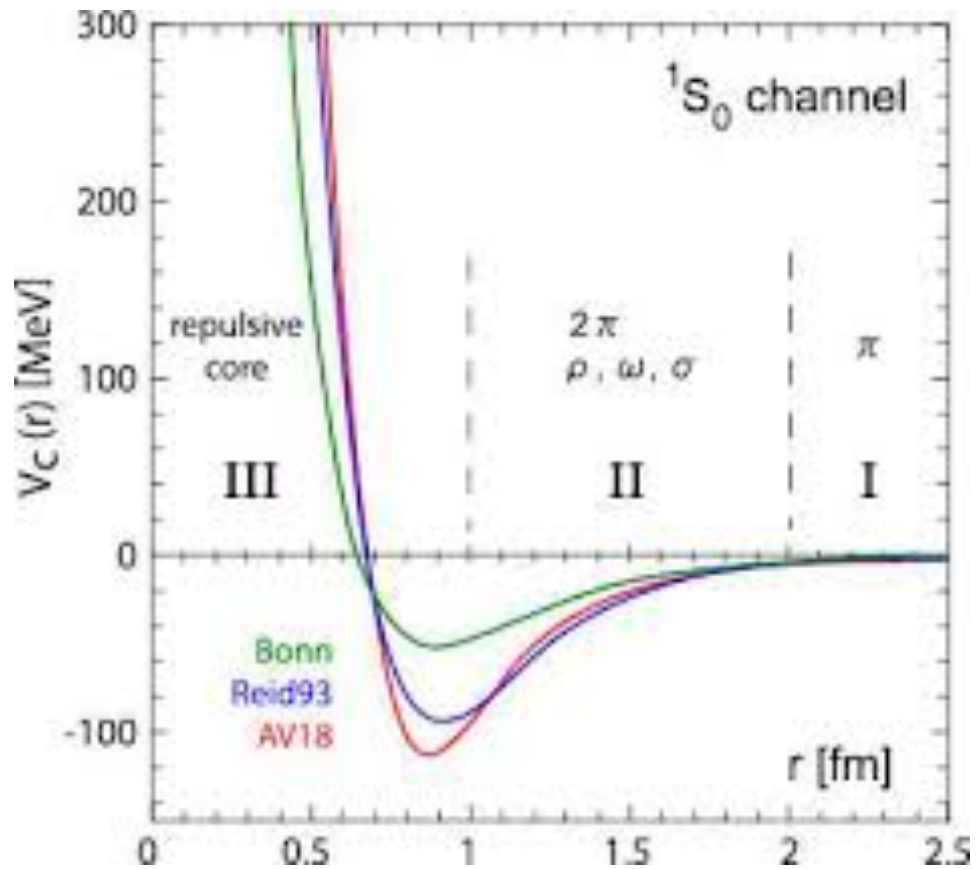
- nuclear binding, Equation of State
- Superfluidity, Beta Decay, ...



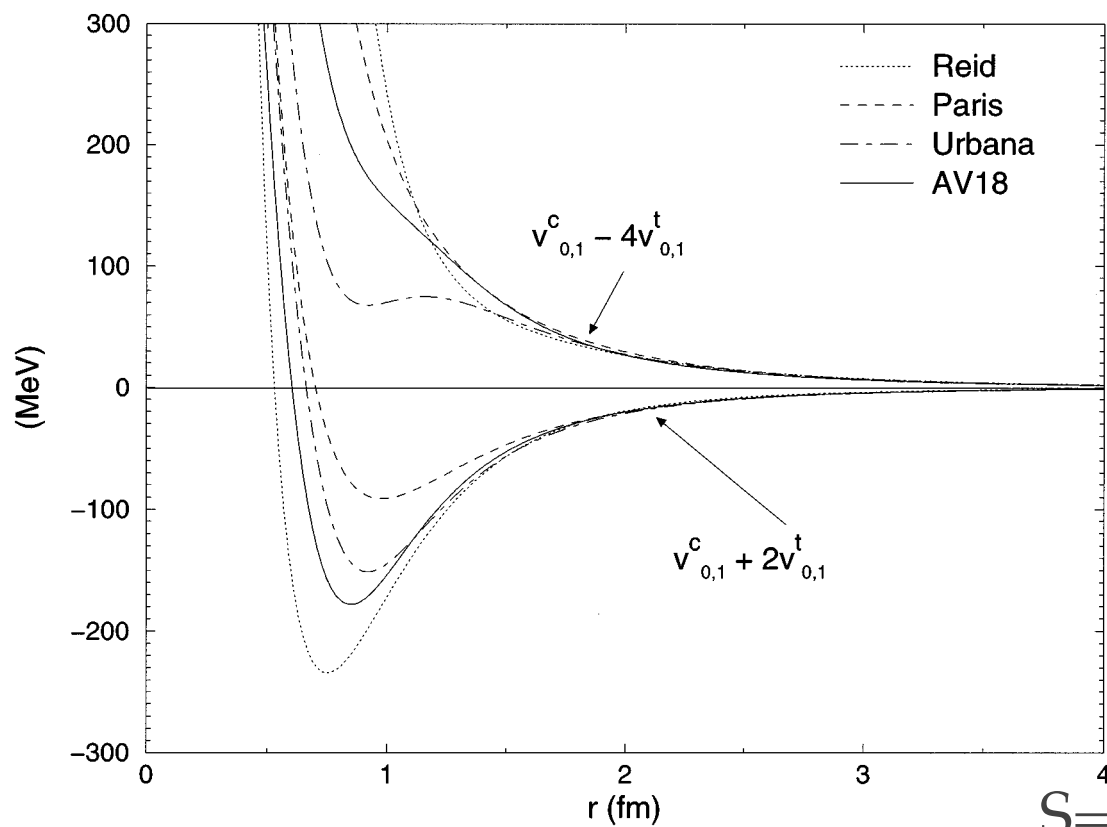
- nucleosynthesis (r-process)
- beta decays
- electron and neutrino scattering
- neutron stars
- fundamental symmetries (EDM, beta decay, etc.)



# Nuclei and Nuclear Interactions



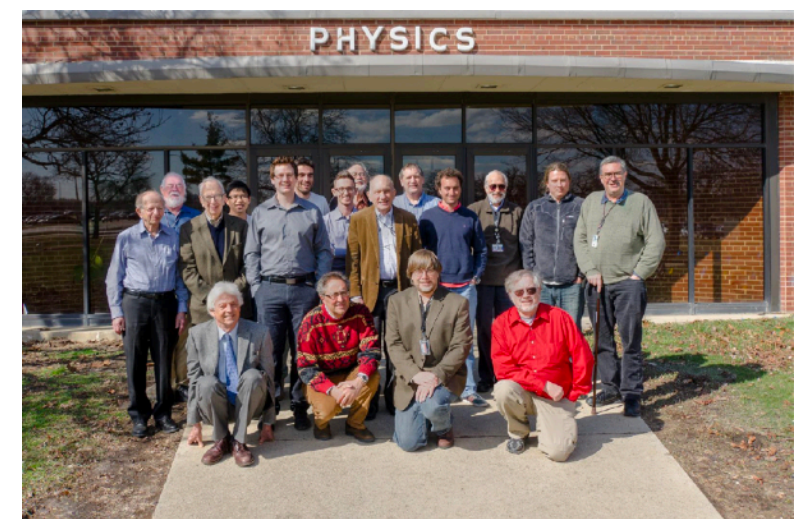
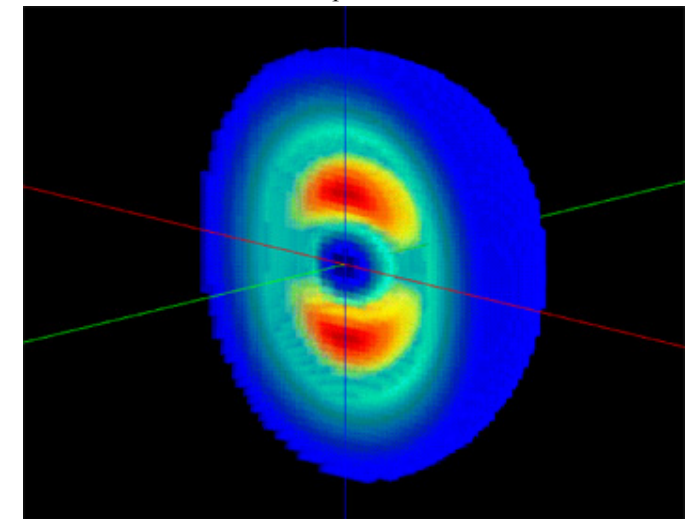
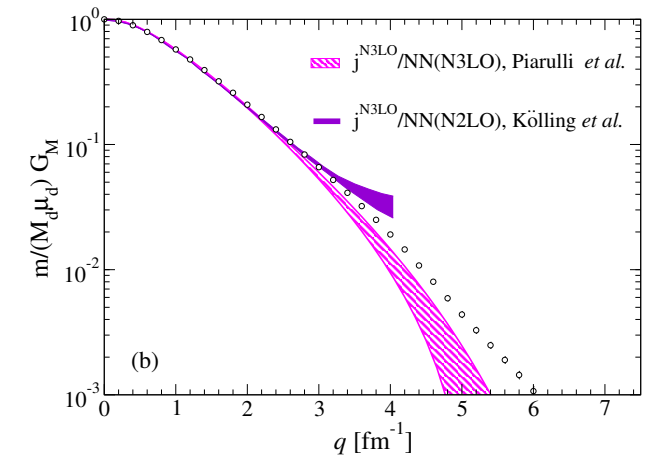
S=0 T=1 channel



S=1 T=0 channel



## Deuteron form factors





# Strongly Correlated Quantum Many-Body Physics

Must solve the quantum many-body problem: structure & dynamics

## *cautions:*

The Schrodinger equation cannot be solved accurately when the number of particles exceeds about 10. No computer existing, or that will ever exist, can break this barrier because it is a catastrophe of dimension ...

Pines and Laughlin (2000)

In general the many electron wave function  $\Psi$  ... for a system of N electrons is not a legitimate scientific concept [for large N]

Kohn (Nobel lecture, 1998)

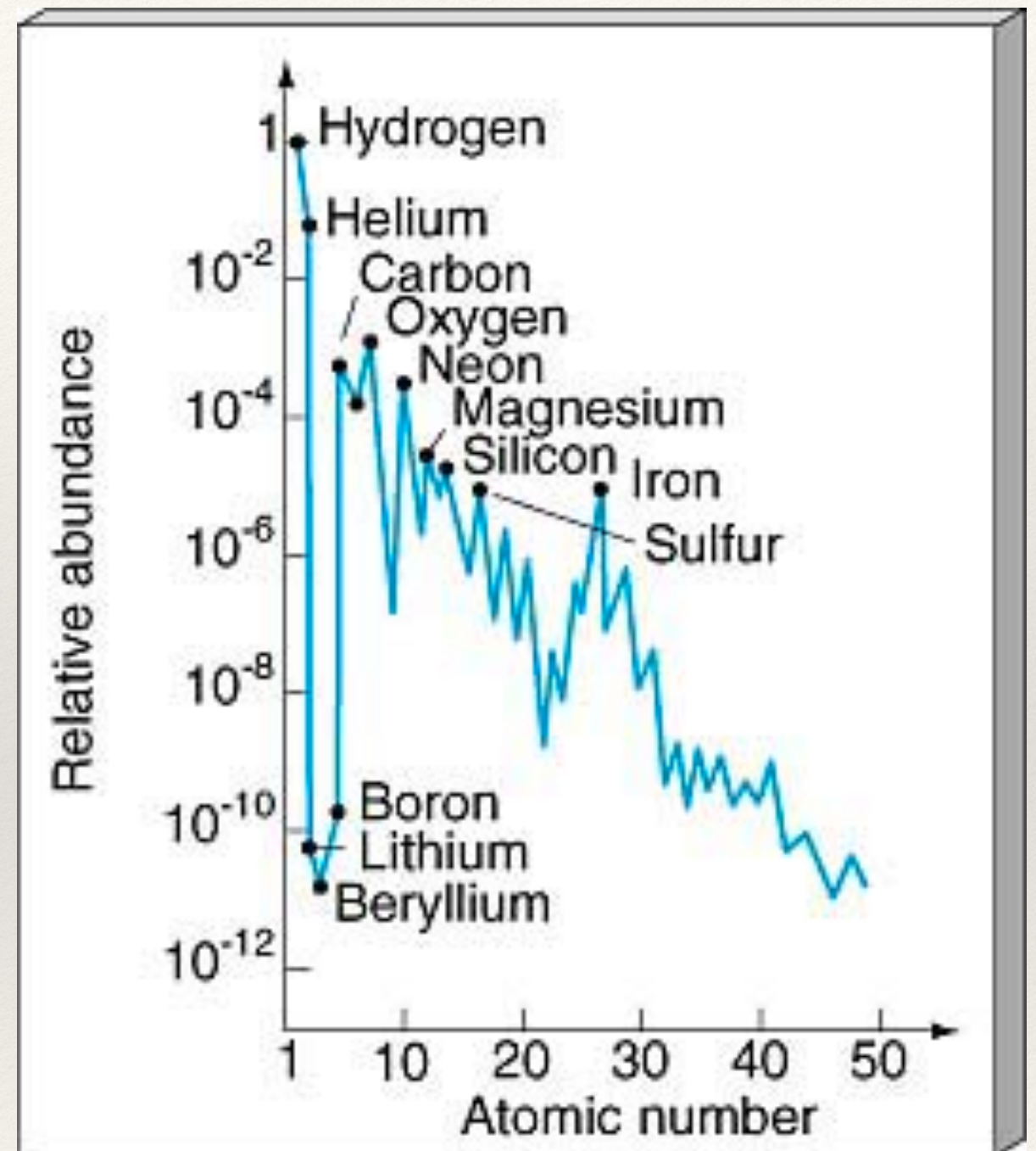
but often we do not need a complete description of the system: thermal properties, samples of path integral, cluster expansions,...

*Quantum Monte Carlo, Coupled Cluster and other many-body methods*



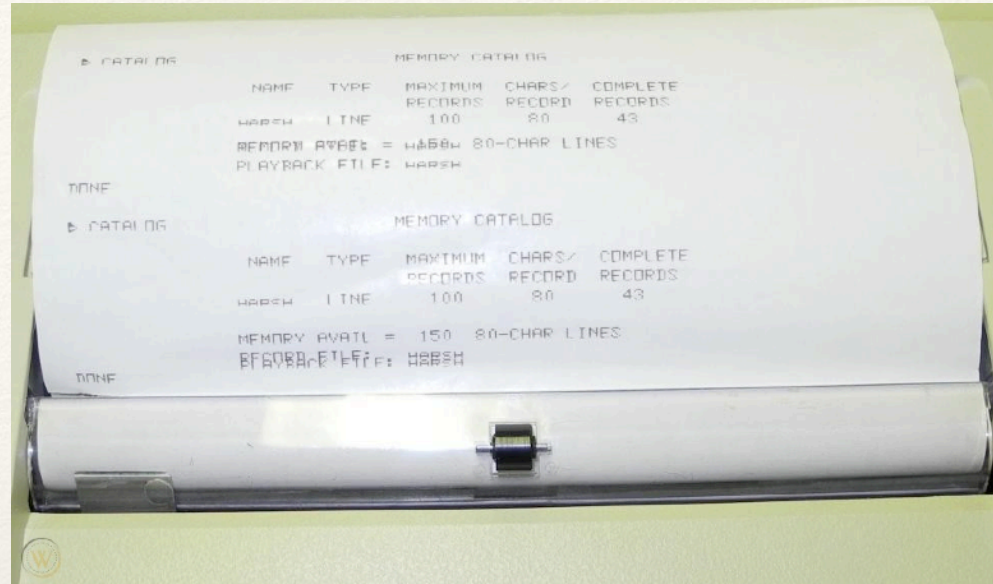
# Abundance of the Elements in the Galaxy

<b>Z</b>	<b>Element</b>	<b>Mass fraction (ppk)</b>
1	<a href="#">Hydrogen</a>	739
2	<a href="#">Helium</a>	240
8	<a href="#">Oxygen</a>	10
6	<a href="#">Carbon</a>	5
	<b>Total</b>	<b>99.4%</b>





# TI Silent 700 terminals



# Apple II for a terminal



# Cyber 175



THE VAST COMPUTATIONAL power of the large-scale Control Data CYBER 176 computer system in the company's Houston Data Center is monitored from this control console. The system is used primarily to run petroleum industry applications.



# *Courant Institute*

**1980s:**

## **The Ultracomputer**

Design Underway:  
Mal Kalos, Allan Gottlieb,  
and Jack Schwartz



*my time as a PD at Courant, with the group of Mal, Paula Whitlock, Jules Moskowitz, w/ frequent visitor Geoffrey Chester around this time*

*Kevin Schmidt returned to NYU as a chemistry professor*

- work on Liquid Helium, nuclear VMC calculations, low-energy nuclear scattering

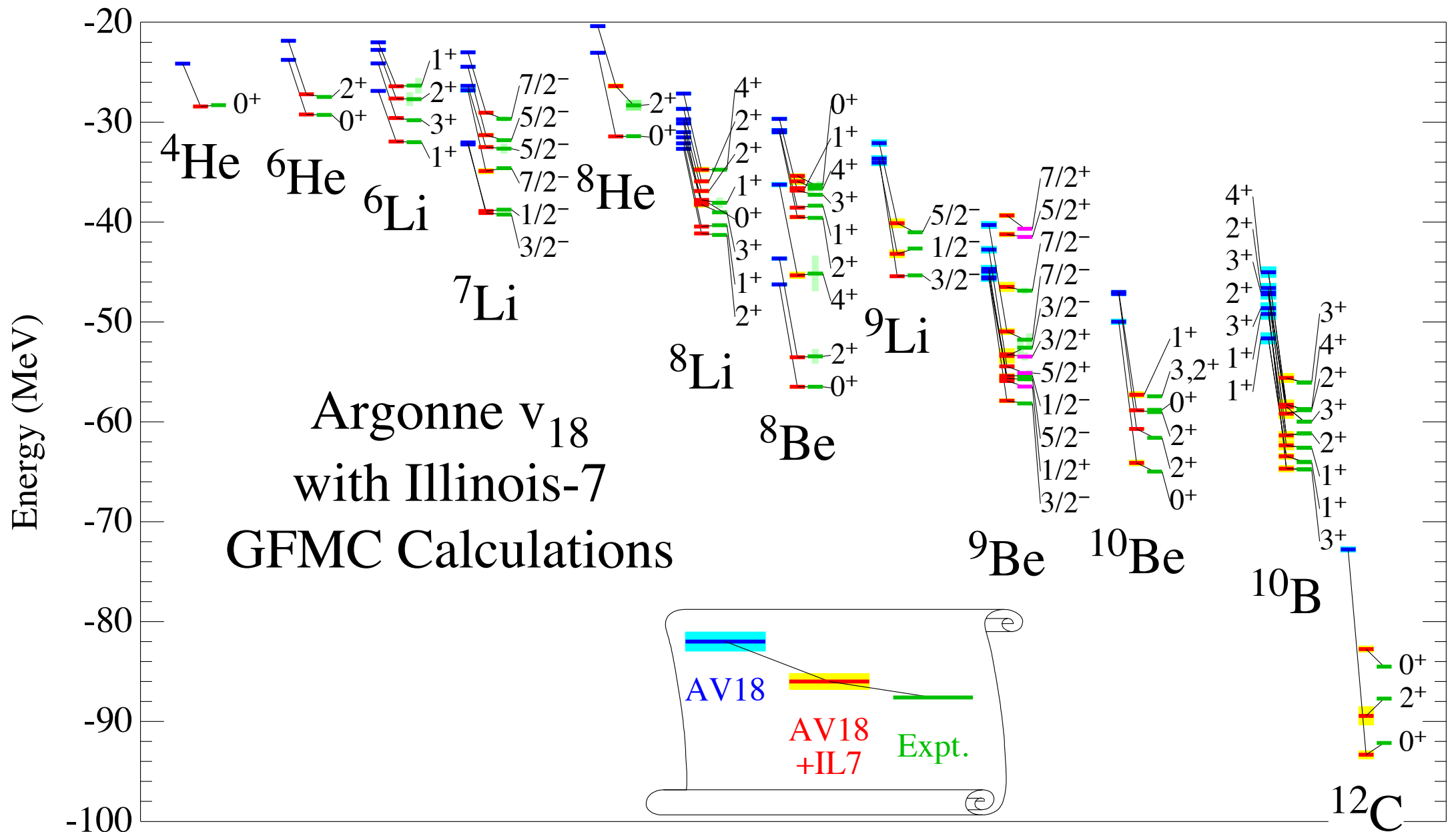


Over the years, much bigger and faster computers  
and the software to use them effectively



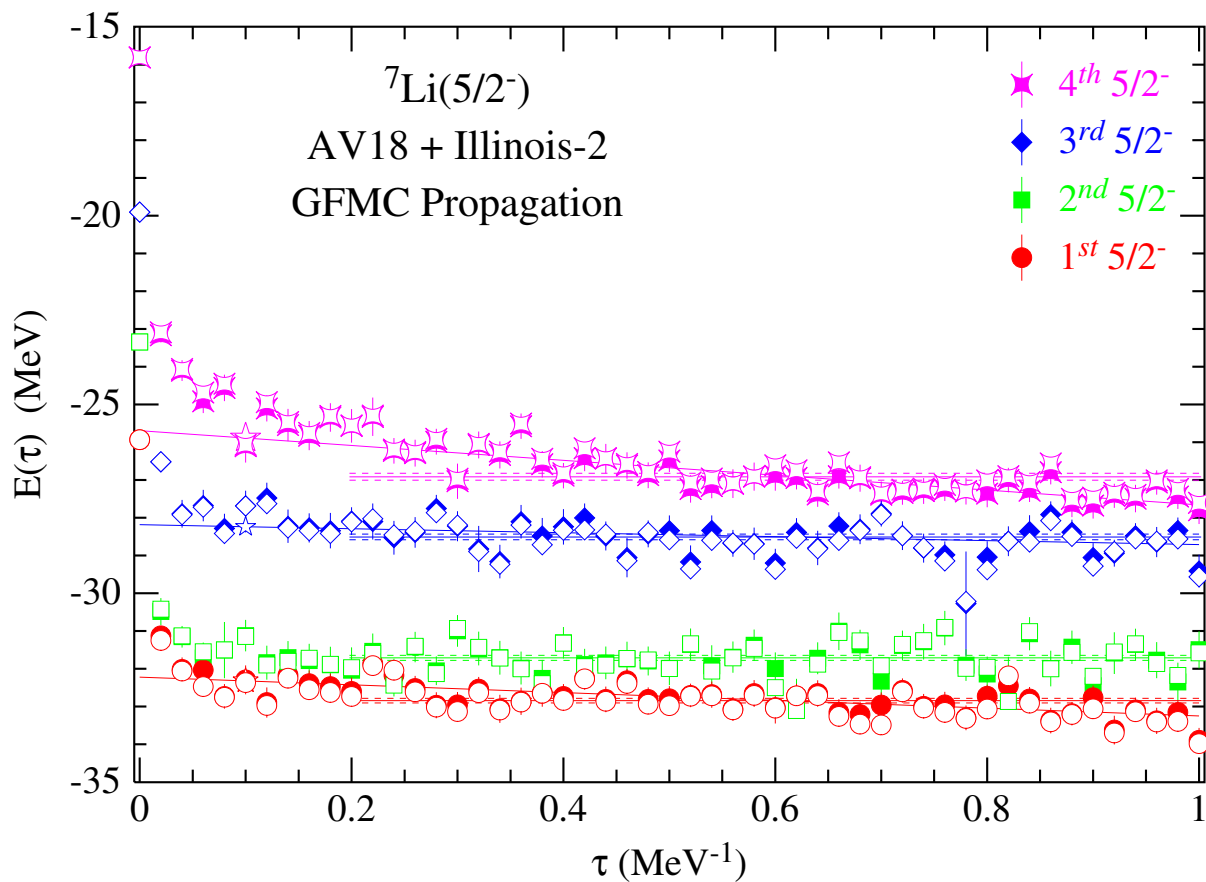


One 'Solution' to problem of dimension:  
solve up to  $A \sim 12$



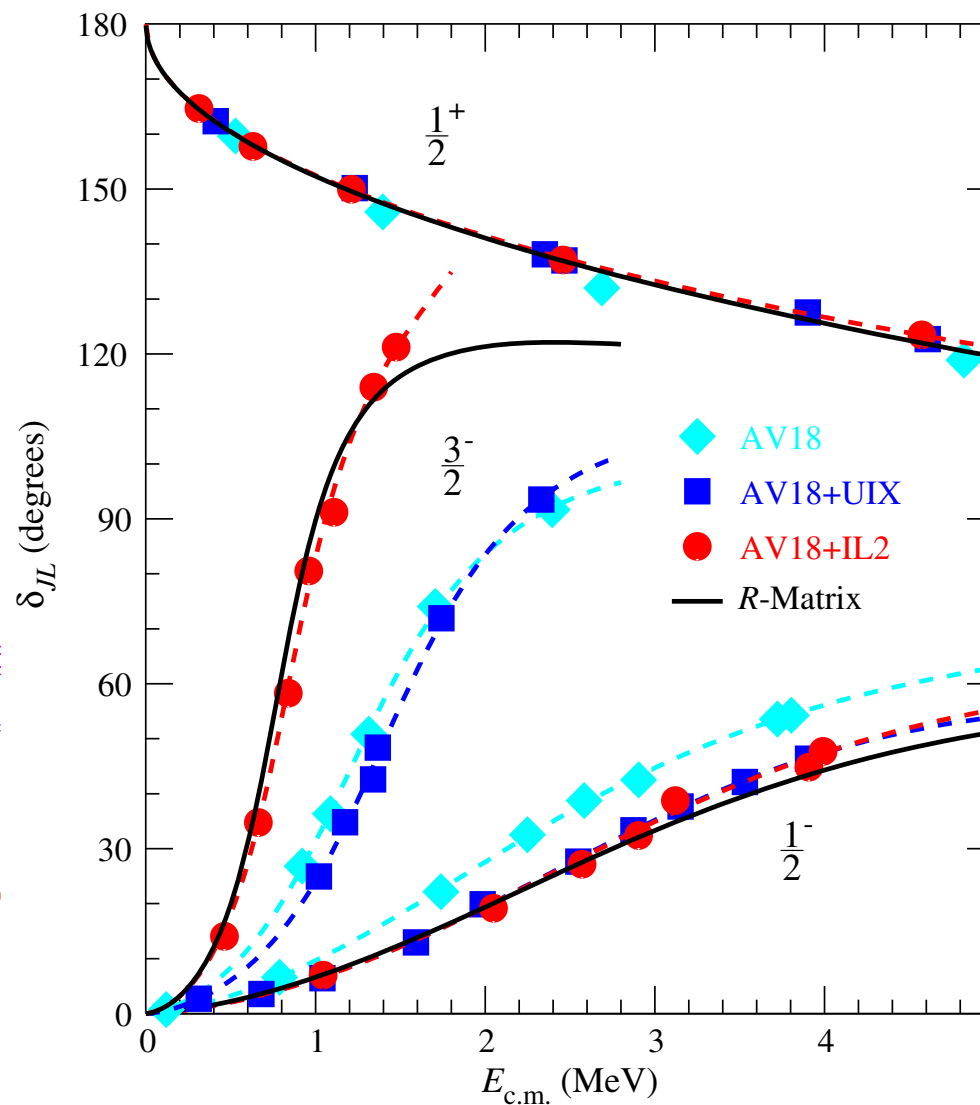


# Low Energy Scattering



States in  ${}^7\text{Li}$

width of resonance  
 correlated to  $E$  vs.  $\tau$   
 Pieper, et al, 2004



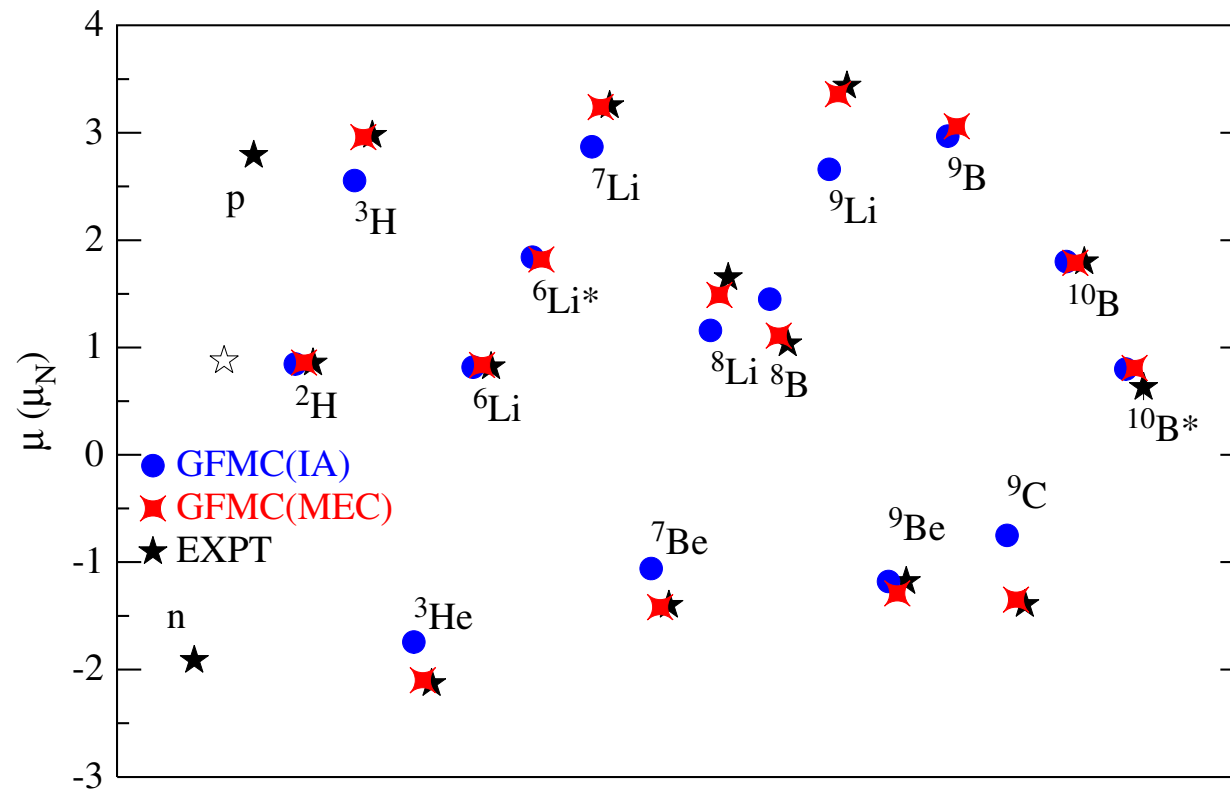
Neutron-alpha scattering  
 Nollett, et al., 2007





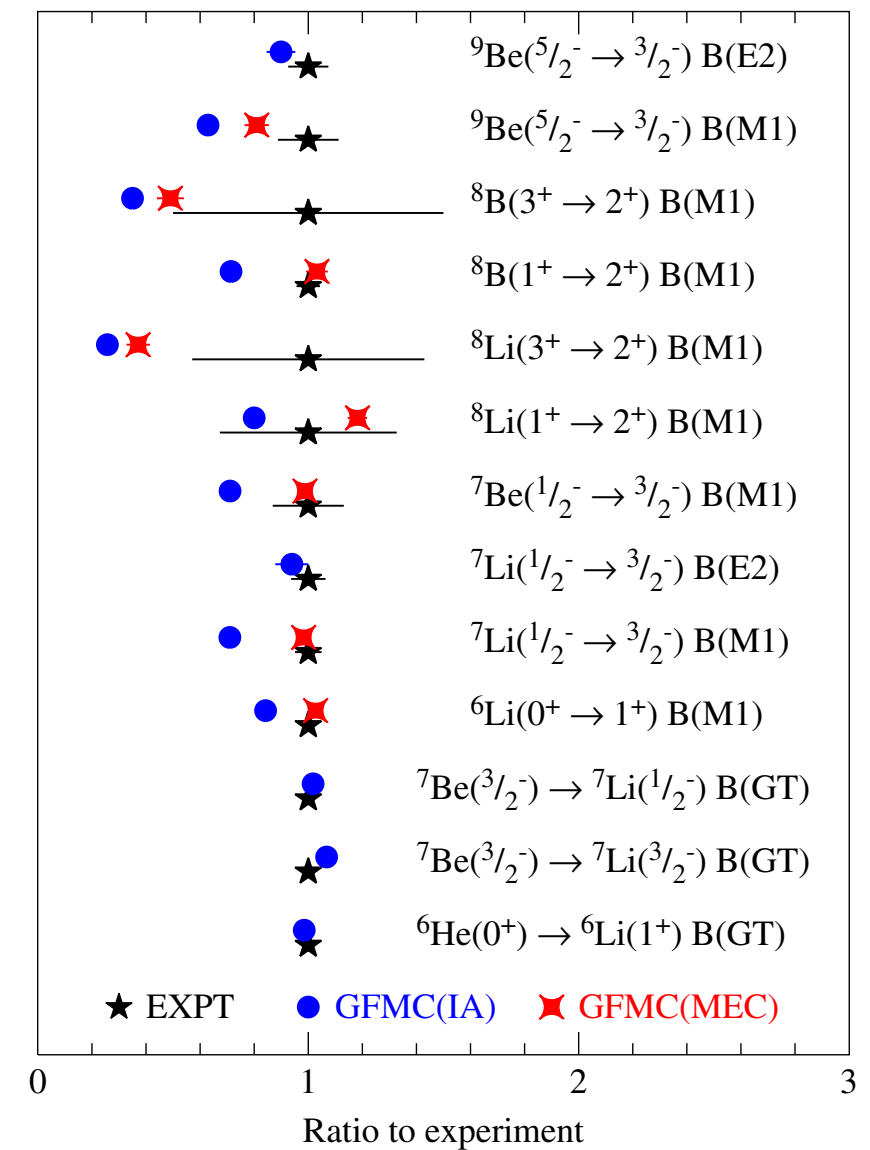
# Electroweak Observables: Transitions

## Magnetic Moments



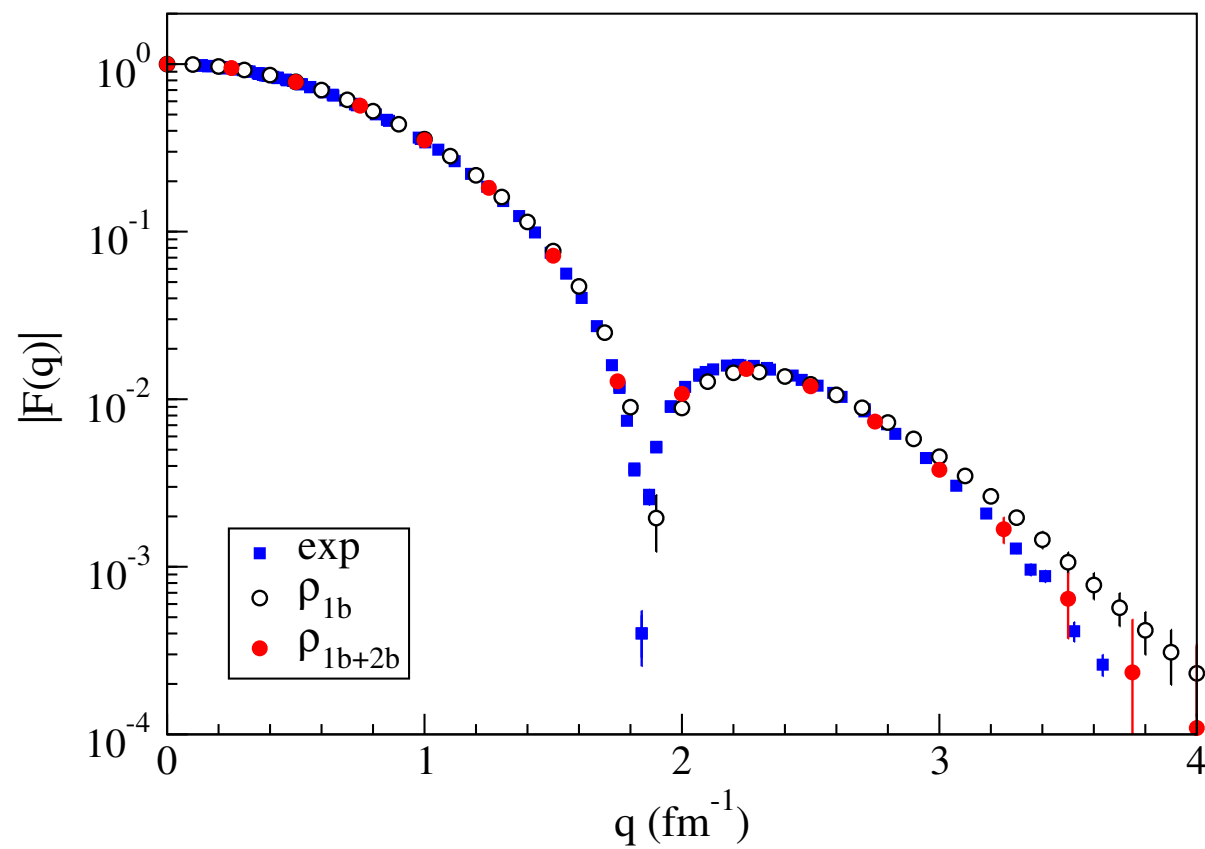
Pastore, et al., 2013 and 2014

## EM transitions

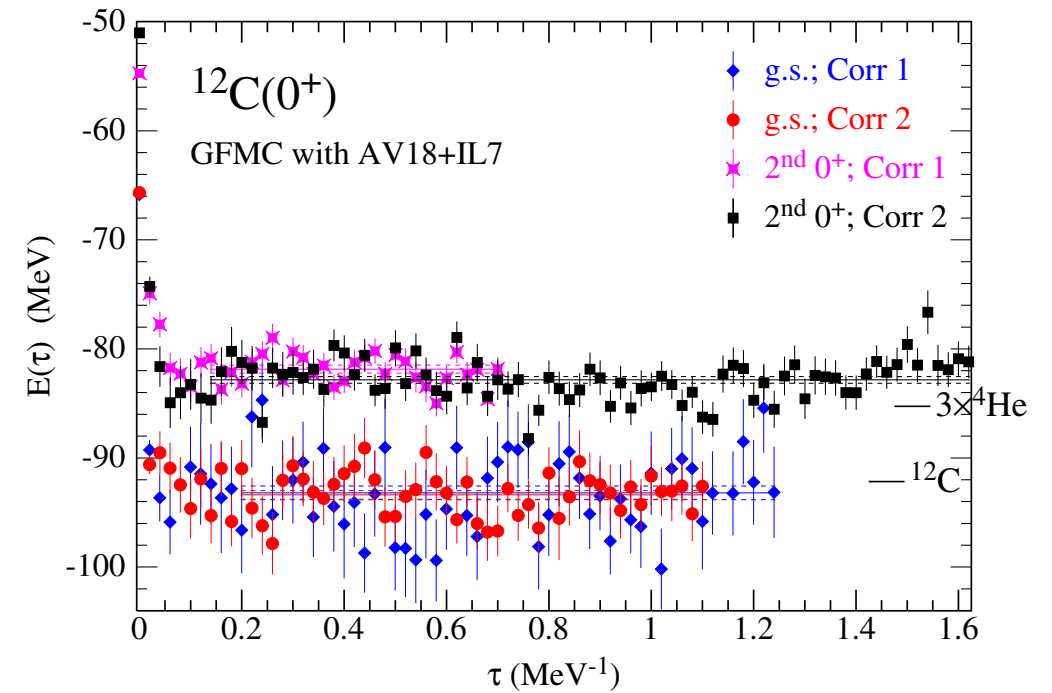




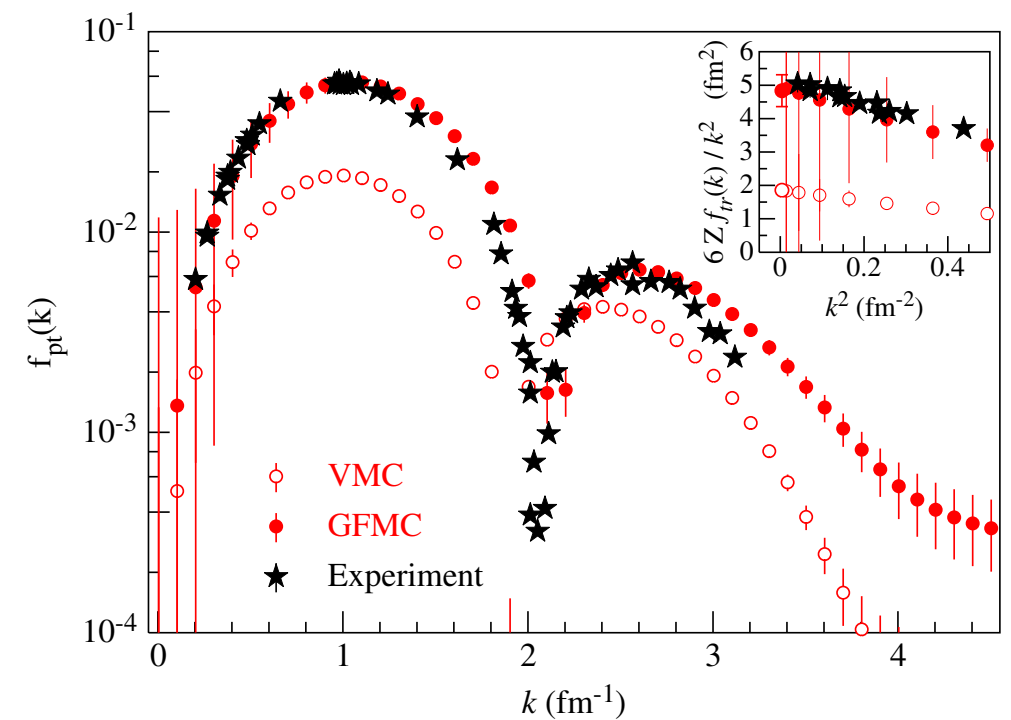
# Nuclear Properties: $A=12$ Ground and Hoyle States



$^{12}\text{C}$  charge form factor  
Lovato, et al., 2013



Hoyle State; Chernykh et al, 2010





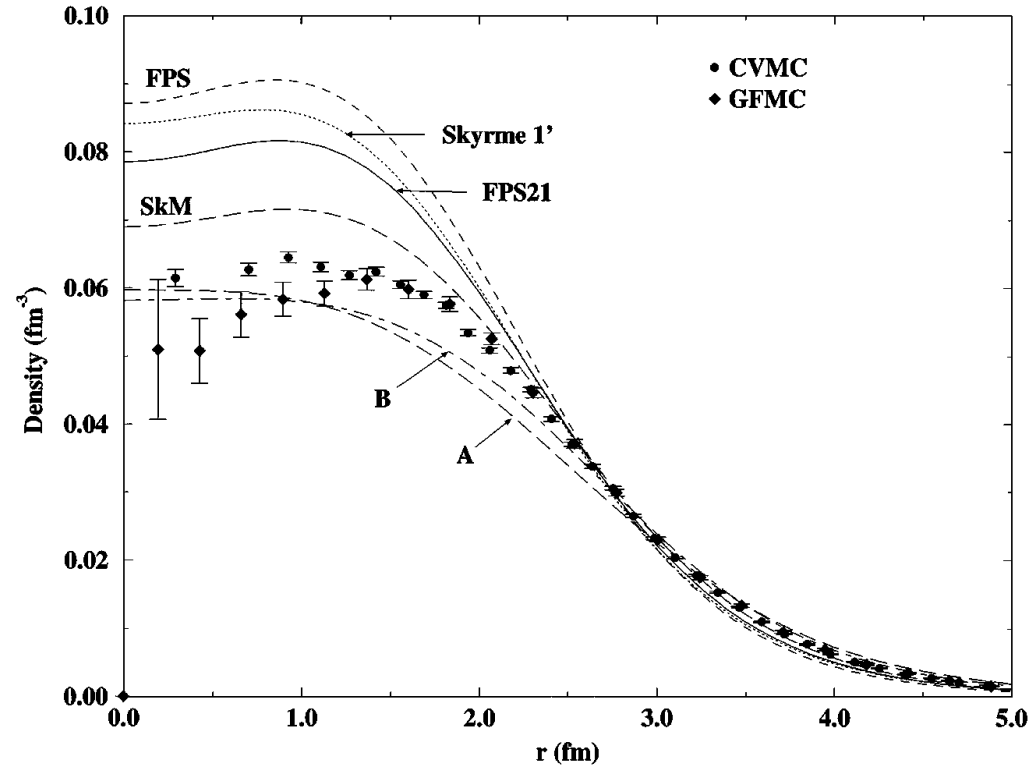
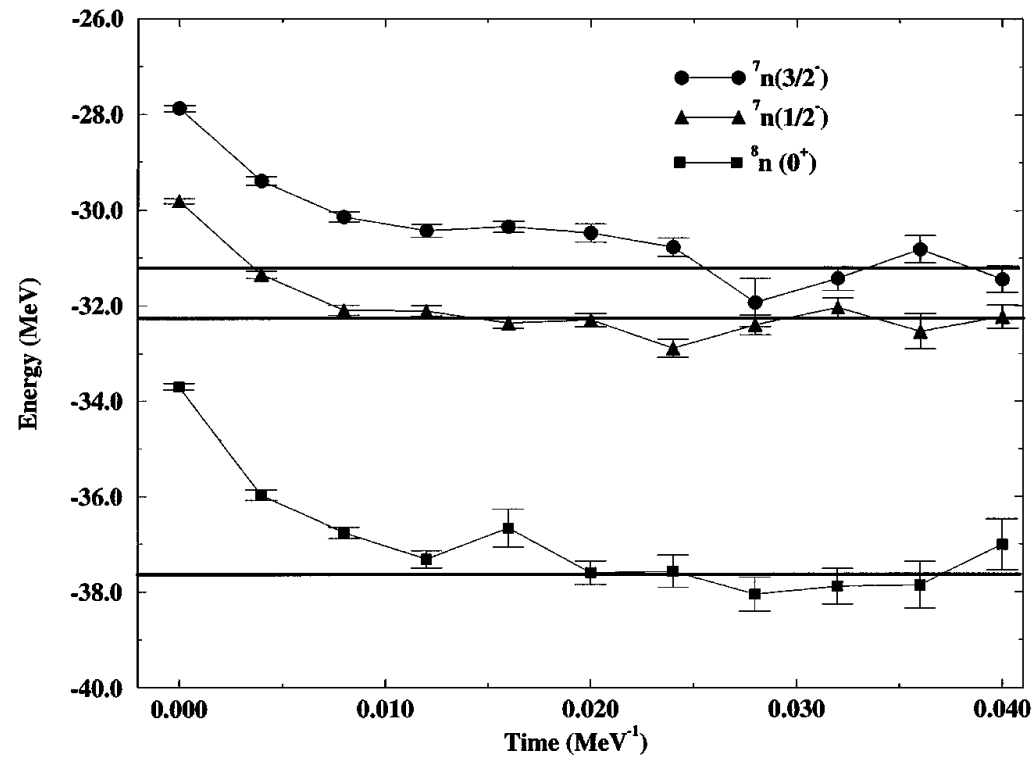
*Auxiliary Field QMC: Monte Carlo also for spins/isospins*

*Applications to larger nuclei:  $^{16}\text{O}$ ,... to neutron matter and stars  
hyperons in neutron star matter*

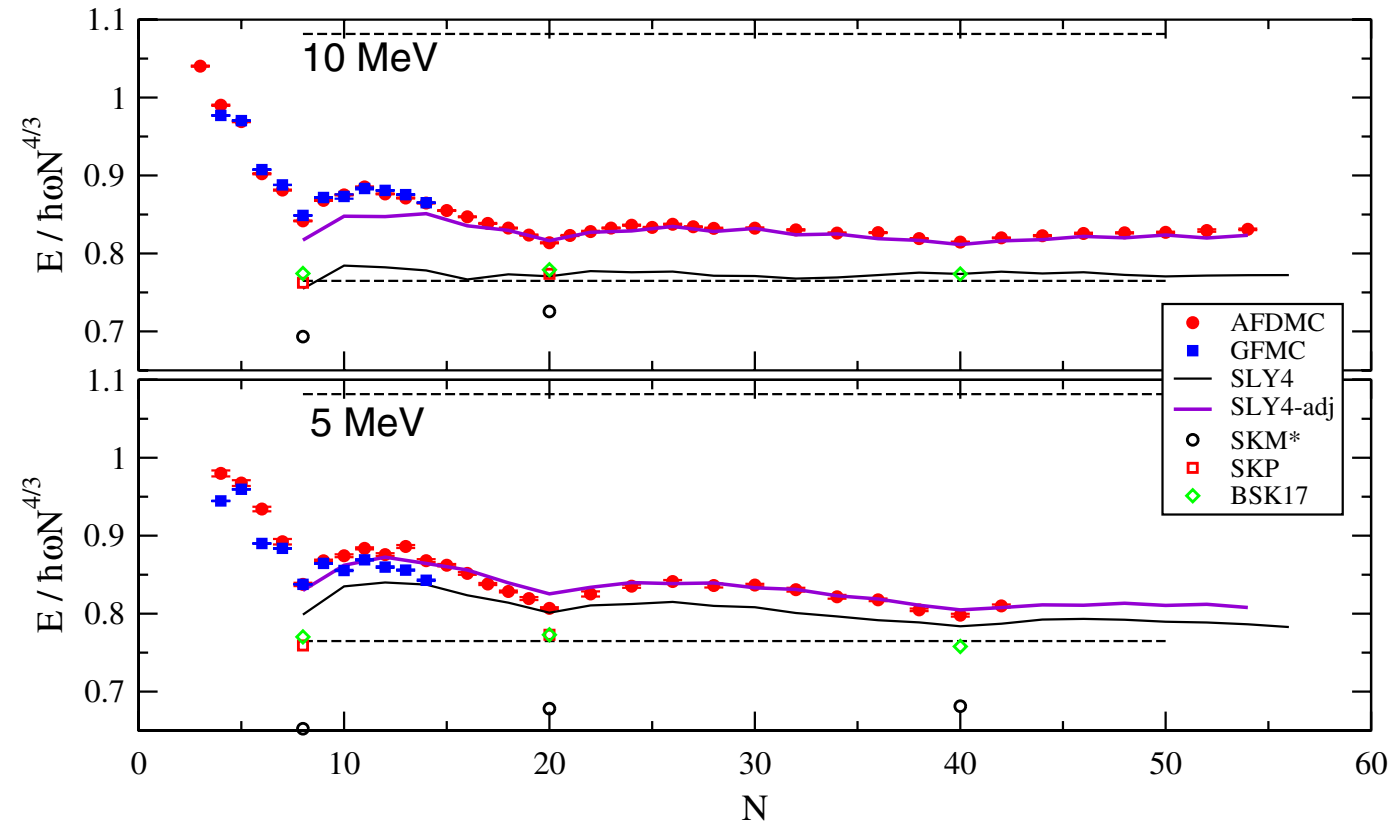




# Neutron Drops and Inhomogeneous Neutron Matter



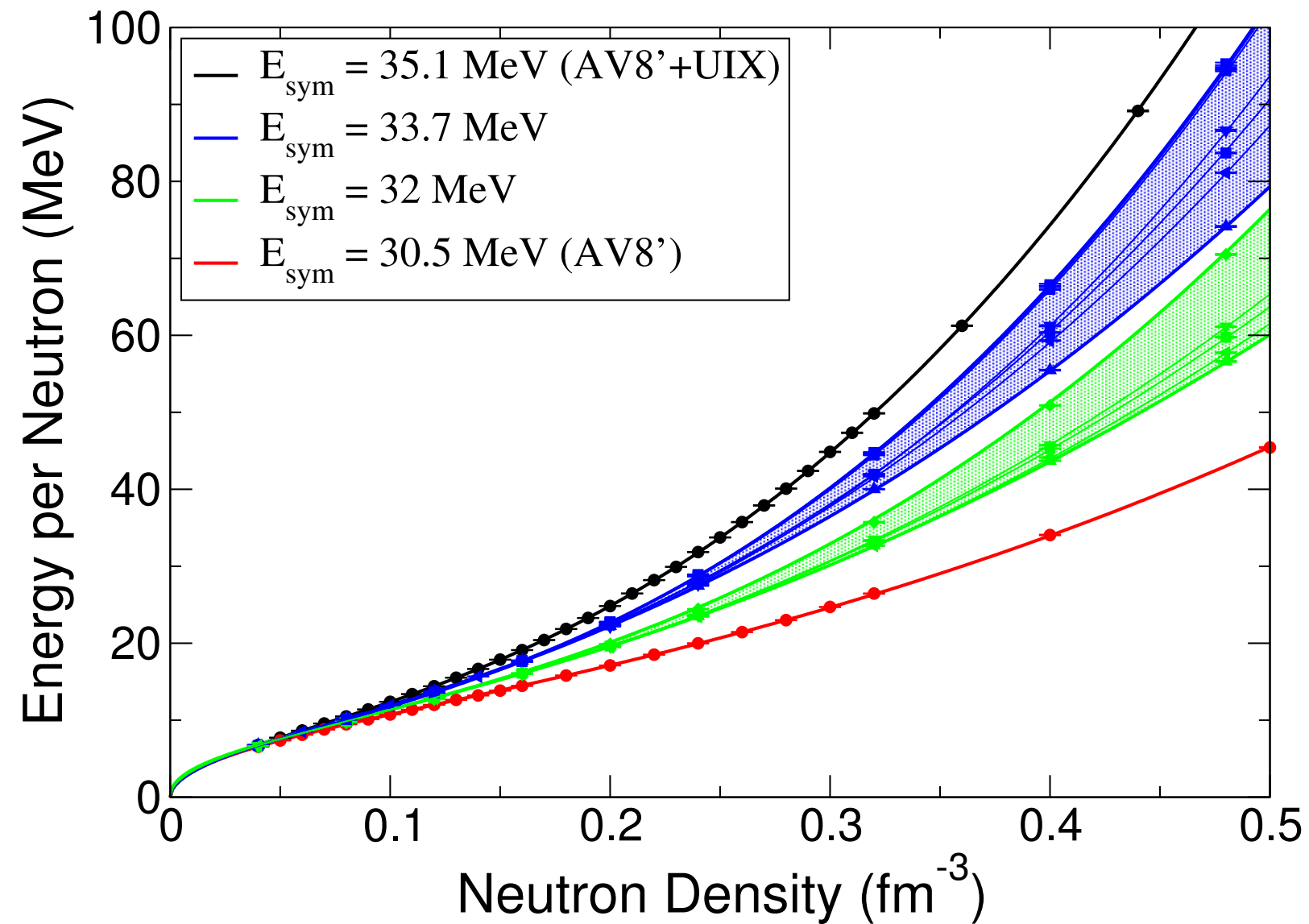
$n = 7, 8$ ; Pudliner, et al, PRL, 1996



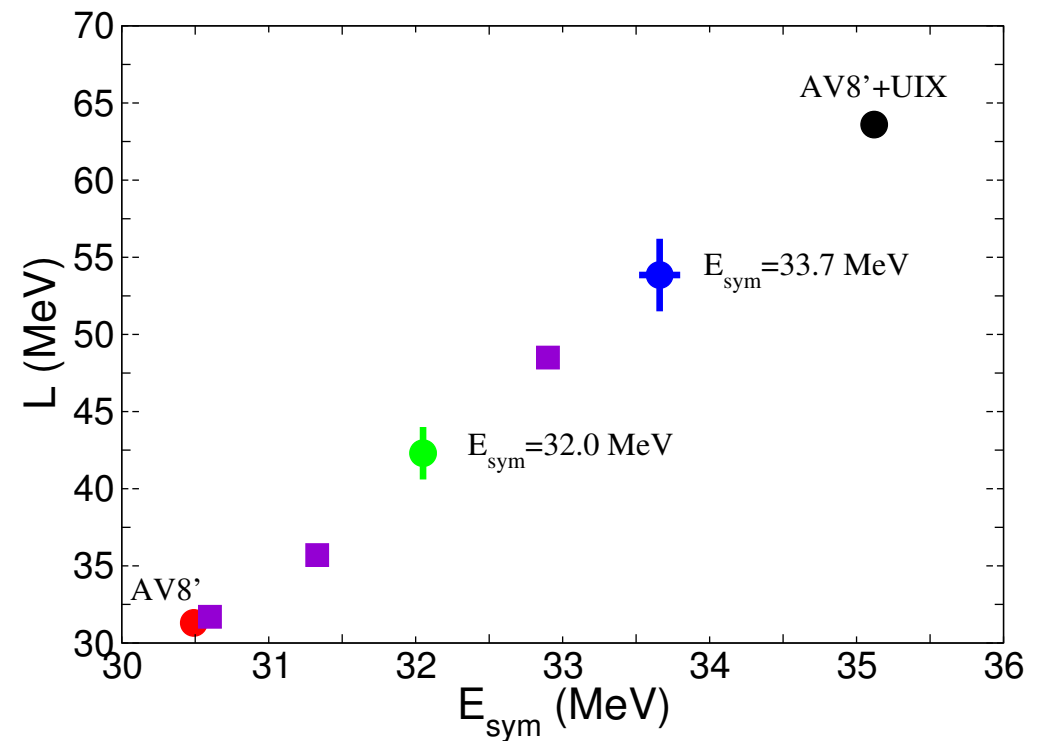
Cold Neutrons Trapped in External Fields; Gandolfi, et al, PRL 2011



# Neutron Matter at higher density and the Three Neutron Interaction



5.

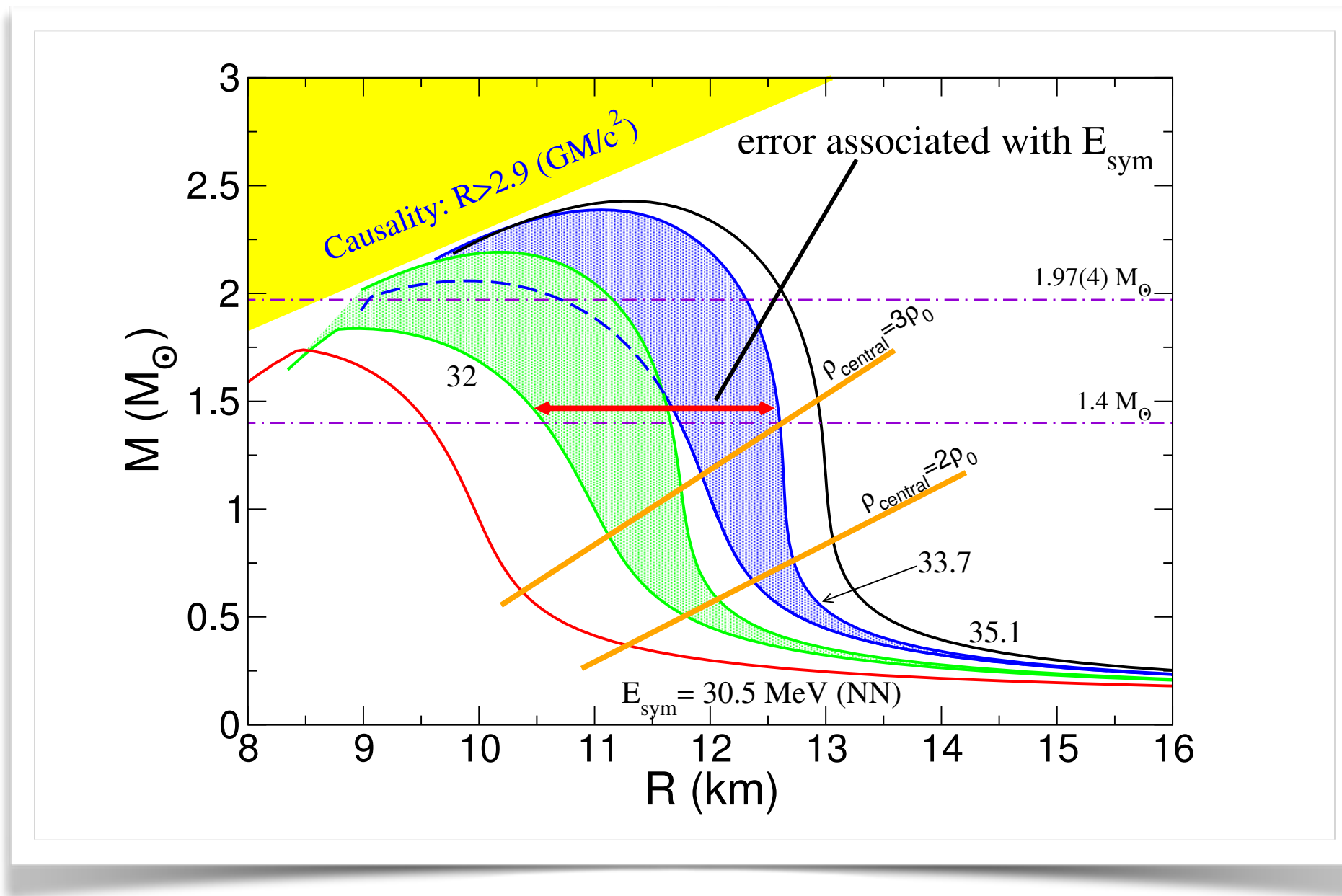


Gandolfi, JC, and Reddy (2012):  
arXiv 1101.1921

Strong Correlation between symmetry energy  
at saturation density and its slope

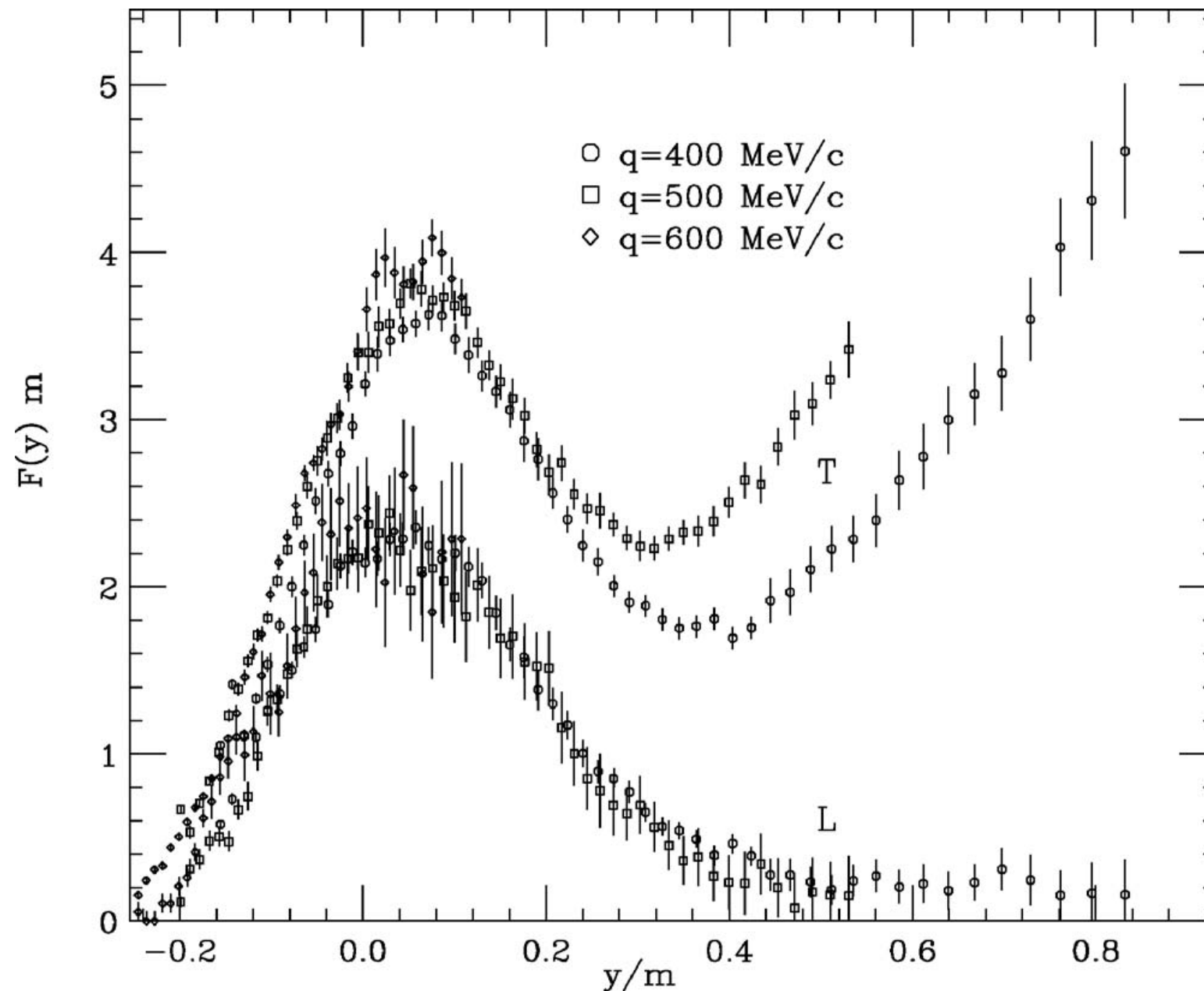


# Neutron Star Mass-Radius Relations testable with neutron star observations, advanced LIGO



How to further reduce uncertainty at 1-4 times saturation density?  
Where is the phase transition?

# Electron and Neutrino Scattering in the QuasiElastic Regime



Benhar, Day, Sick; RMP 2007

Observations:

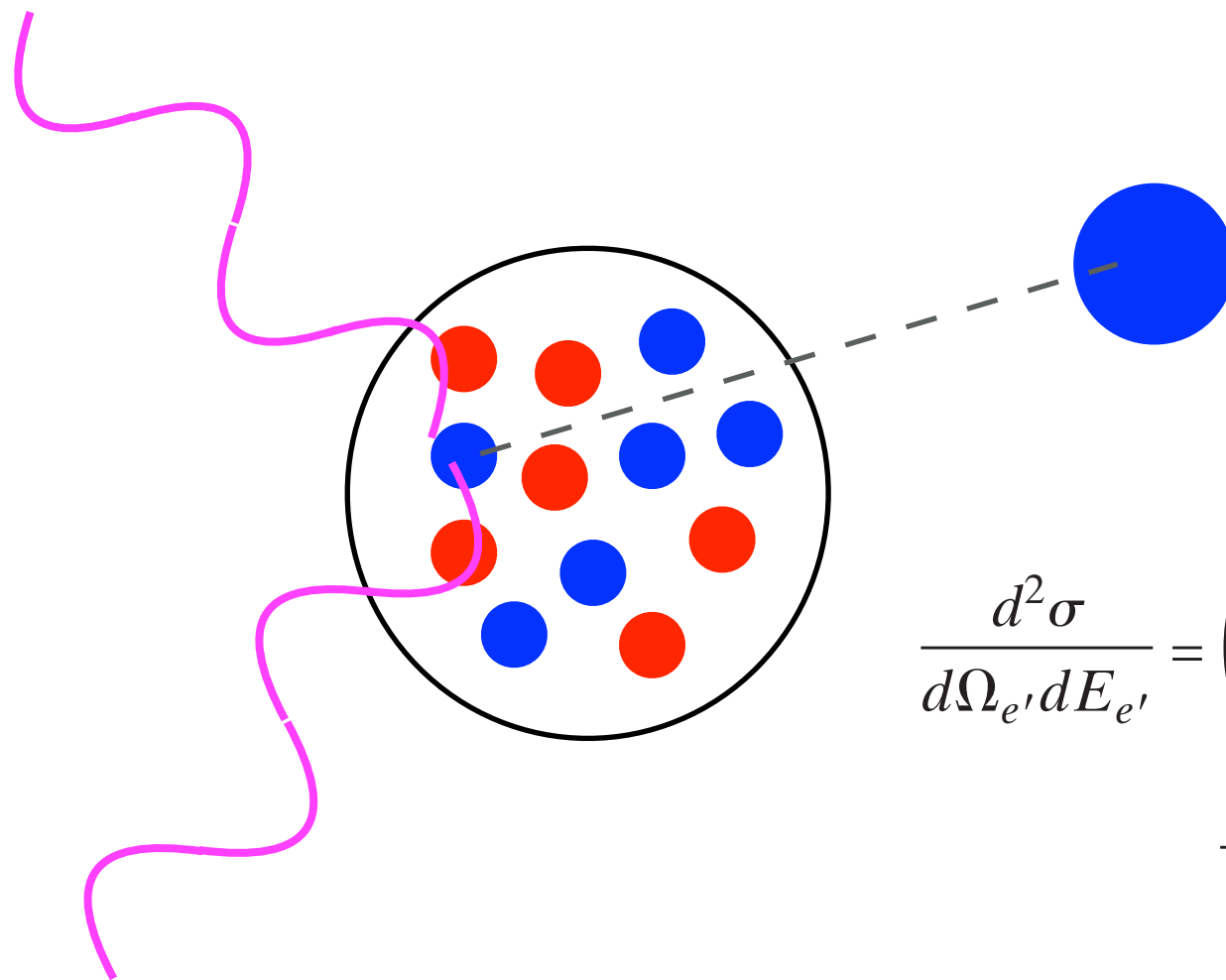
- Scaling with momentum transfer - 'y' scaling
- Scaling w/ number of nucleons

But Longitudinal and Transverse Response are very different



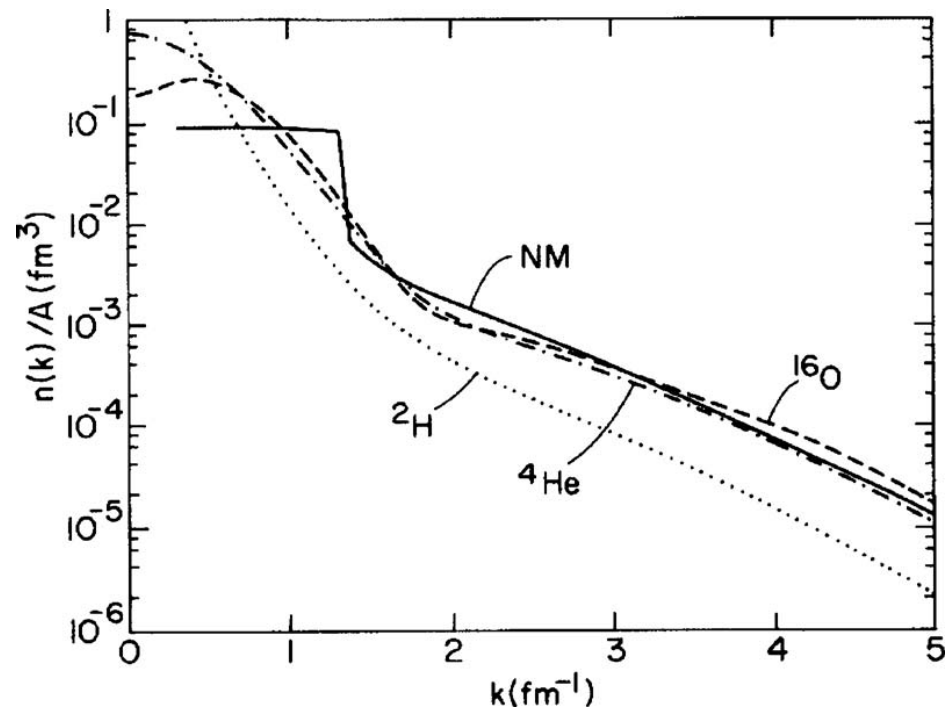
# Quasi-elastic scattering: higher $q$ , $E$

Scaling with momentum transfer: 'y'-scaling  
incoherent sum over scattering from single nucleons  
- scaling of 1st kind-



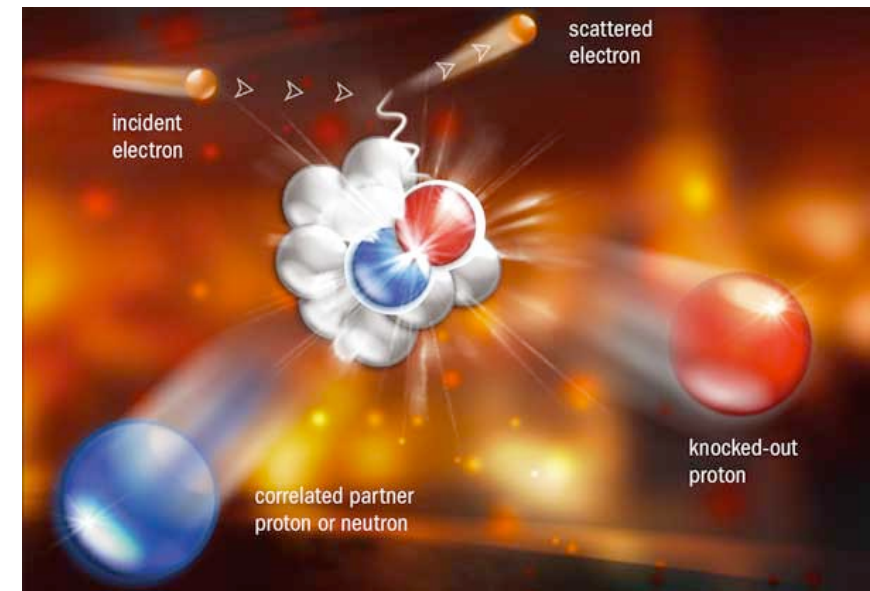
$$\frac{d^2\sigma}{d\Omega_{e'}dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}}\right)_M \left[ \frac{Q^4}{|\mathbf{q}|^4} R_L(|\mathbf{q}|, \omega) + \left( \frac{1}{2} \frac{Q^2}{|\mathbf{q}|^2} + \tan^2 \frac{\theta}{2} \right) R_T(|\mathbf{q}|, \omega) \right]$$

# Nuclei: 1- and 2-nucleon momentum distributions

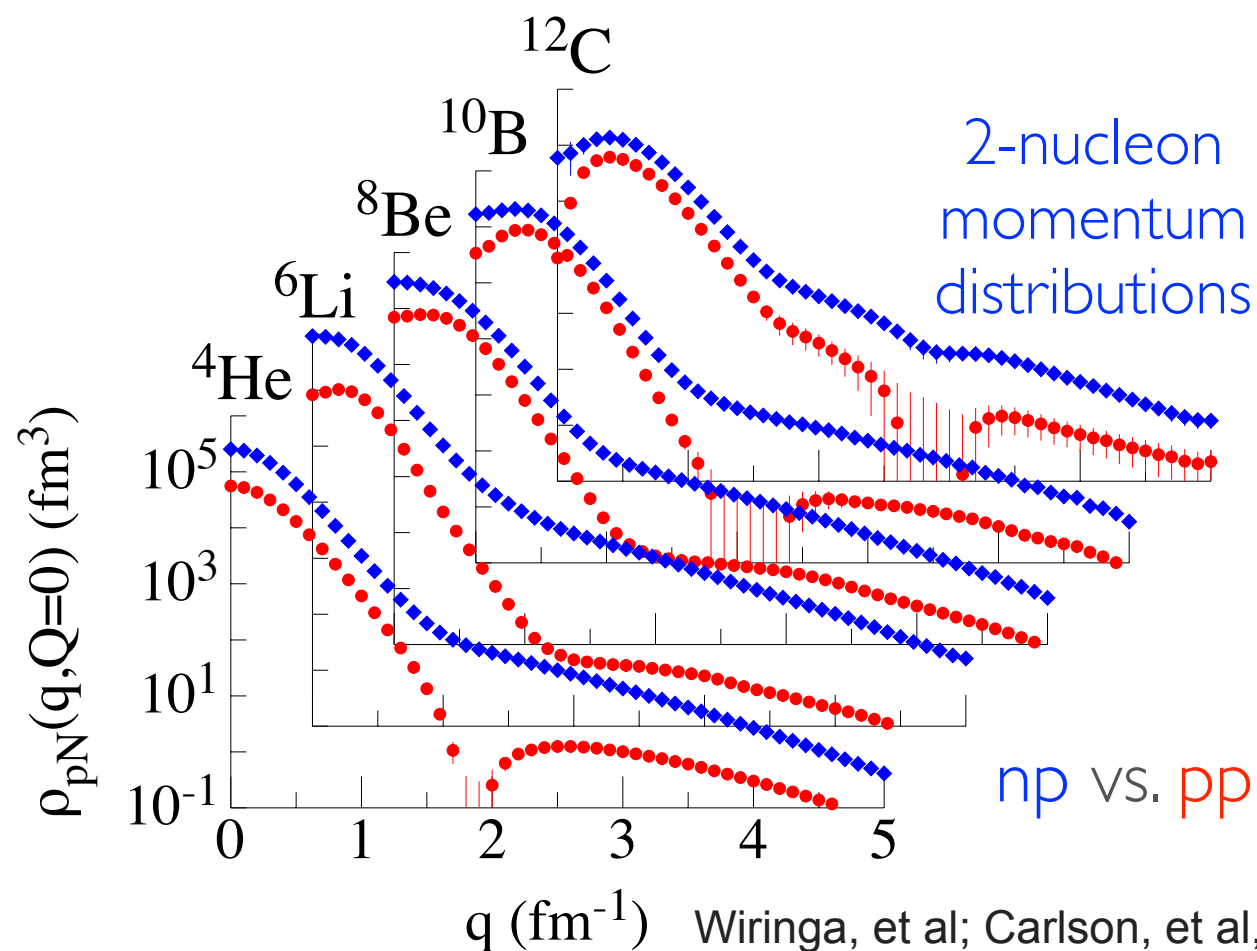


Schiavilla, et al 1986, Benhar, et al 1993

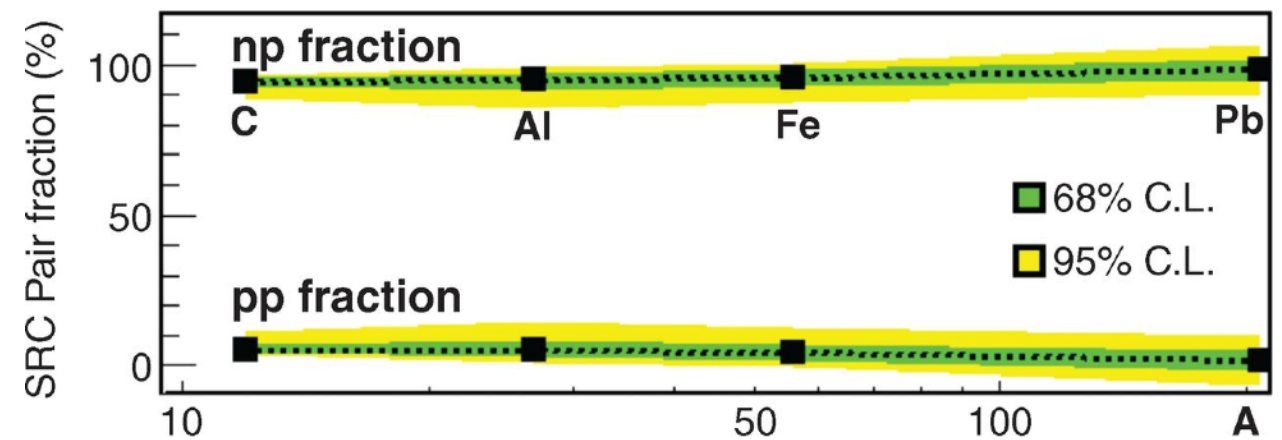
Experimental measurement of back-to-back nucleons dominance of np pairs



Subedi, et al, Science, 2008, ... relation to EMC effect



Wiringa, et al; Carlson, et al, RMP 2015

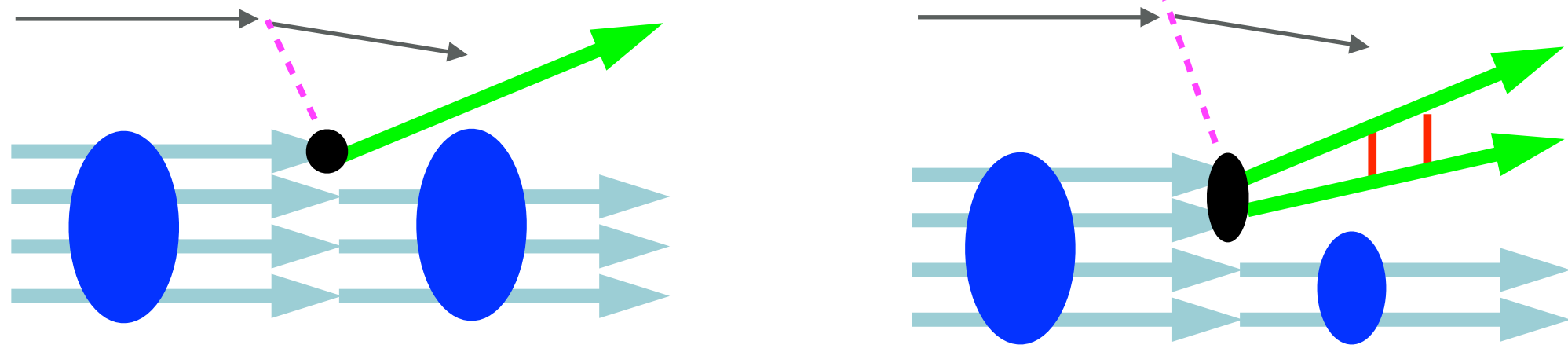


Hen, et al., Science, 2014



# Quasi-elastic electron and neutrino scattering: 1- and 2-nucleon processes

Both correlations and currents are important



Single Nucleon Scattering  
plus scattering from correlated pairs

# Euclidean Response

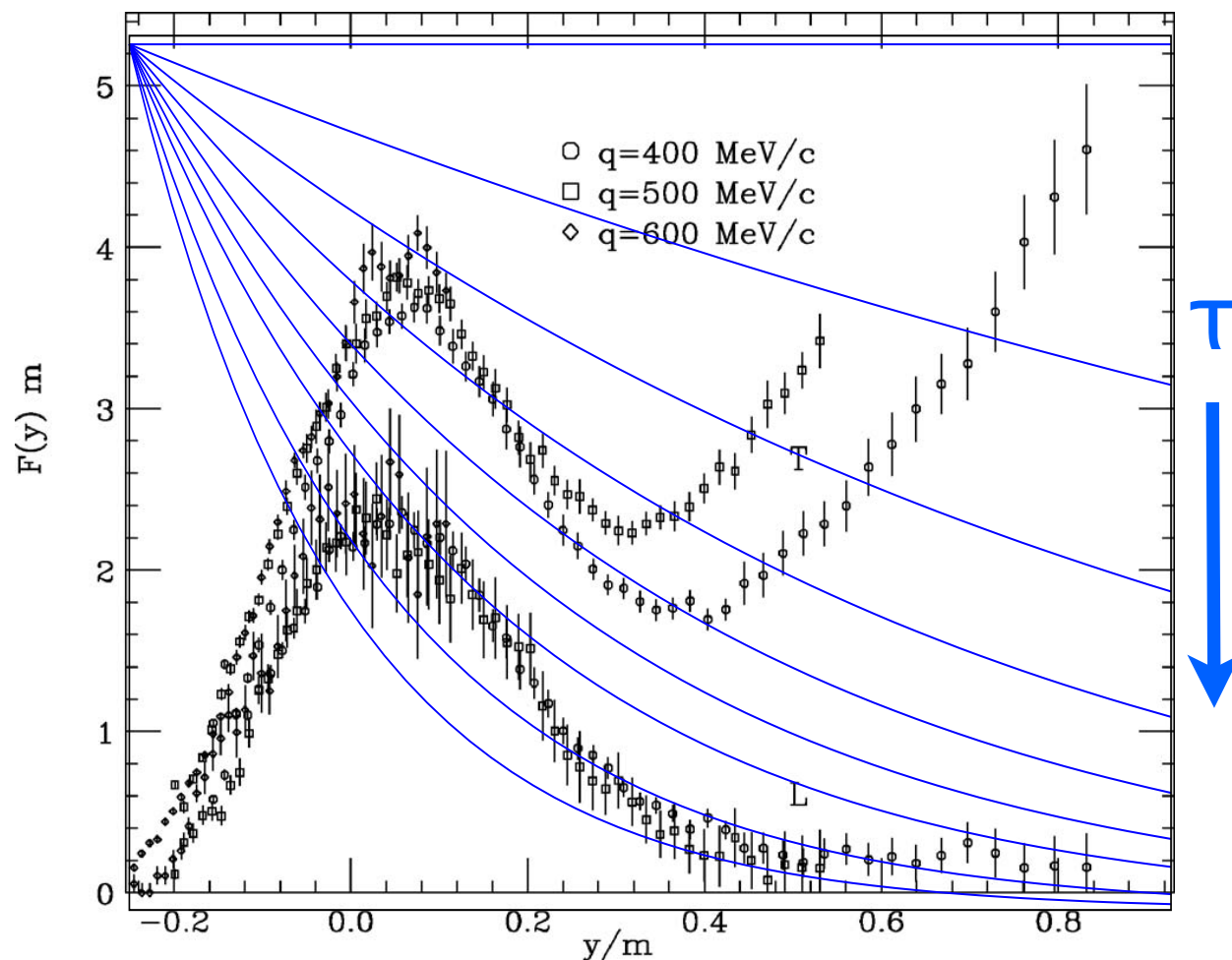
Want to calculate

$$R(q, \omega) = \int dt \langle 0 | \mathbf{j}^\dagger \exp[i(H - \omega)t] \mathbf{j} | 0 \rangle$$

Can calculate

$$\tilde{R}(q, \tau) = \langle 0 | \mathbf{j}^\dagger \exp[-(\mathbf{H} - \mathbf{E}_0 - \mathbf{q}^2 / (2\mathbf{m}))\tau] \mathbf{j} | \mathbf{0} \rangle$$

- Exact given a model of interactions, currents
- 'Thermal' statistical average
- Full final-state interactions
- All contributions included - elastic, low-lying states, quasi elastic, ...



Excellent agreement  
w/ EM (L & T)  
response in A=4,12  
Lovato, 2015, PRL 2016

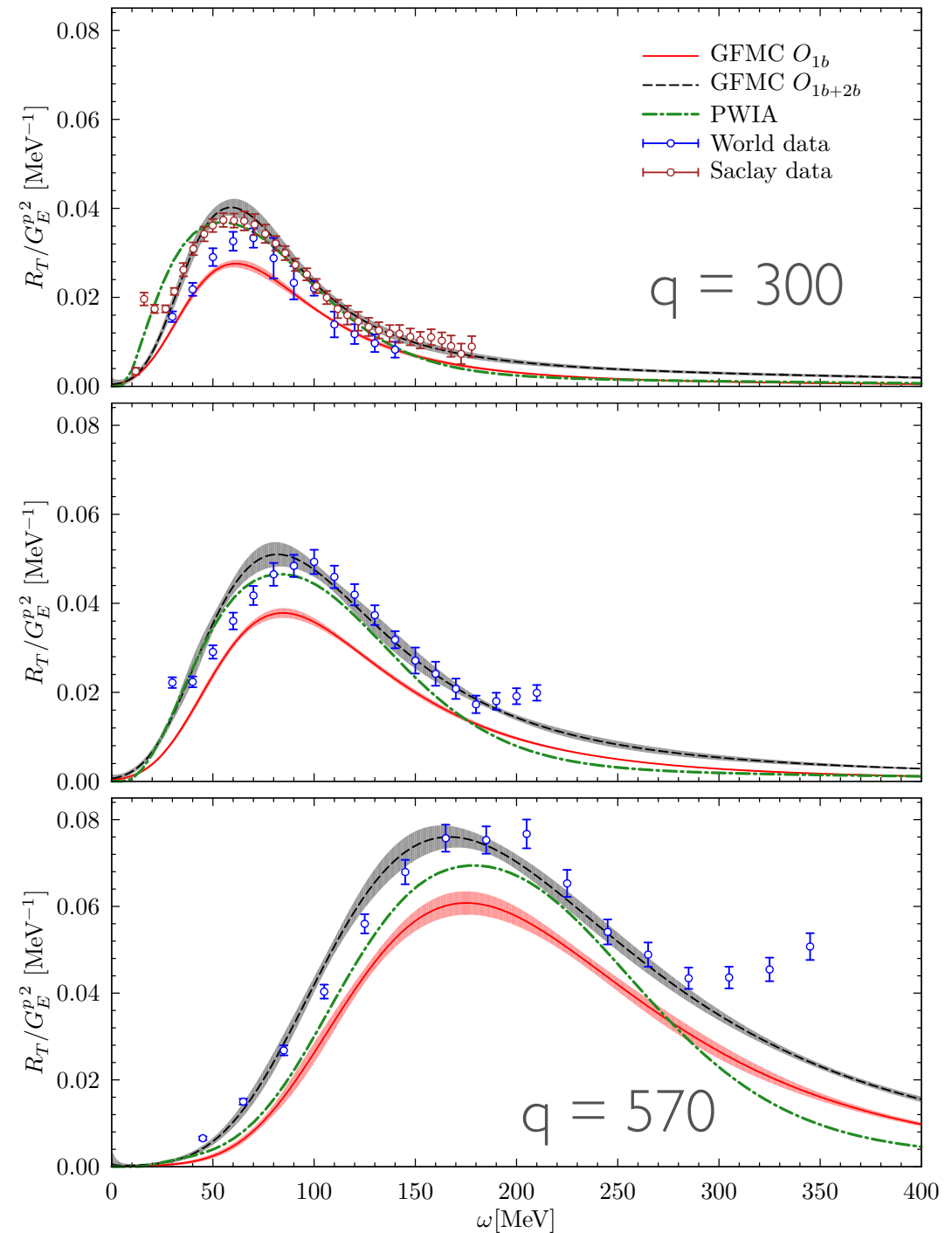
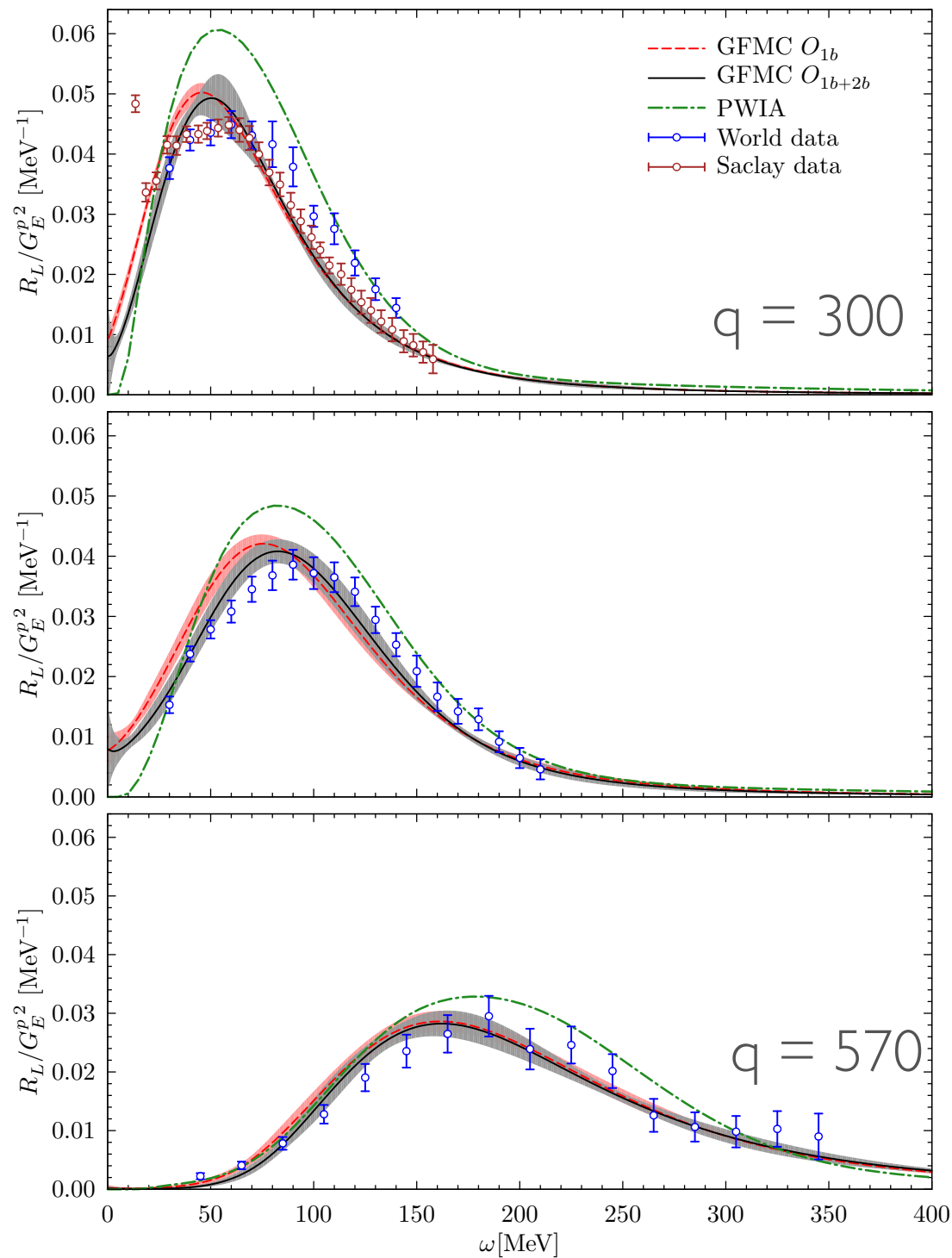
Sum rule → elastic FF<sup>2</sup> w/ increasing



# Nuclei: inclusive electron scattering: Carbon

Longitudinal (charge) Response

Transverse (current) Response

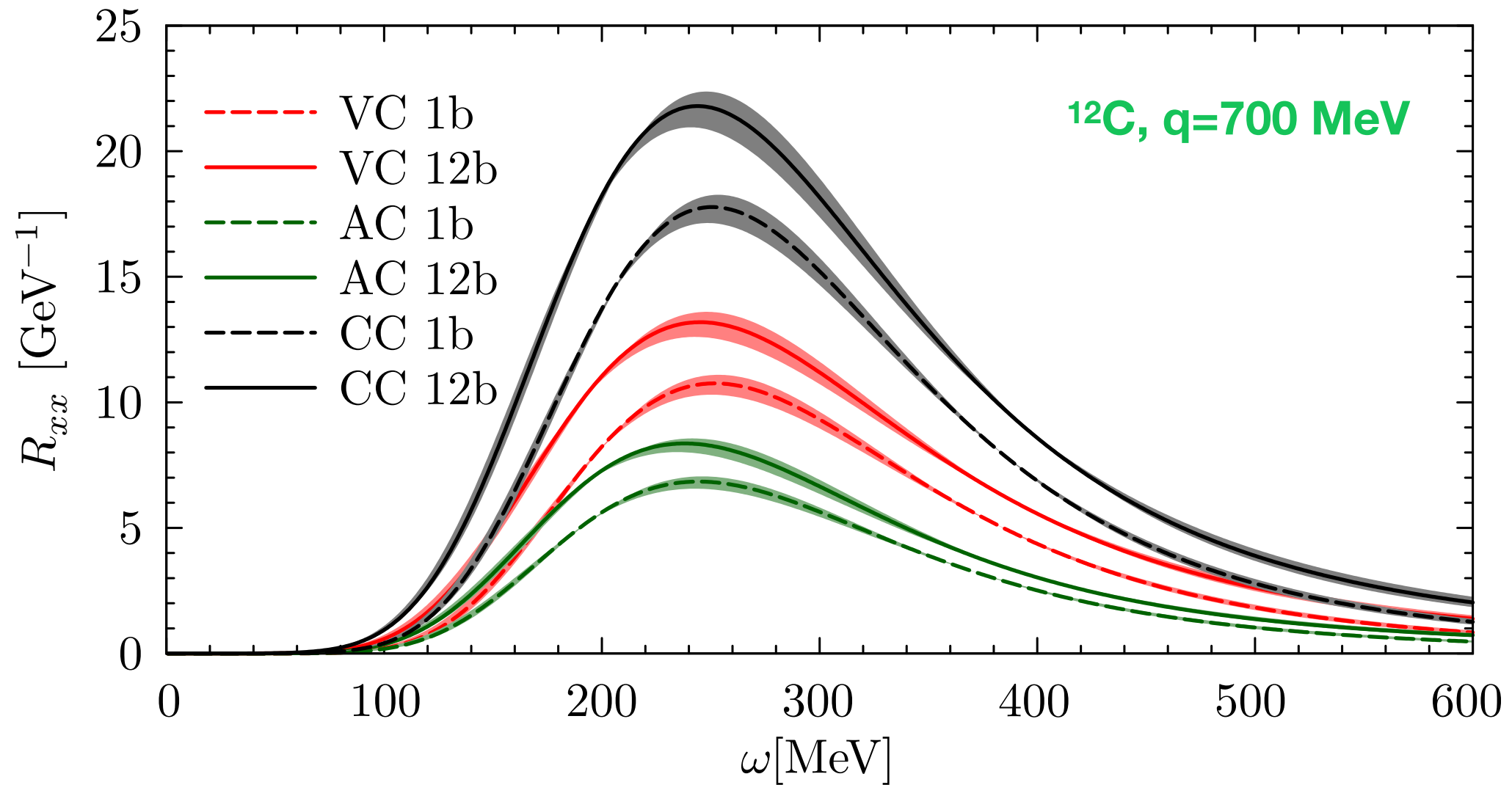


PWIA dashed lines  
 One-nucleon currents: red band  
 Full result: gray band

Lovato, et al, PRL 2016

similar calculations for  $^4\text{He}$

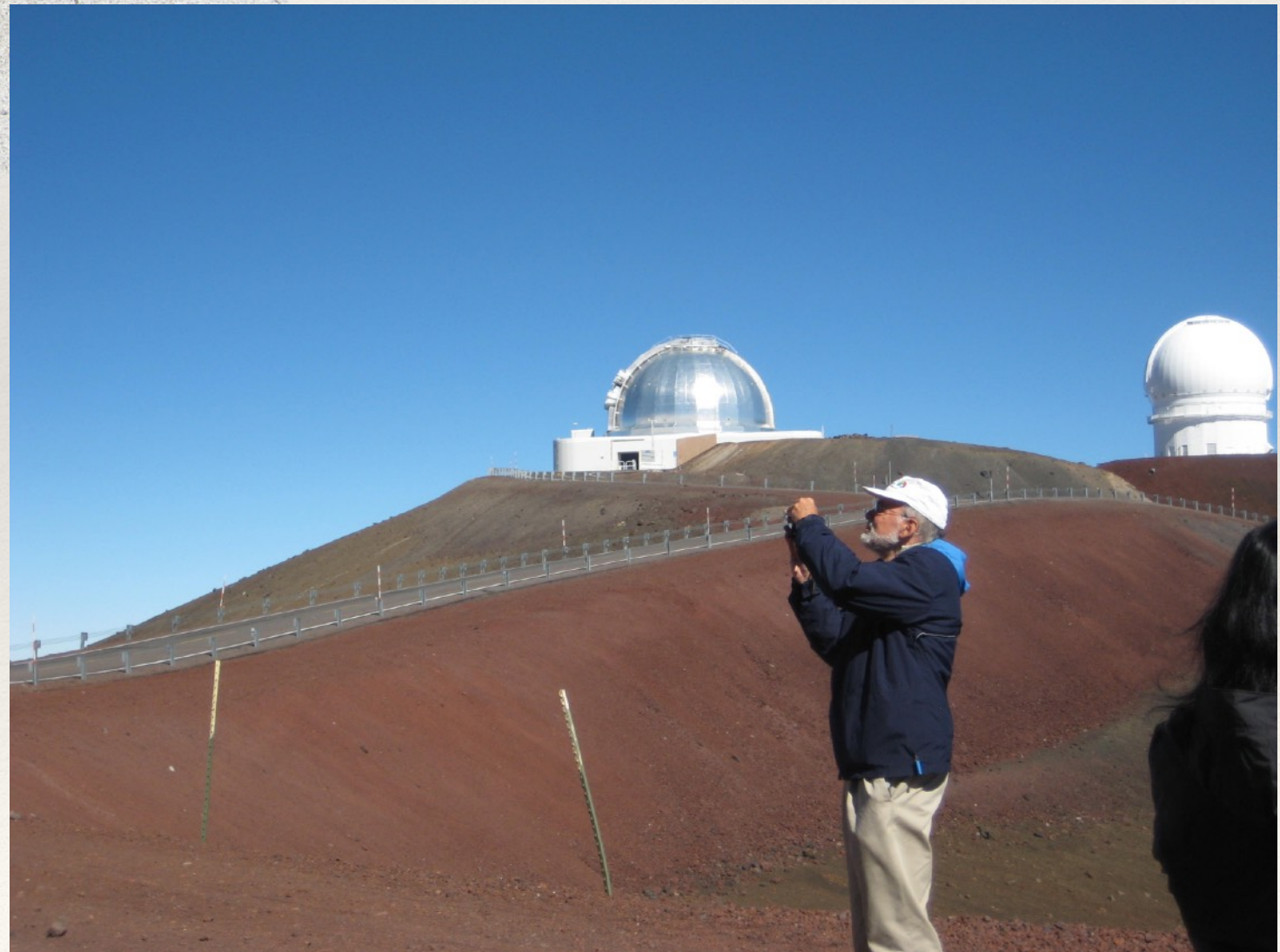
charged current neutrino scattering from  $^{12}\text{C}$



Alessandro Lovato, 2019



# Some time for Fun DNP in Hawaii





Present status : Outstanding combination of ingredients:

Many important physics problems:

electron and neutrino scattering - accelerators

astrophysical neutrinos

double beta decay

neutron star structure and mergers

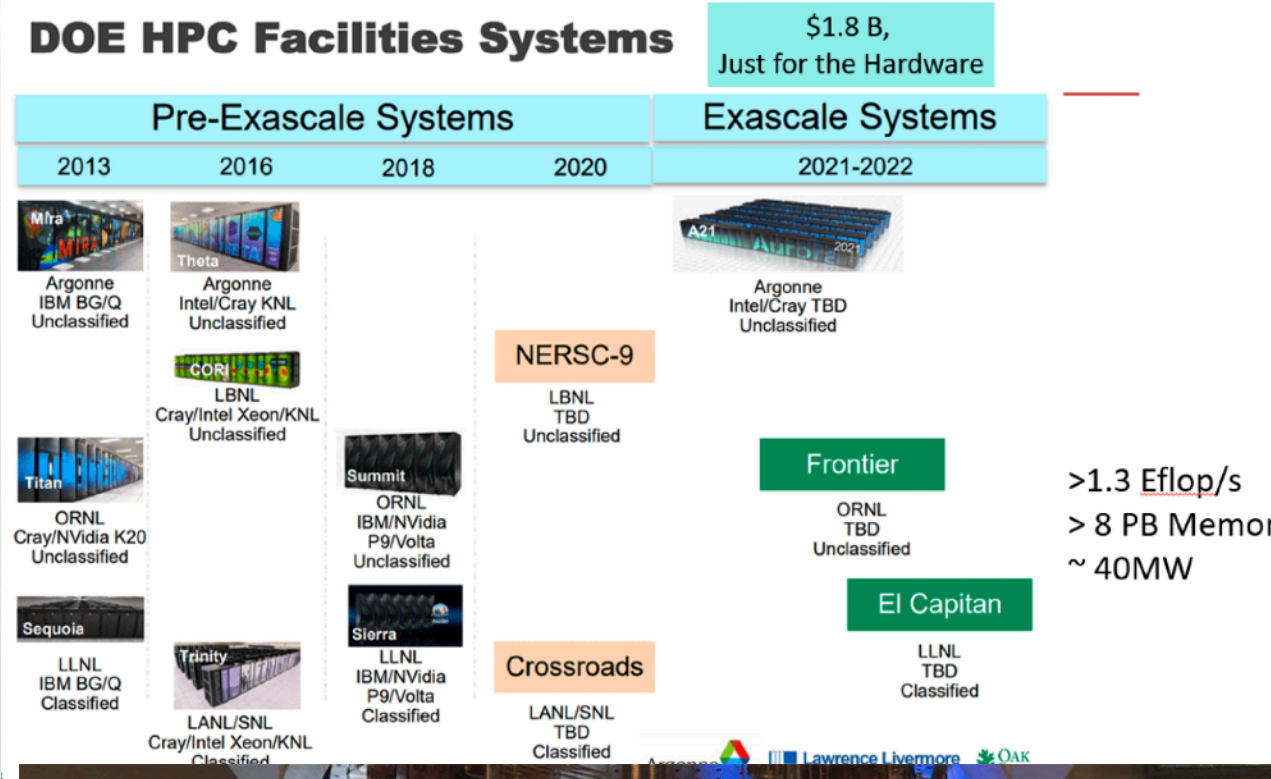
larger and neutron rich nuclei

tests of BSM physics

connections to other areas:

cold atoms, .....

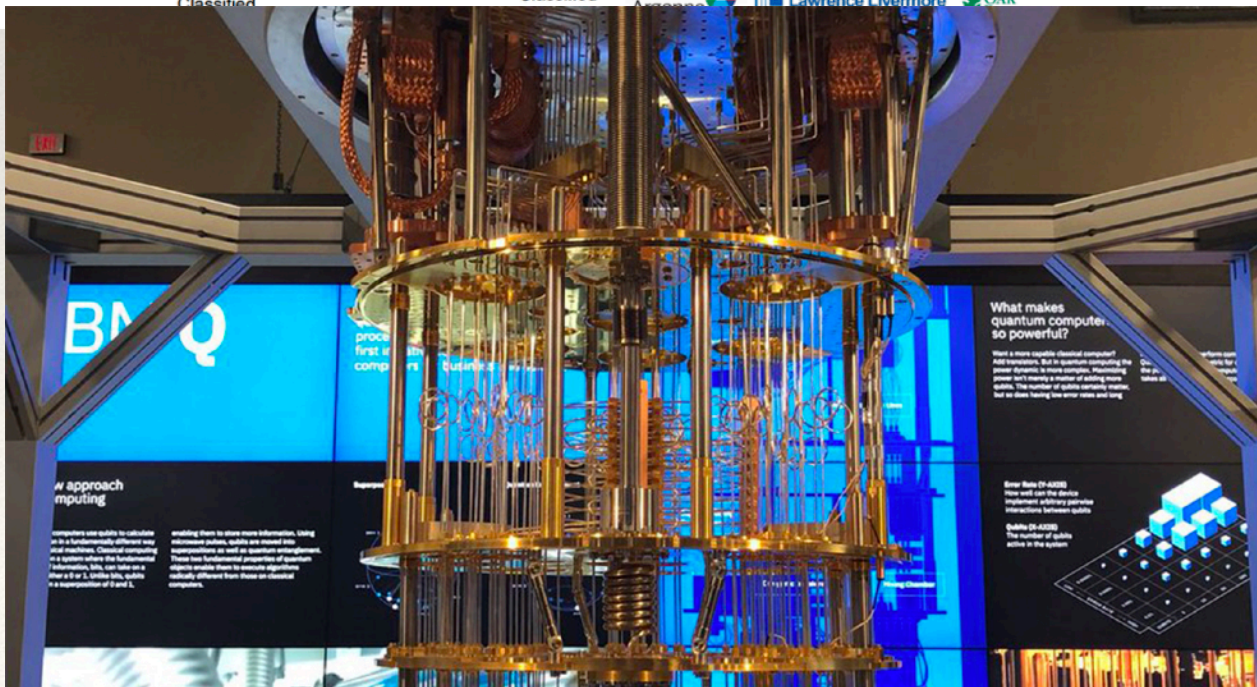
Quantum (Nuclear) Dynamics



Rapidly advancing tools:

Exascale Computers (Cray, Intel, AMD)

Quantum Computers (IBM, Google, Microsoft, ...)









Thank you for many wonderful years working together: Steve Pieper

