The Nuclear Symmetry Energy

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The Equation of State
at Varying Density and Temperature
and Its Application in Astrophysics

Theory Group in Physics Division
Argonne National Laboratory
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Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under $n \leftrightarrow p$ interchange

An isoscalar quantity F does not change under $n \leftrightarrow p$ interchange. Example: nuclear energy. Expansion in $\eta = (N - Z)/A$ for smooth F, has even terms only:

$$F(\eta) = F_0 + F_2\,\eta^2 + F_4\,\eta^4 + \dots$$

An isovector quantity G changes sign. Example: $\rho_{np}(r) = \rho_n(r) - \rho_p(r)$. Expansion with odd terms only:

$$G(\eta) = G_1\,\eta + G_3\,\eta^3 + \dots$$

Note: $G/\eta = G_1 + G_3\,\eta^2 + \dots$.

Charge invariance: invariance of nuclear interactions under rotations in n-p space

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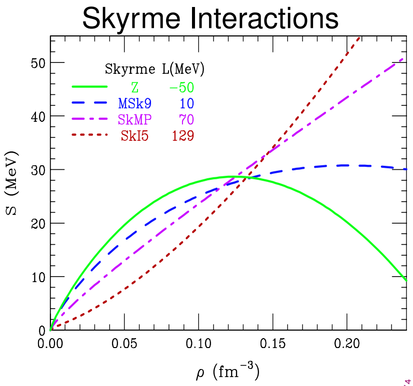
Tools

- Qualitative Considerations/Semiempirical Energy Formula
- Hohenberg-Kohn Energy Functional
- Spherical and Half-Infinite Matter Skyrme-HF
- Spherical and Half-Infinite Matter Thomas-Fermi
- Energies of Isobaric Analog States
- Asymmetry Skins
- Charge Radii & Distributions

Symmetry Energy: From Finite to ∞ System



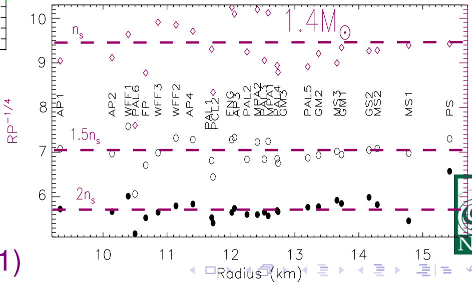
$\eta = (\rho_n - \rho_p)/\rho$-expansion under $n \leftrightarrow p$ symmetry

$$E(\rho_n, \rho_p) = E_0(\rho) + S(\rho) \left(\frac{\rho_n - \rho_p}{\rho}\right)^2$$

$$S(\rho) = a_a^V + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0} + \ldots$$

In neutron matter: $E \simeq E_0 + S$

& $P \simeq \rho^2\, dS/d\rho \simeq L\,\rho^2/(3\rho_0)$

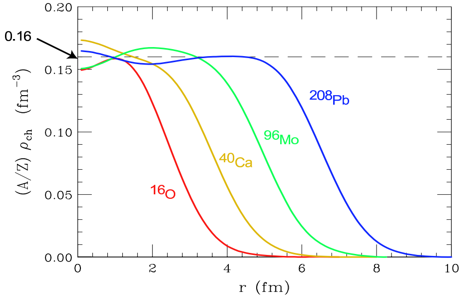


Empirical correlation for neutron stars
$R\,P^{-1/4} \approx \text{const}$
Lattimer&Prakash ApJ550(01)

Finite System

Interrelation between nucleonic densities…

Net density: $\rho(r) \stackrel{?}{=} \frac{A}{Z}\rho_p(r)$



Bethe-Weizsäcker formula:

$$E = -a_V\,A + a_S\,A^{2/3} + a_C\,\frac{Z^2}{A^{1/3}} + a_a(A)\,\frac{(N-Z)^2}{A} + E_{mic}$$

$a_a \stackrel{?}{=} a_a^V$ $\qquad \frac{A}{a_a} = \frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}$ $\qquad a_a(A) = ?$

minimally finite system $\Rightarrow$ half-infinite matter

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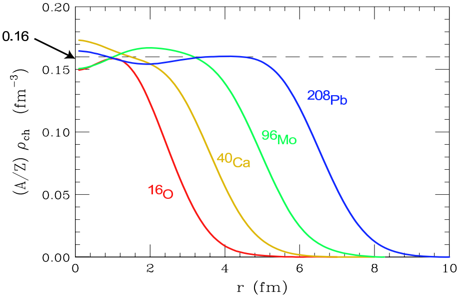

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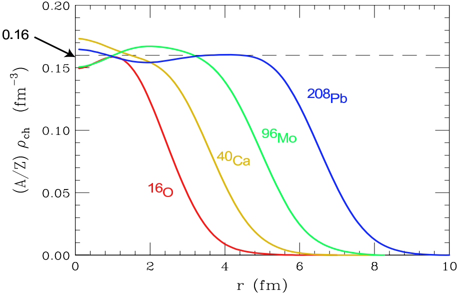

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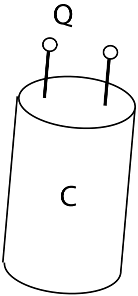
Nucleus as Capacitor for Asymmetry

$$E = -a_v A + a_s A^{2/3} + \frac{a_a}{A}(N-Z)^2$$
$$= E_0(A) + \frac{a_a(A)}{A}(N-Z)^2$$



Capacitor analogy

$$E = E_0 + \frac{Q^2}{2C} \Rightarrow \begin{cases} Q \equiv N - Z \\ C \equiv \dfrac{A}{2a_a(A)} \end{cases}$$

Asymmetry chemical potential

$$\mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{2a_a(A)}{A}(N-Z)$$

Analogy

$$V = \frac{Q}{C} \Rightarrow V \equiv \mu_a$$

Note: for connected capacitors, charge (asymmetry) distributes itself *in proportion to capacitance*

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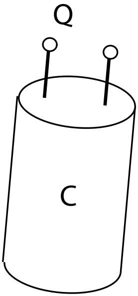

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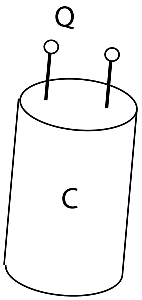

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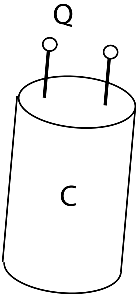

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Invariant Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N-Z)$ for given A. (Coulomb suppressed...)

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A\,\rho_{np}(r)/(N-Z)$ isoscalar!

$A/(N-Z)$ normalizing factor global... Similar local normalizing factor, in terms of intense quantities, $2a_a^V/\mu_a$, where $a_a^V \equiv S(\rho_0)$

Asymmetric density (formfactor for isovector density) defined:

$$\rho_a(r) = \frac{2a_a^V}{\mu_a}\left[\rho_n(r) - \rho_p(r)\right]$$

Normal matter $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on η!

In any nucleus:

$$\boxed{\rho_{n,p}(r) = \frac{1}{2}\left[\rho(r) \pm \frac{\mu_a}{2a_a^V}\rho_a(r)\right]}$$

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Net isoscalar density ρ usually parameterized w/Fermi function

$$\rho(r) = \frac{1}{1+\exp\left(\frac{r-R}{d}\right)} \qquad \text{with} \quad R = r_0\, A^{1/3}$$

Isovector density ρ_a?? Related to $a_a(A)$ & to $S(\rho)$!

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NSCL

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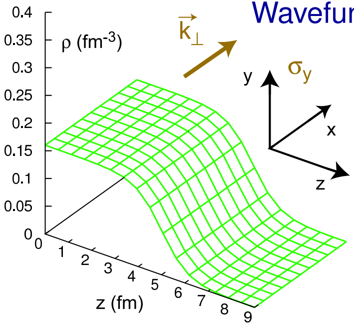
Half-Infinite Matter in Skyrme-Hartree-Fock

To one side infinite uniform matter & vacuum to the other



Wavefunctions: $\Phi(\boldsymbol{r}) = \phi(z)\, e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp}$

matter interior/exterior:

$$\phi(z) \propto \sin\left(k_z\, z + \delta(\boldsymbol{k})\right)$$

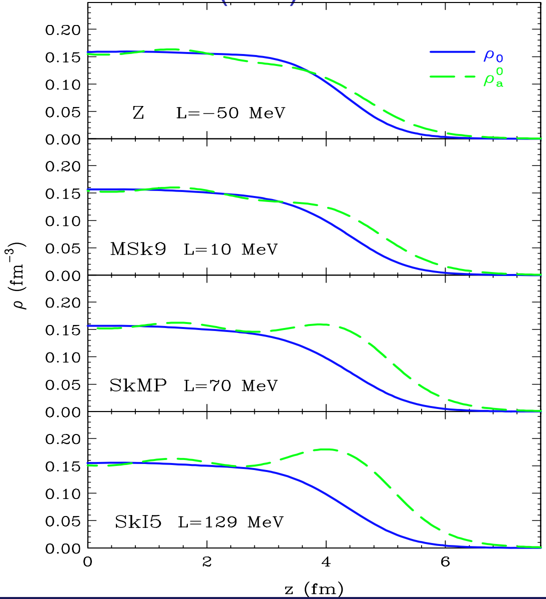
$$\phi(z) \propto e^{-\kappa(\boldsymbol{k}) z}$$

Discretization in $\boldsymbol{k}$-space. Set of 1D HF eqs solved using Numerov's method until self-consistency:

$$-\frac{d}{dz} B(z) \frac{d}{dz} \phi(z) + \left(B(z)\, k_\perp^2 + U(z) \right) \phi(z) = \epsilon(\boldsymbol{k})\, \phi(z)$$

Before: Farine *et al*, NPA338(80)86

Isoscalar (Net) & Isovector Densities from SHF

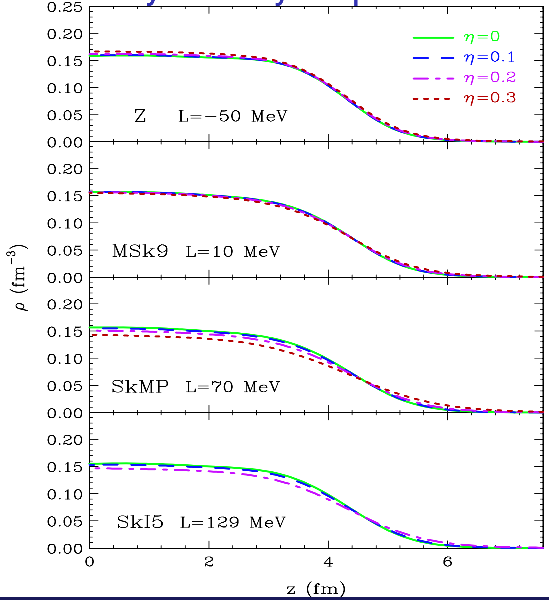


Results for different Skyrme interactions in half-infinite matter.

Net & isovector densities displaced relative to each other.

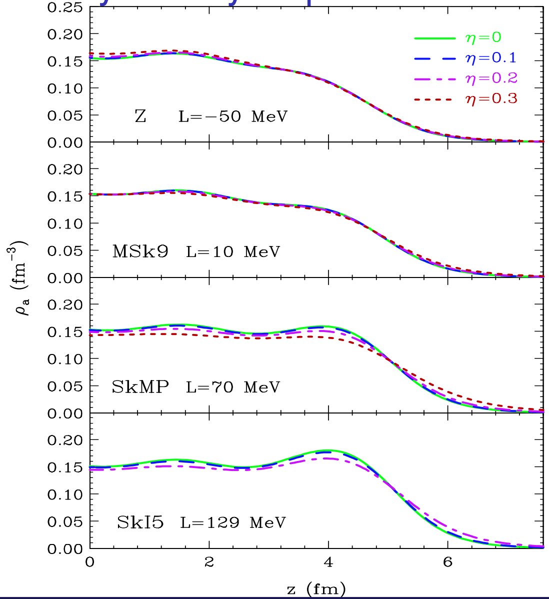
As symmetry energy changes gradually, so does the displacement.

Asymmetry Dependence of Net Density



Results for different asymmetries

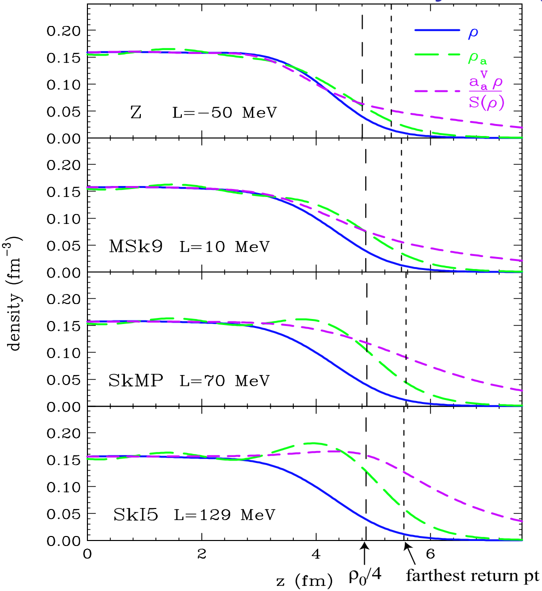
Asymmetry Dependence of Isovector Density



$$\rho_a = \frac{2a_a^V}{\mu_a}\,(\rho_n - \rho_p)$$

Results for different asymmetries

Sensitivity to $S(\rho)$



For weakly nonuniform matter, expected asymmetric density $\rho_a = \rho\, a_a^V / S(\rho)$

Isovector density ρ_a oscillates around the expectation down to $\rho \simeq \rho_0/4$

WKB Analysis

Semiclassical wavefunction

$$\phi_{\boldsymbol{k}^\infty}(z) \propto \begin{cases} \sin\left(\int_z^{z_0} k_z(z')\,\mathrm{d}z'\right) & \text{allowed} \quad z < z_0 \\ \exp\left(-\int_{z_0}^{z} \kappa_z(z')\,\mathrm{d}z'\right) & \text{forbidden} \quad z > z_0 \end{cases}$$

Density from

$$\rho_\alpha(z) = \int \mathrm{d}\boldsymbol{k}^\infty \, |\phi_{\boldsymbol{k}^\infty\alpha}(z)|^2$$

Findings:

At $z < z_0$

$$\rho_a(z) \approx \frac{a_a^V \, \rho}{S(\rho)} \left(1 + \frac{\rho^{2/3}}{S(\rho)} \mathcal{F}\right)$$

where $\mathcal{F}(z) \propto \sin\left(2k_F(z_0 - z)\right)$, describing Friedel oscillations around ρ/S, up to the classical return point z_0.

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Skyrme Parameters

Name	a_V	m^*/m	K	a_a^V	L	a_S	a_a^S	ΔR
SkT	-15.40	0.602	333	24.8	28.2	14.2	17.5	0.477
SkT1	-15.98	1.000	236	32.0	56.2	18.2	14.6	0.799
SkT2	-15.94	1.000	235	32.0	56.2	18.0	14.7	0.794
SkT3	-15.94	1.000	235	31.5	55.3	17.7	15.3	0.776
SkT4	-15.95	1.000	235	35.4	94.1	18.1	11.5	0.986
SkT5	-16.00	1.000	201	37.0	98.5	18.1	10.9	1.084
SkM1	-15.77	0.789	216	25.1	-35.3	17.4	59.6	0.180
SkI1	-15.95	0.693	242	37.5	161.0	17.4	11.4	1.126
Gσ	-15.59	0.784	237	31.3	94.0	16.0	10.1	0.929
Rσ	-15.59	0.783	237	30.5	85.7	16.0	10.5	0.888
T	-15.93	1.000	235	28.3	27.2	17.7	22.6	0.587
Z	-15.97	0.842	330	26.8	-49.7	17.7	51.5	0.213
Zσ	-15.88	0.783	233	26.6	-29.3	17.0	46.6	0.233
Z$\sigma\sigma$	-15.96	0.775	234	28.8	-4.5	17.3	29.3	0.406

Symmetry Coefficient

surface region

$$\frac{A}{a_a(A)} = \frac{1}{a_a^V}\int d^3r\,\rho_a(r) = \frac{1}{a_a^V}\int d^3r\,\rho(r) + \frac{1}{a_a^V}\int d^3r\,(\rho_a-\rho)(r)$$

$$\simeq \frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}$$

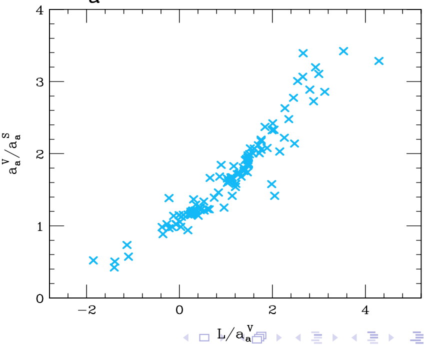
where

$$\frac{a_a^V}{a_a^S} = 4\pi\, r_0^2 \int dr\,(\rho_a-\rho)(r) \simeq \frac{3\,\Delta R}{r_0}$$

and ΔR is displacement of isovector and isoscalar surfaces.

$$\Rightarrow \frac{1}{a_a(A)} \simeq \frac{1}{a_a^V} + \frac{A^{-1/3}}{a_a^S}$$

$a_a^S - L$ Correlation f/SHF



Charge Invariance

?$a_a(A)$? Conclusions on sym-energy details, following E-formula fits, interrelated with conclusions on other terms in the formula: asymmetry-dependent Coulomb, Wigner & pairing + asymmetry-independent, due to $(N-Z)/A$ - A correlations along stability line (PD NPA727(03)233)!

Best would be to study the symmetry energy in isolation from the rest of E-formula! Absurd?!

Charge invariance to rescue: lowest nuclear states characterized by different isospin values (T, T_z), $T_z = (Z-N)/2$. Nuclear energy scalar in isospin space:

sym energy $$E_a = a_a(A)\,\frac{(N-Z)^2}{A} = 4\,a_a(A)\,\frac{T_z^2}{A}$$

$$\rightarrow E_a = 4\,a_a(A)\,\frac{T^2}{A} = 4\,a_a(A)\,\frac{T(T+1)}{A}$$

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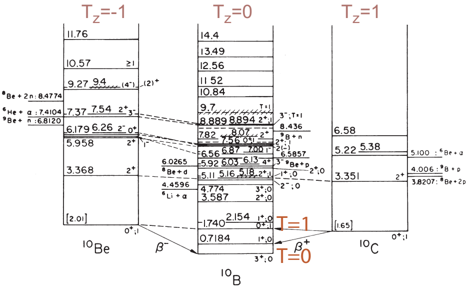
$a_a(A)$ Nucleus-by-Nucleus

$$\rightarrow E_a = 4\, a_a(A)\, \frac{T(T+1)}{A}$$

In the ground state T takes on the lowest possible value $T = |T_z| = |N - Z|/2$. Through '+1' most of the Wigner term absorbed.

Formula generalized to the lowest state of a given T (e.g. Jänecke *et al.*, NPA728(03)23). Pairing depends on evenness of T.

?Lowest state of a given T: isobaric analogue state (IAS) of some neighboring nucleus ground-state.



Study of changes in the symmetry term possible nucleus by nucleus

IAS Data Analysis

In the same nucleus:

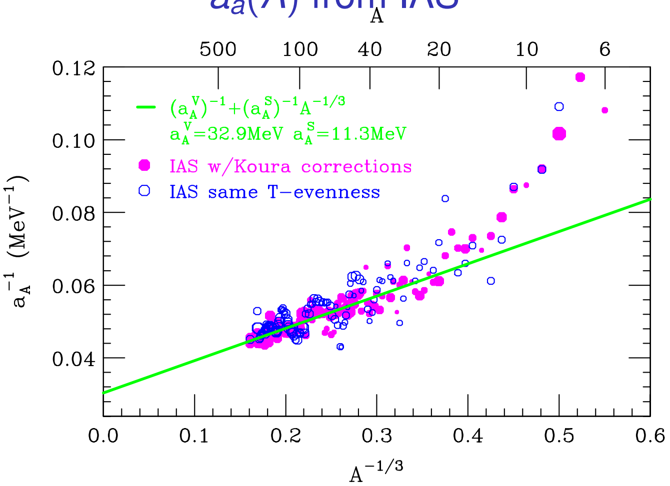
$$E_2(T_2) - E_1(T_1) = \frac{4\,a_a}{A}\{T_2(T_2+1) - T_1(T_1+1)\} + E_{\text{mic}}(T_2, T_z) - E_{\text{mic}}(T_2, T_z)$$

$$a_a^{-1}(A) = \frac{4\,\Delta T^2}{A\,\Delta E} \overset{?}{=} (a_a^V)^{-1} + (a_a^S)^{-1}\,A^{-1/3}$$

Data: Antony *et al.* ADNDT66(97)1

E_{mic}: Koura *et al.*, ProTheoPhys113(05)305
v Groote *et al.*, AtDatNucDatTab17(76)418
Moller *et al.*, AtDatNucDatTab59(95)185

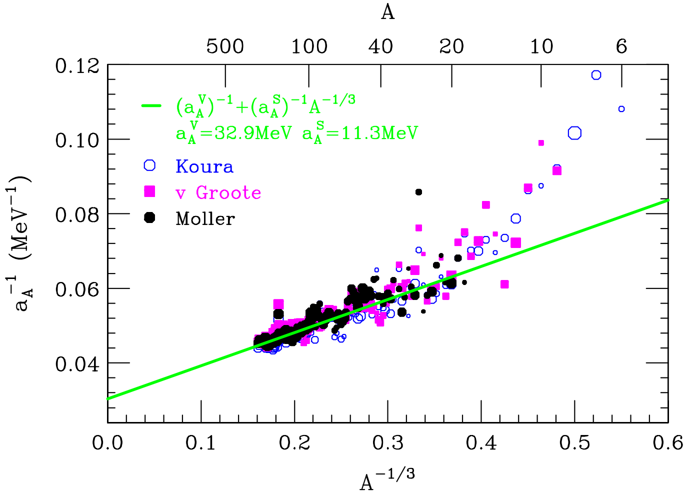
$a_a(A)$ from IAS



Symbol size proportional to relative significance.

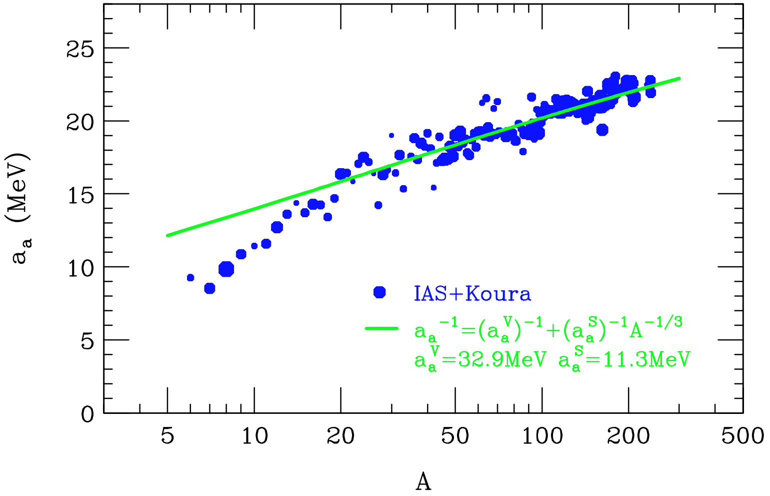
∼Linear dependance from $A \gtrsim 20$ on.

Different Shell Corrections



Line: best fit to Koura at $A > 20$.

a_a vs A



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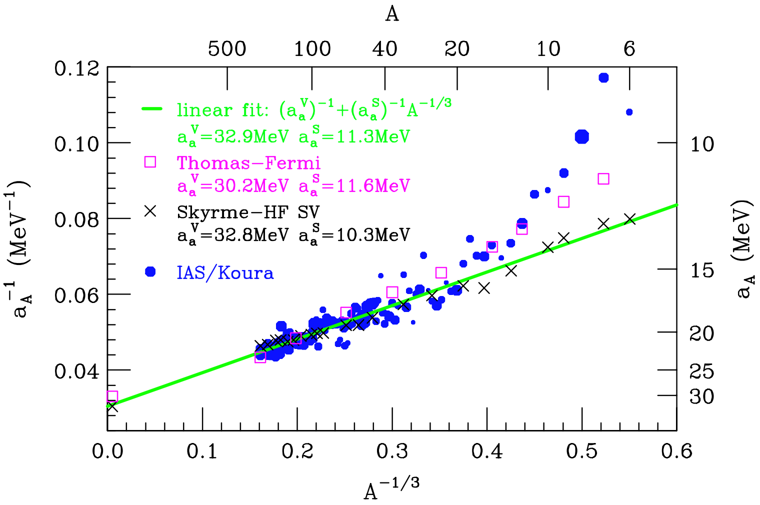
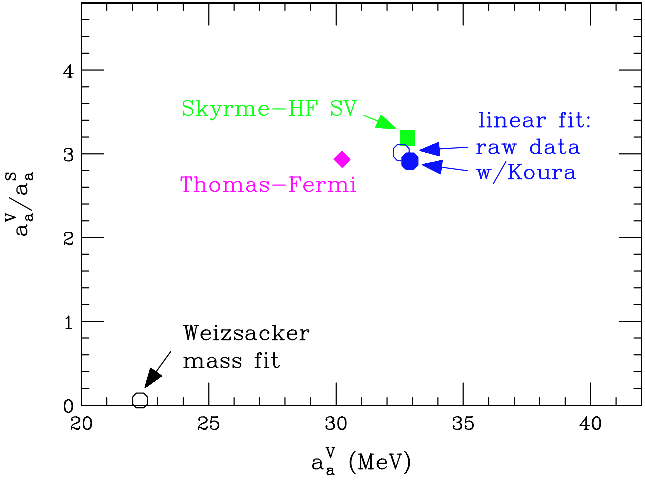
Best $a(A)$-Descriptions



. . . some problems w/extracting $a_a(A)$ from SHF for finite nuclei

Symmetry-Energy Parameters

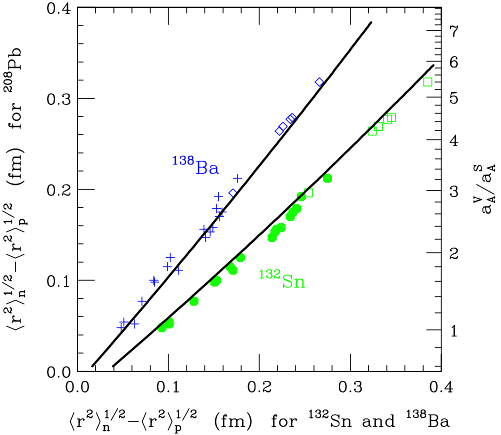


$a_a^V = (31.5 - 33.5)\,\text{MeV}$, $a_a^S = (9.5 - 12)\,\text{MeV}$, $L \sim 95\,\text{MeV}$

Analytic Expression for Skin Size

symmetry energy only | Coulomb correction

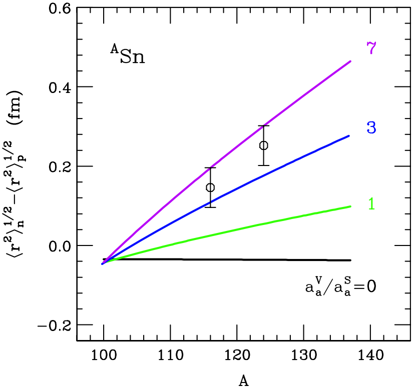
$$\frac{\langle r^2\rangle_n^{1/2}-\langle r^2\rangle_p^{1/2}}{\langle r^2\rangle^{1/2}} = \frac{A}{6NZ}\,\frac{N-Z}{1+A^{1/3}\,a_a^S/a_a^V} - \frac{a_C}{168a_a^V}\,\frac{A^{5/3}}{N}\,\frac{\frac{10}{3}+A^{1/3}\,a_a^S/a_a^V}{1+A^{1/3}\,a_a^S/a_a^V}$$

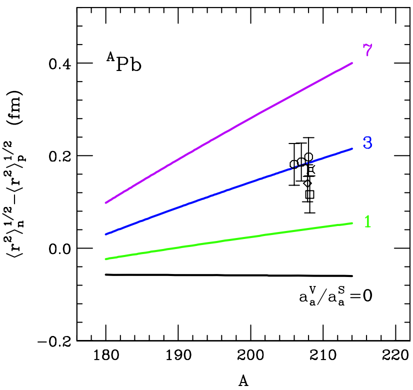


Formula (lines) vs Typel & Brown results (symbols) from nonrelativistic & relativistic mean-field calculations, PRC64(01)027302

Skin Sizes for Sn & Pb Isotopes

Lines - formula predictions, PD NPA727(03)233

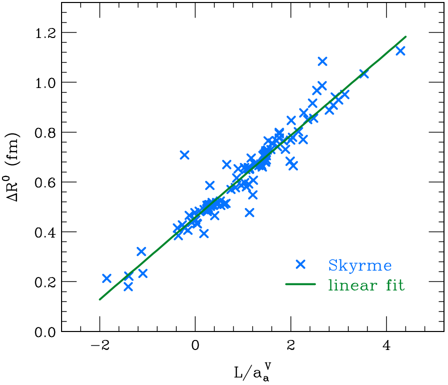


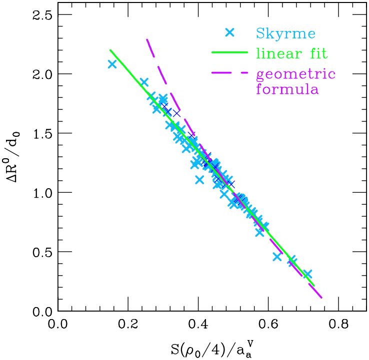


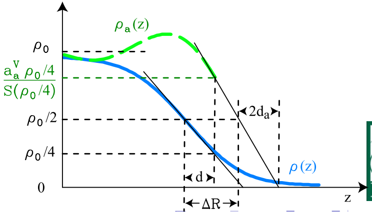
Favored ratio $a_a^V/a_a^S \simeq 32.5/10.8 \simeq 3.0$

Displacement of Isovector Surface

$$\langle r^2\rangle_n^{1/2} - \langle r^2\rangle_p^{1/2} \simeq \frac{A\,(N-Z)}{4NZ}\,\frac{\langle r^2\rangle_a - \langle r^2\rangle}{\langle r^2\rangle^{1/2}}$$







Conclusions

- Symmetry energy weakens as nuclear mass number decreases; for $A \gtrsim 20$, $a_a(A) \simeq a_a^V/(1 + a_a^V/a_a^s A^{1/3})$, where $a_a^V = (31.5 - 33.5)\,\text{MeV}$, $a_a^S = (9.5 - 12)\,\text{MeV}$.
- Skin sizes in all nuclei quantifiable in terms of single ratio, already known, $a_a^V/a_a^S \simeq 3.0$. Corresponding $L \sim 95\,\text{MeV}$.
- Systematic of proton densities for one A should principally contain as much info as the skins and even more: $S(\rho)$ for $\rho \gtrsim \rho_0/4$.
  Issues: shell, pairing, deformation, Coulomb effects.
- Two fundamental densities characterize nucleon distributions in nuclei: isoscalar & isovector. Their surfaces are displaced from each other by $\Delta R \simeq 0.95\,\text{fm}$ and different diffusenesses, $d \sim 0.54\,\text{fm}$ and $d_a \sim 0.40\,\text{fm}$.
- Outlook: finite nuclei - Coulomb & shell effects, learning on finer $S(\rho)$-details from $\rho_p(r)$

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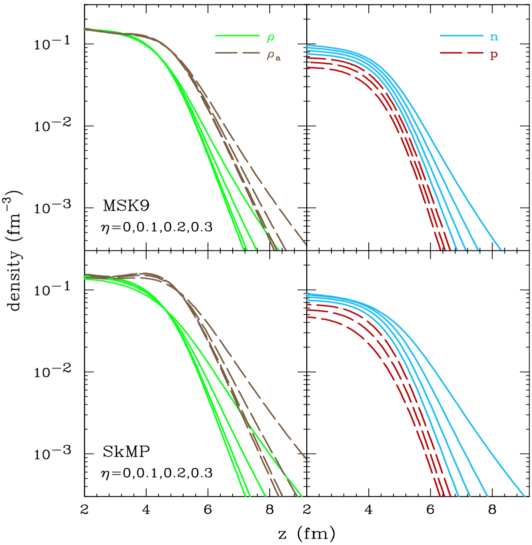
Density Tails



Two Skyrme interactions + different asymmetries

Modified Binding Formula

$$E = -a_V\,A + a_S\,A^{2/3} + a_C\,\frac{Z^2}{A^{1/3}} + \frac{a_a^V}{1 + A^{-1/3}\,a_a^V/a_a^S}\,\frac{(N-Z)^2}{A}$$

Energy Formula Performance:

Fit residuals f/light asymmetric nuclei, either following the Bethe-Weizsäcker formula (open symbols) or the modified formula with $a_a^V/a_a^S = 2.8$ imposed (closed), i.e. the same parameter No.