# The rms-radius of $^6Li$

## .... a close interaction with Steve

**As experimentalist**

**measure data**
**analyze, preferably with *world* data, extract most accurate observable**

**Then compare to theory**

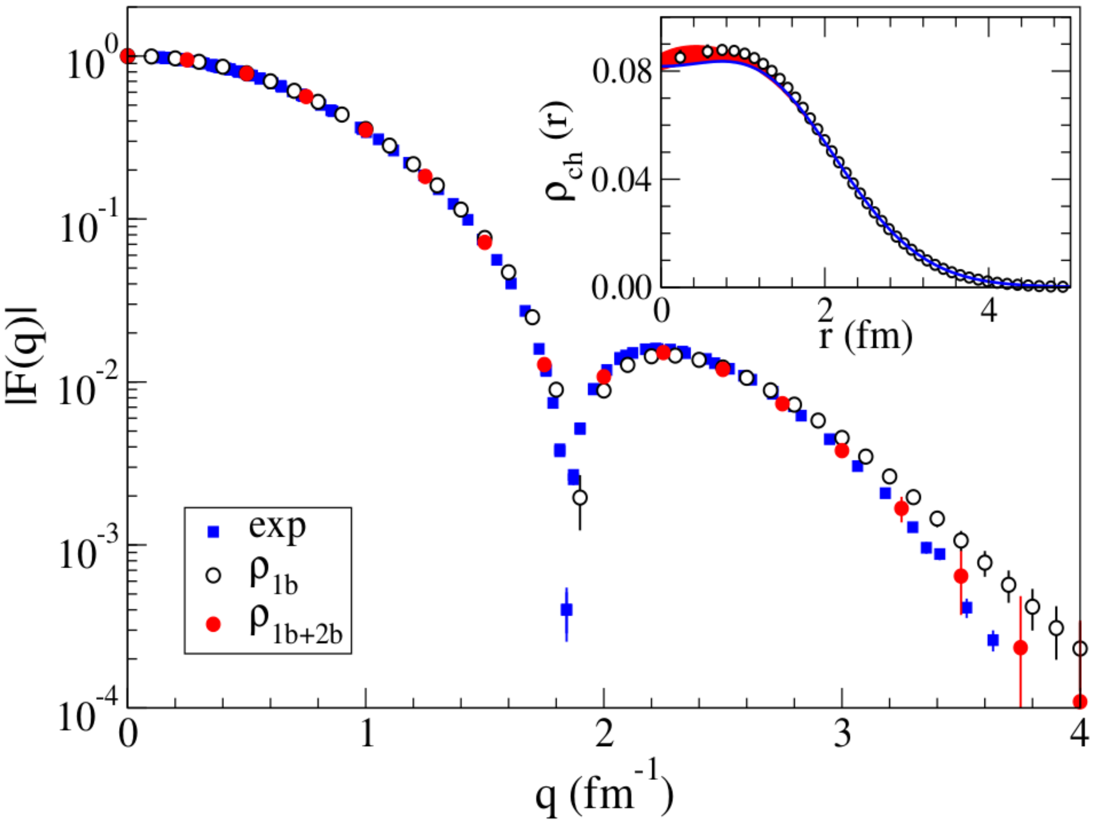
**want most fundamental approach**
**least approximations, *ab initio* calculation**
**best NN interaction (fit to highest-energy NN data), AV18+3BF**
**nucleonic+non-nucleonic degrees of freedom**

**→ natural customer of results of ANL group, in particular GFMC**

**have obtained many results from different members of theory group**
**calculated with codes developed by Steve**
**written to exploit most powerful computers becoming available**

## Quick reminder of observables studied

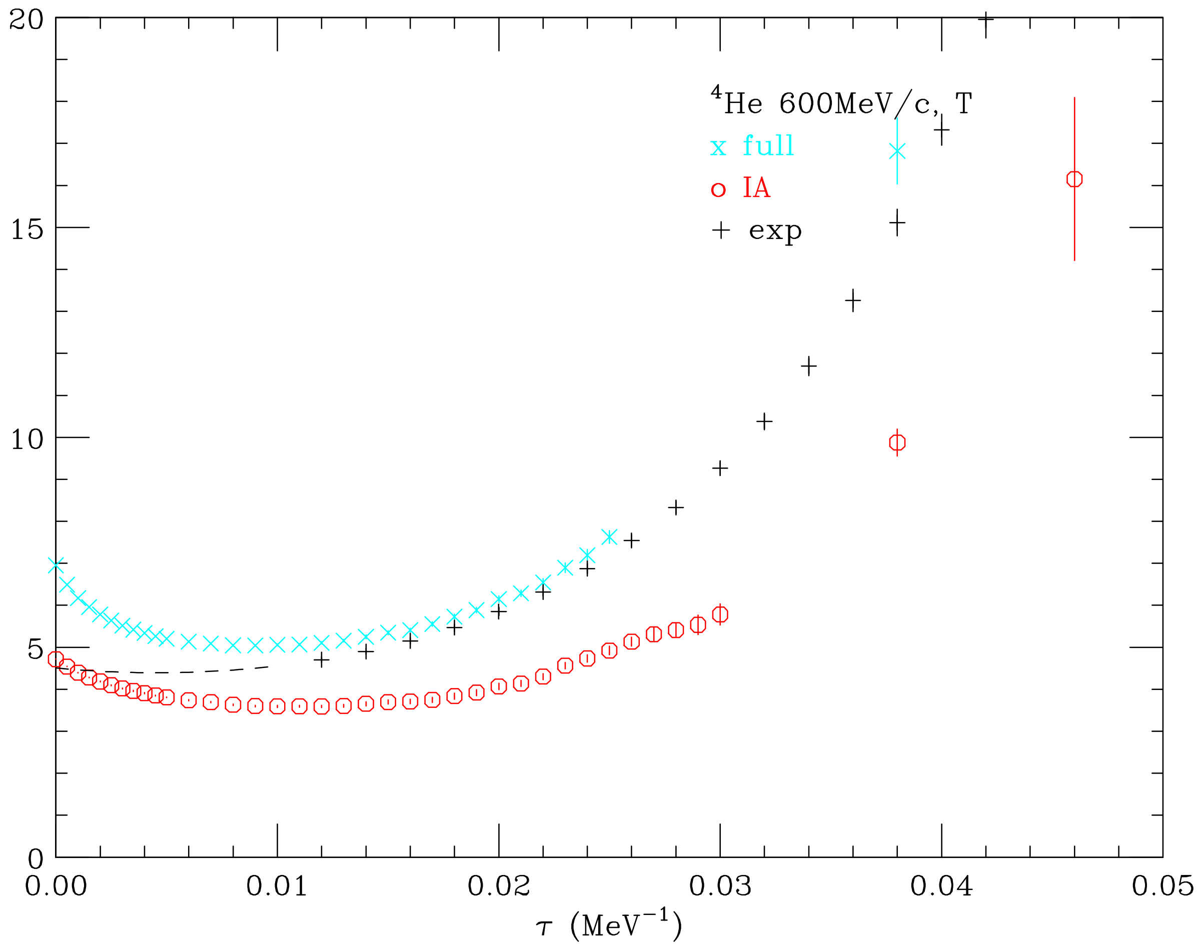
### Densities and form factors: $^{12}C$, GFMC, AV18+3-body

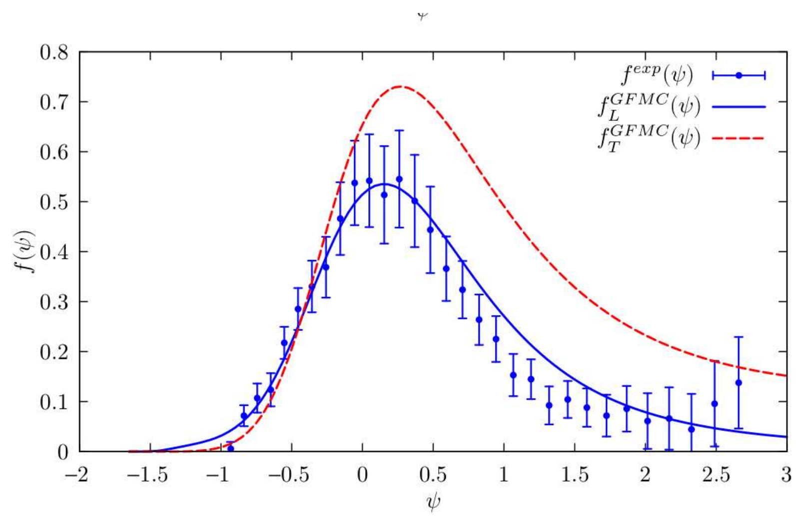



**amazingly good agreement for *ab initio* calculation**

**large mesonic effects for *light* nuclei A=2-4, magnetic form factors**

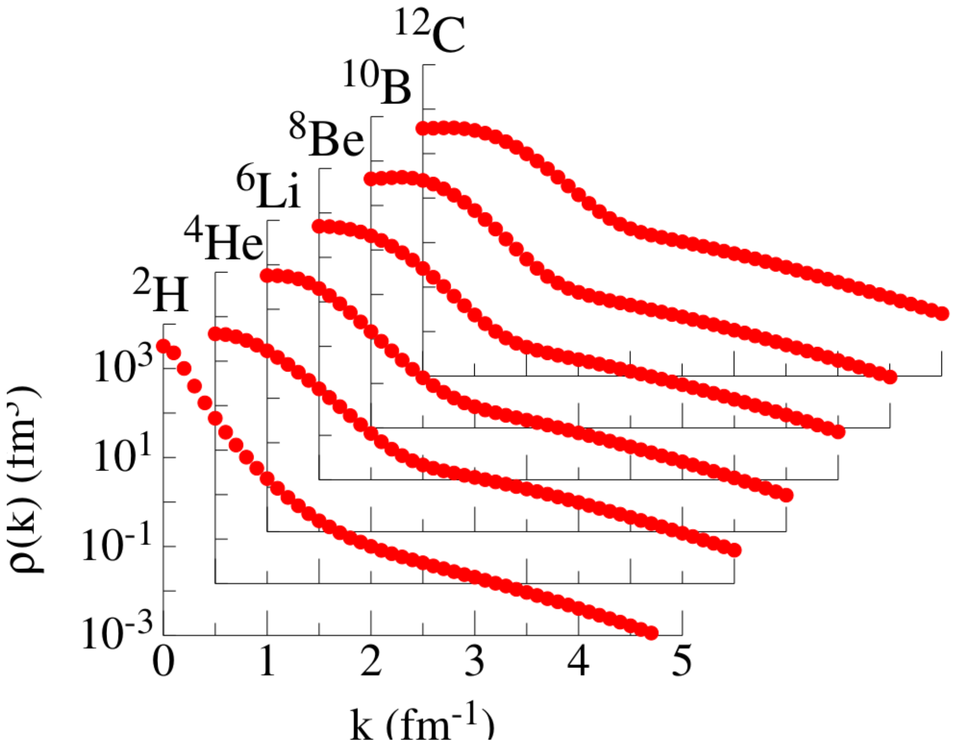
**Euclidian response and (e,e'):** understanding of excess-T response
importance of MEC and connection of tensor correlations
role of final state interactions, ...

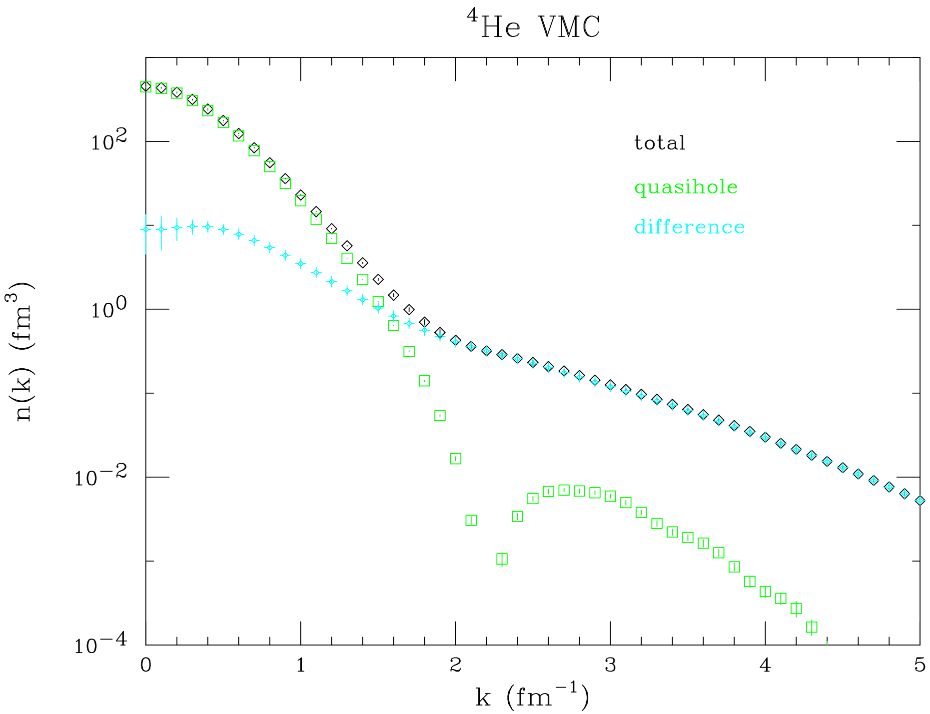



only very recently: inversion to response as function of $\omega$ (max. entropy)

## Momentum distributions and occupations



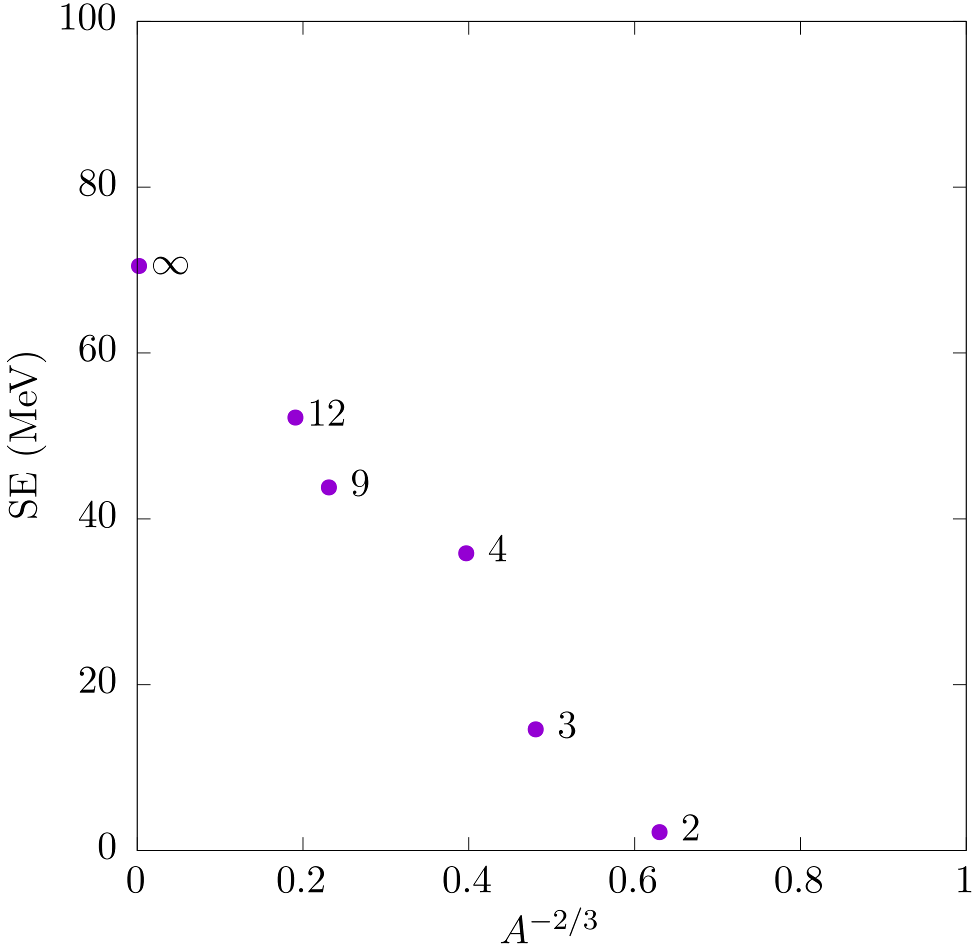



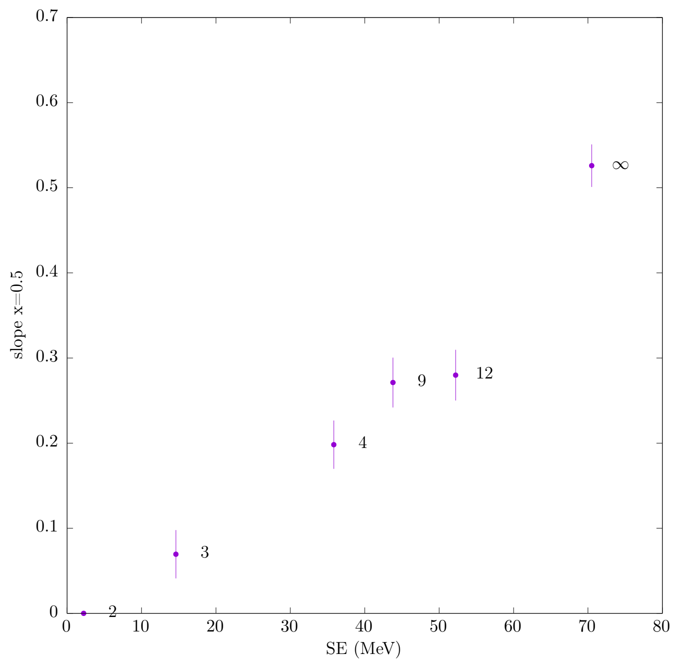

**universal high-momentum tails, split into correlated/mean-field part, occupations**

**For derived quantities:** **see next page**

## Average separation energies

**>2 times as large as usually assumed (from mean field calculations)**
**shows that binding vastly underestimated in standard calculations of EMC effect**




## ... spectral functions (still in the making)

**Important: not only best theory results**

**leading to best agreement with data**

**Steve and collaborators always willing to go the extra mile**

**perform calculation with modified input**
**explore which ingredient really important**

**$\longrightarrow$ much better physics understanding**

**Today: discuss very different example**

**importance of Steve's input at first sight not obvious**

**but in the end decisive**

**involved many back-and-forth discussions with Steve**

**Determination of the rms-radius of $^6Li$**

**a very modest project, but characteristic of interaction with Steve**

# Motivation

## Request of R. Wiringa for accurate radius

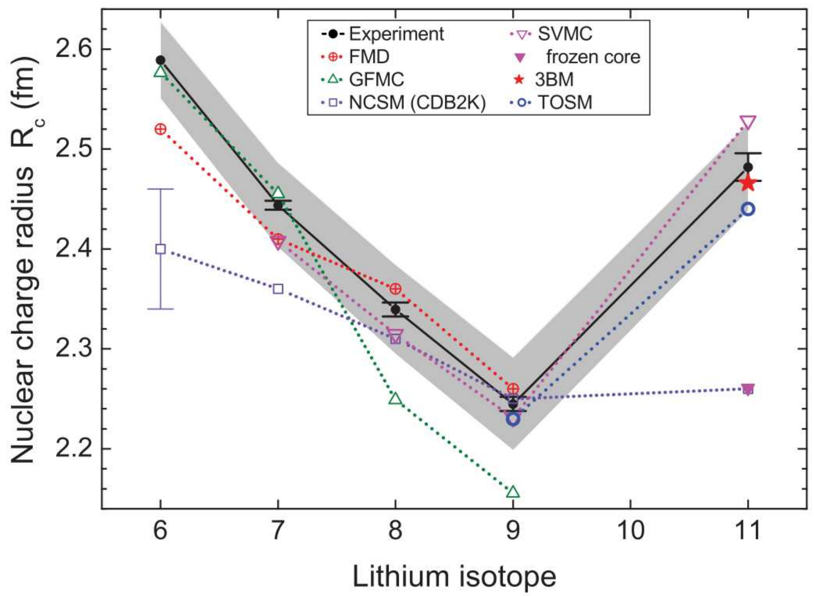
only old model-dependent radii from individual experiments available

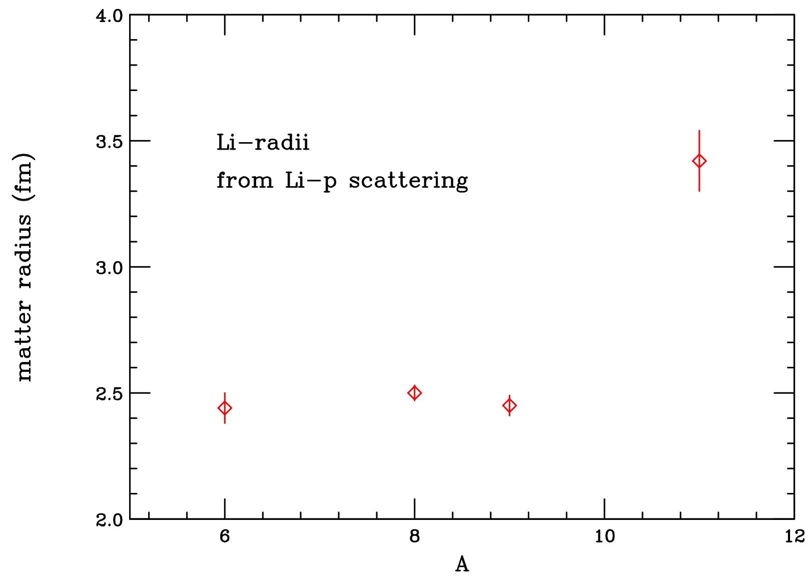
## Experiments on 2s-3s transitions underway at CERN

shifts for *many* (unstable) isotopes, A = 6 ... 11

desire to get absolute radii

need radius of *one* reference nucleus



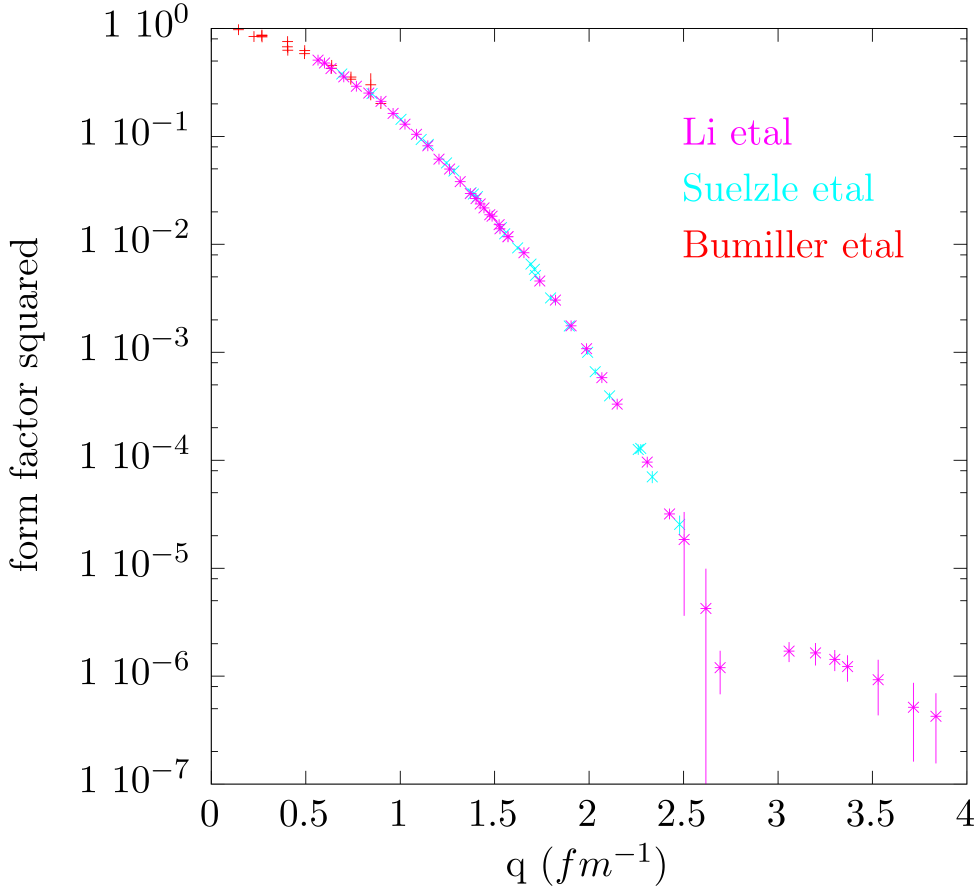



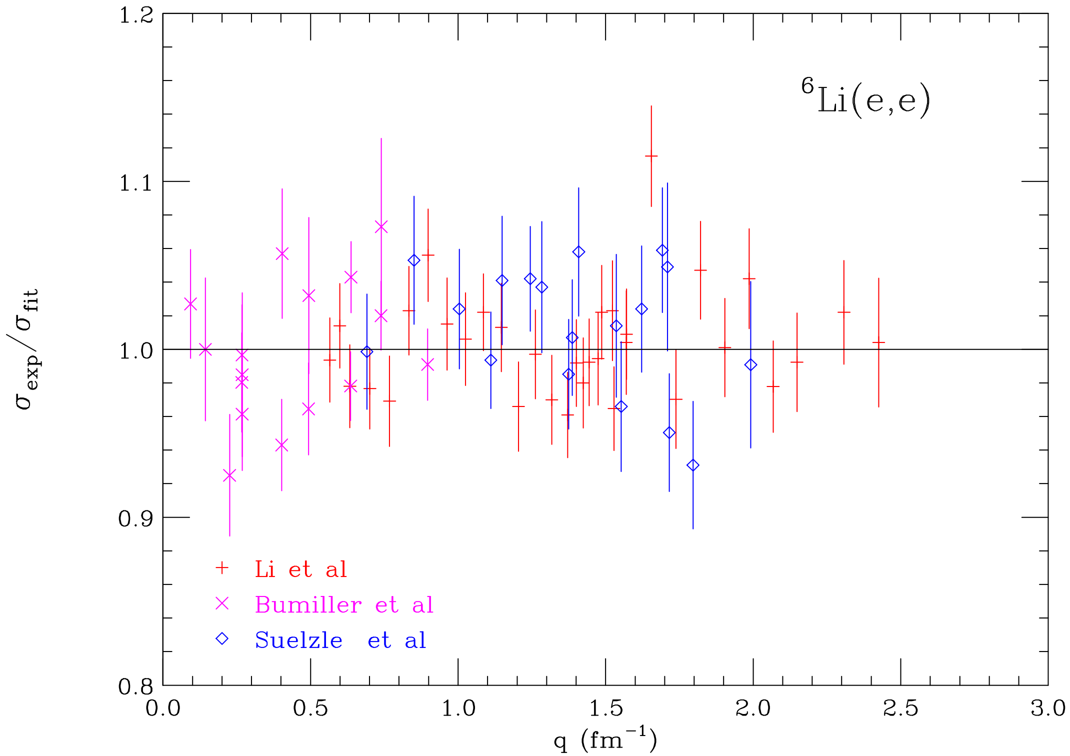

Matter shifts from p(Li,Li)p (inverse kinematics) from Glauber analysis

also need R from elastic electron scattering as check

## Problem 1: data from (e,e) not particularly accurate

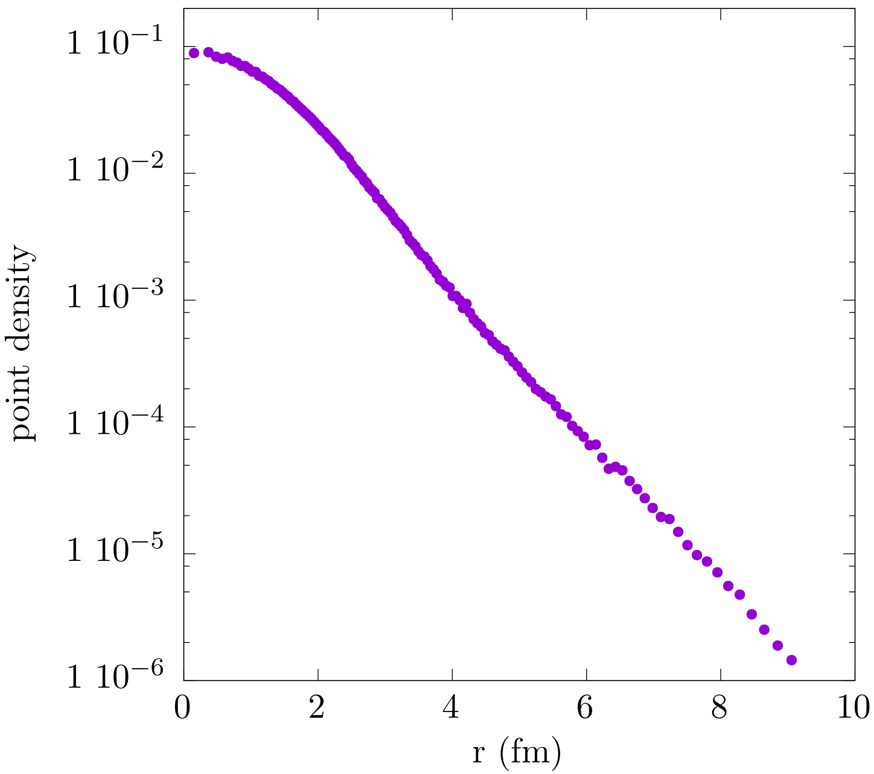
**systematic errors 2%**



## Problem 2: long tail of density $\rho(r)$

**d-$\alpha$ separation energy only 1.47 MeV (lower than for deuteron!) $\rightarrow$ long tail**

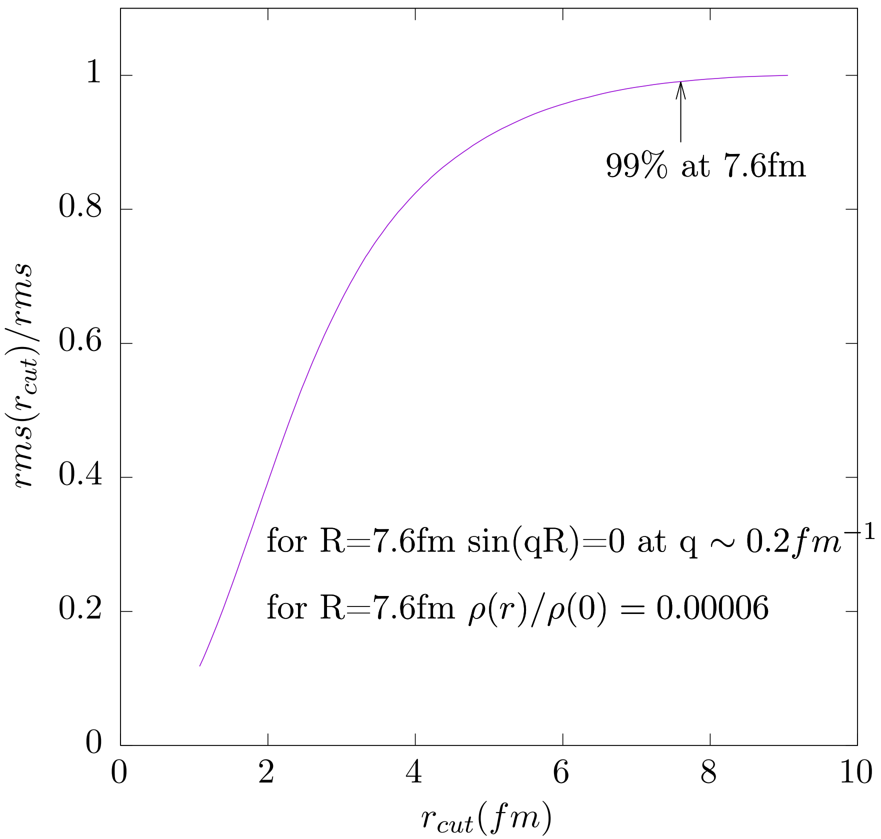



**does this tail create a problem?**

## Illustration of tail-problem

**calculate rms-radius R as function of cut-off**

$$R(r_{cut}) = \left[ \int_0^{r_{cut}} \rho(r)\ r^4\ dr \ \Big/ \int_0^{\infty} \rho(r)\ r^4\ dr \right]^{1/2}$$

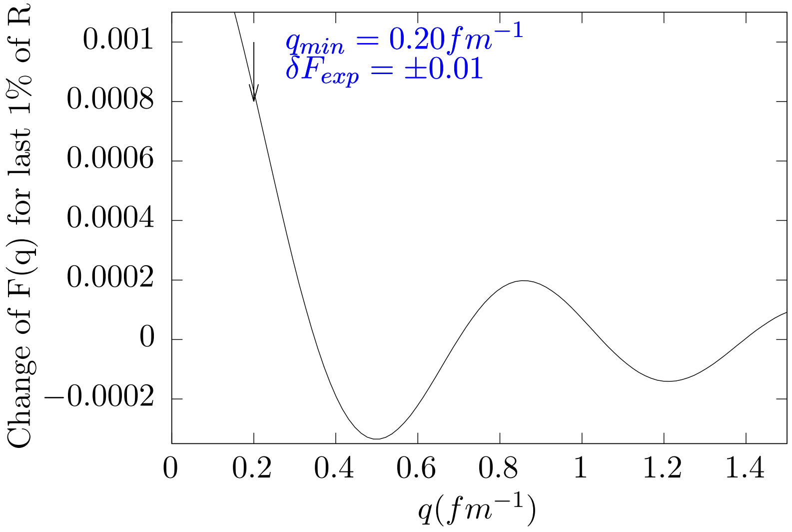



**R∼2.6fm**

**for 99% must integrate to 7.6$fm$**
**at r=7.6$fm$ $\rho(r)/\rho(0) = 0.00006$!**

**charge at such large radii:**
**tiny contribution to $F(q)$ despite effect upon $R$**

## Contribution to F(q) of last % outside $7.6 fm$



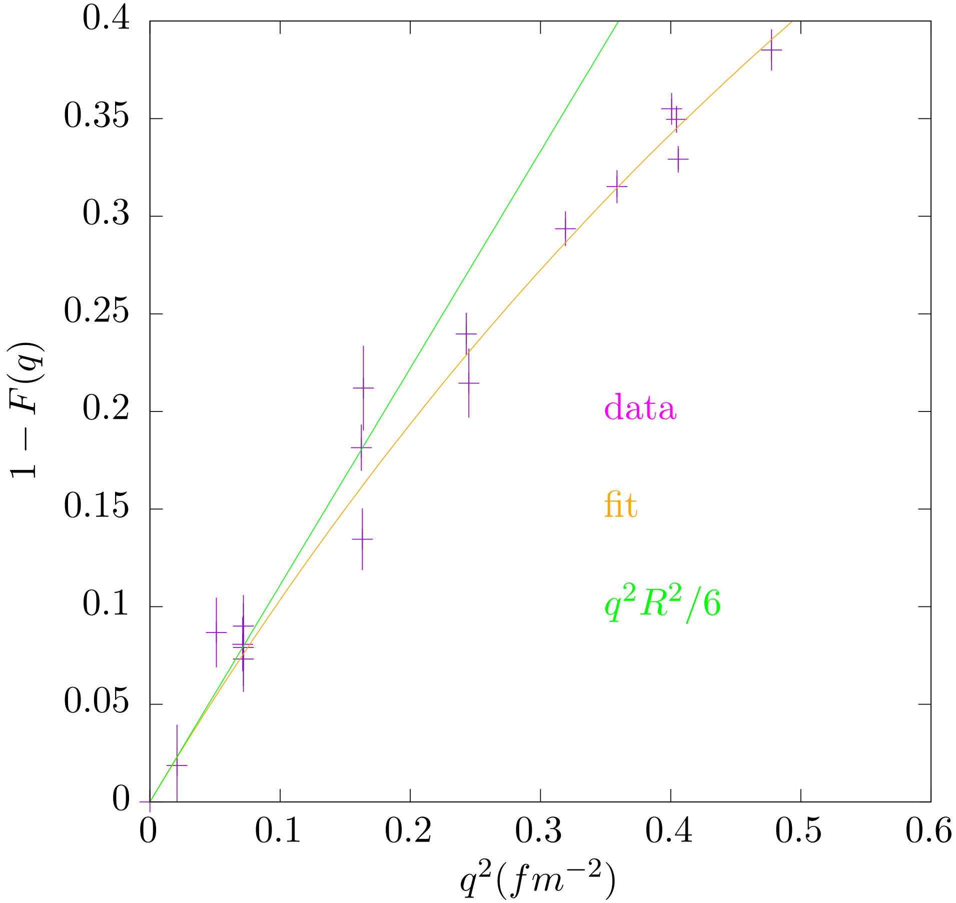

**maximal contribution 8% of $\delta F$**

**even 10 times larger contribution not measurable**

**same problem, expressed differently: higher moments $< r^N >$ large**

**Standard idea: R from $q = 0$ slope of $F(q) = 1 - q^2R^2/6 + q^4\langle r^4\rangle/120 + \ldots$**

**but.. higher moments give large contribution**
**illustrated by (small) finite size effect $= 1 - F(q)$**




**Illusion to get R to %-type accuracy from data alone (true for all A)**
**curvature of $F(q)$ at low $q$ prevents extrapolation**

## Consequences for uncertainty of $R$

Model-independent analysis of world data (using SOG parameterization)

yields error bar of R $\pm 0.17 fm$

model-dependent analyses gave $\pm$ 0.05 $fm$

My usual approach to get accurate R

parameterize $\rho(r)$, not $F(q)$

supplement data with *physics* constraint on large-$r$ behavior

this constrains curvature of $F(q)$ at low $q$

$\rightarrow$ this allows for more accurate (implicit) extrapolation to $q = 0$

Normal case

large-$r$ density given by least-bound proton shell

there $\rho(r)$ dominated by Fock state p + (A–1)

p wave function = Whittaker function

only input QM + separation energy

+corrections for p, n finite size, spin-orbit, ...

$\rightarrow$ shape of $\rho(r)$ (for $^4He$ $\rho(r)$)

Works perfectly, yields the most accurate radii from (e,e)

d ($\pm$0.5%), $^4$He ($\pm$0.25%), $^{12}C$ ($\pm$0.5%), all agree with recent $\mu$X

## Special problem with $^6Li$

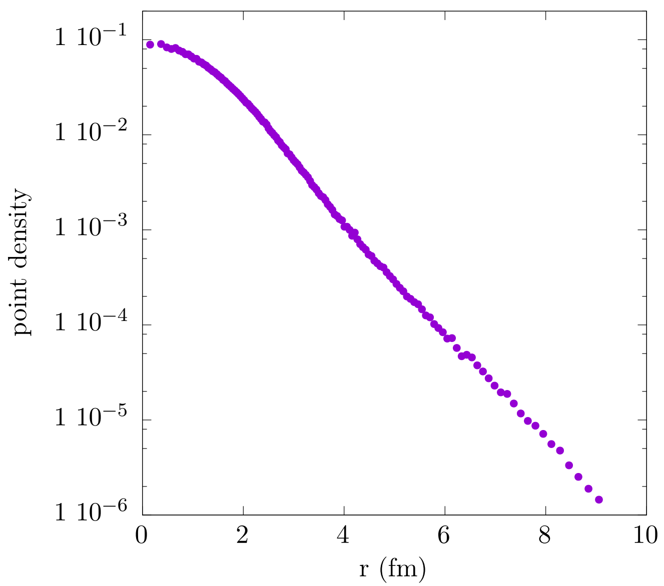
a) structure = $^4He$+d or $^4He$+p+n (SE = 3.7MeV) ?
b) folding $\Psi^2_{\alpha d} \to \rho(r)$: density of d $\neq$ free d, smaller (Wildermuth)?
radius of d almost as large as radius of $^6Li$

## Solution: GFMC Steve Pieper $\to$ shape of tail

available calculation: not satisfactory, large-$r$ slope $\sim$ SE=8MeV (Steve)
normally not a problem: $\rho < 10^{-4}$
need better calculation since interest is in *extreme* tail

## Improvements of Steve

better statistics
longer GFMC propagation
newly developed propagation scheme




Find expected behavior out to $8.5fm$, good enough!
(for $r > 8.5fm$ slope still too large, ignore)

## Continuing discussion with Steve

does GFMC converge well enough?

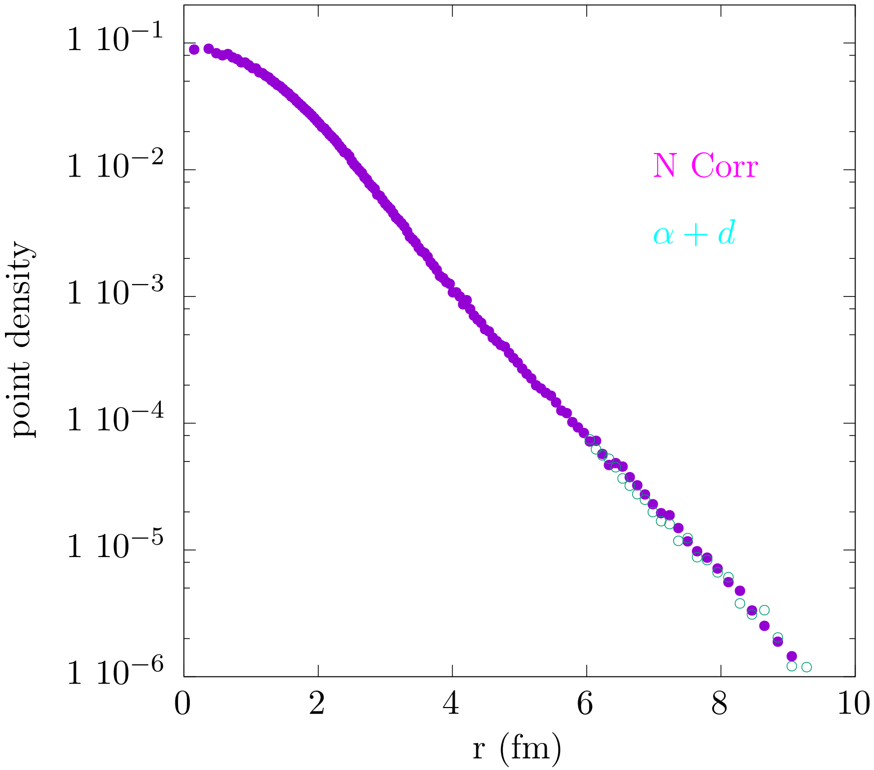
## Main concern:

- starting wave function based on shell-model WF + correlations
- end-result more like $\alpha$ + d cluster WF

## New calculation by Steve

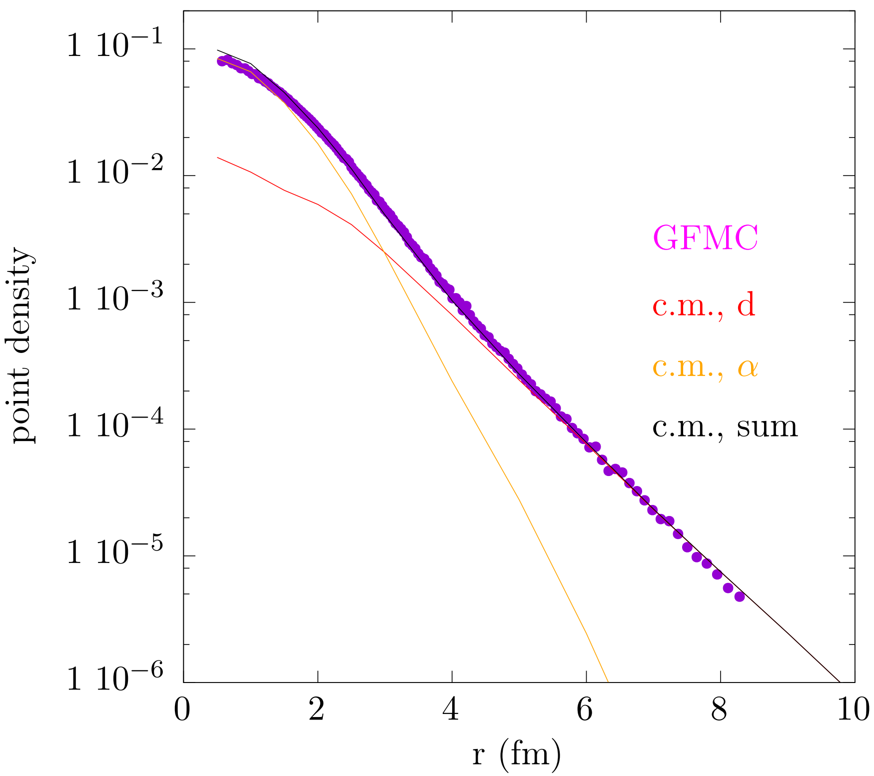
construct cluster wave function
use as starting point
run GFMC

find *same* result within error bars



## Extra step taken by Steve

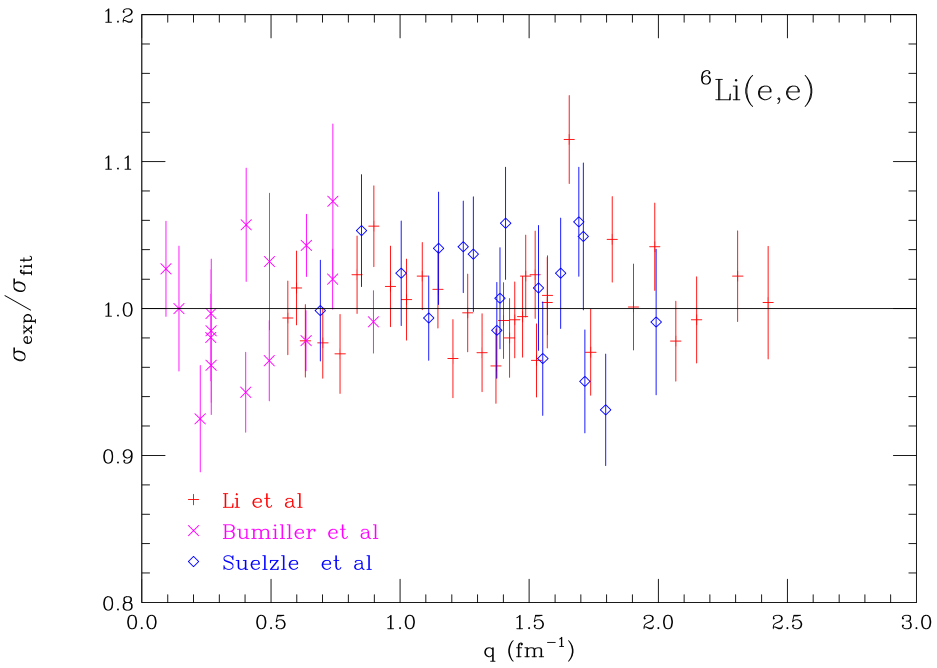
**locates $\alpha$-d potential from cluster model (Langanke)**
**calculates density (folded with *free* d-density)**
**finds almost identical large-$r$ fall-off as GFMC**
**confirms dominance of $\alpha$-d structure**



Consequence: know *shape* of $\rho_p(r)$ out to $8.5 fm$

can use as constraint when fitting form factor data

shape used for $r > 3.5 fm$ where $\rho(r) < 0.01\rho(0)$ get good fit




Find R $= 2.582 \pm 0.027\ fm$, uncertainty dominated by syst. error of data

Final result: factor of 6 reduction of error bar only due to input from Steve

Pastore, Pieper, Schiavilla, Wiringa 2013: $R = 2.55 fm$

This was only *one* small example of the interactions with Steve

a great physicist and wonderful colleague

whom we all are going to dearly miss!