The Equation of State in Astrophysical Applications

James M. Lattimer

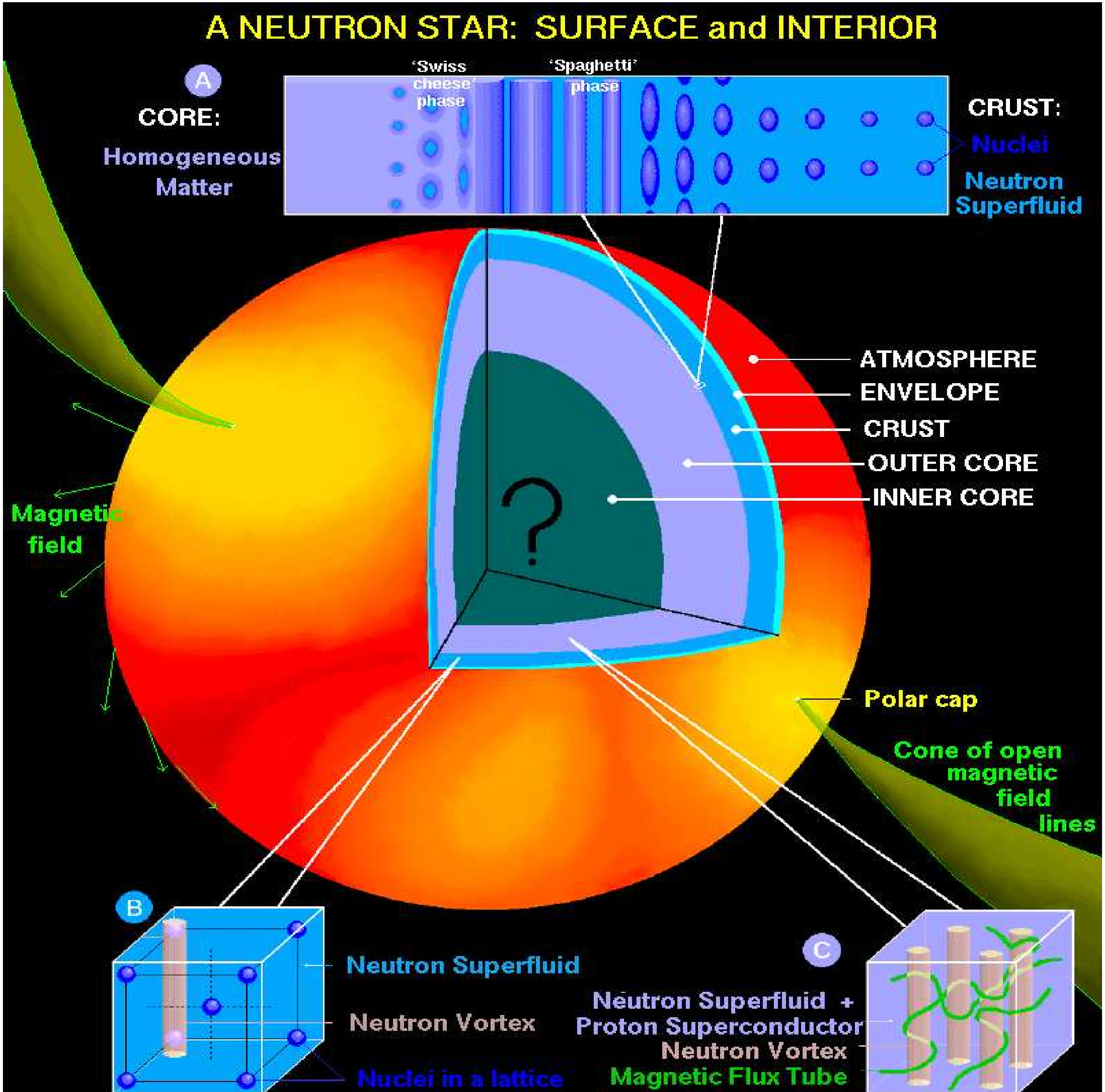
lattimer@astro.sunysb.edu

Department of Physics & Astronomy

Stony Brook University

Outline

- Modeling Dense Matter
  - Uniform Hadronic Matter
  - Exotic Components
  - Matter at Subsaturation Densities
- Laboratory Constraints on Matter Properties
  - Nuclear Structure
  - Heavy Ion Collisions
- Astrophysical Constraints on Dense Matter
  - Theoretical Extremes
  - Pulsars
  - Cooling Neutron Stars
  - Giant Flares and Accreting Sources
  - Neutrinos
  - Gravitational Radiation



Credit: Dany Page, UNAM

Modeling Dense Matter – Uniform Matter

- Phenomenological
  - Contact Non-Relativistic Interactions (Skyrme Forces)
  - Finite-Range Non-Relativistic Interactions
    - Gogny
    - Seylar-Blanchard, modified by Myers & Swiatecki
  - Relativistic Mean Field Models
    - Walecka (meson-exchange) models
    - Density-dependent couplings or masses
- Variational or *ab initio* Calculations
  - DBHF
  - Hypernetted chain

Skyrme Interactions

$$\mathcal{H} = \mathcal{H}_B + \frac{1}{2}\left[Q_{nn}(\vec{\nabla} n_n)^2 + 2Q_{np}\vec{\nabla} n_n \cdot \vec{\nabla} n_p + Q_{pp}(\vec{\nabla} n_p)^2\right] + \mathcal{H}_C + \mathcal{H}_J\,,$$

$$\begin{aligned}
\mathcal{H}_B &= \sum_t \frac{\tau_t}{2M} + U\,,\\
U &= n\left(\tau_n + \tau_p\right)\left[\frac{t_1}{4}\left(1 + \frac{x_1}{2}\right) + \frac{t_2}{4}\left(1 + \frac{x_2}{2}\right)\right]\\
&\quad + \left(\tau_n n_n + \tau_p n_p\right)\left[\frac{t_2}{4}\left(\frac{1}{2} + x_2\right) - \frac{t_1}{4}\left(\frac{1}{2} + x_1\right)\right]\\
&\quad + \frac{t_0}{2}\left[\left(1 + \frac{x_0}{2}\right)n^2 - \left(\tfrac{1}{2} + x_0\right)\left(n_n^2 + n_p^2\right)\right]\\
&\quad + \frac{t_3}{12}\left[\left(1 + \frac{x_3}{2}\right)n^2 - \left(\frac{1}{2} + x_3\right)\left(n_n^2 + n_p^2\right)\right] n^{\epsilon}\,,\\
n &= \sum_t n_t = \sum_t \int \frac{d^3p}{(2\pi)^3} f_t\,, \qquad \tau_t = \int \frac{d^3p}{(2\pi)^3} p^2 f_t\,,\\
f_t^{-1} &= 1 + \exp\left[\frac{1}{T}\left(\frac{p^2}{2m_t^*} + V_t - \mu_t\right)\right],\\
\frac{1}{2m_t^*} &= \frac{\delta\mathcal{H}}{\delta\tau_t}\,, \qquad V_t = \frac{\delta\mathcal{H}}{\delta n_t}\,,
\end{aligned}$$

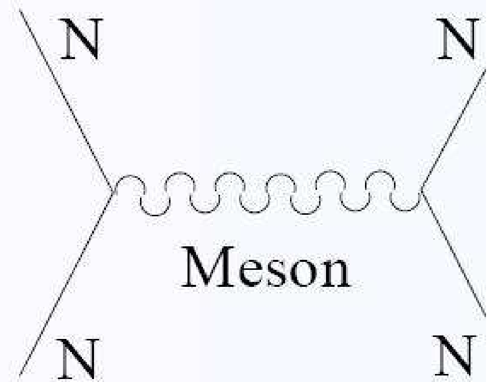
$$Q_{nn} = Q_{pp} = \frac{3}{16}\left[t_1\left(1 - x_1\right) - t_2\left(1 + x_2\right)\right]\,, \quad Q_{np} = Q_{pn} = \frac{1}{8}\left[3t_1\left(1 + \frac{x_1}{2}\right) - t_2\left(1 + \frac{x_2}{2}\right.\right.$$

Walecka model for dense nuclear matter

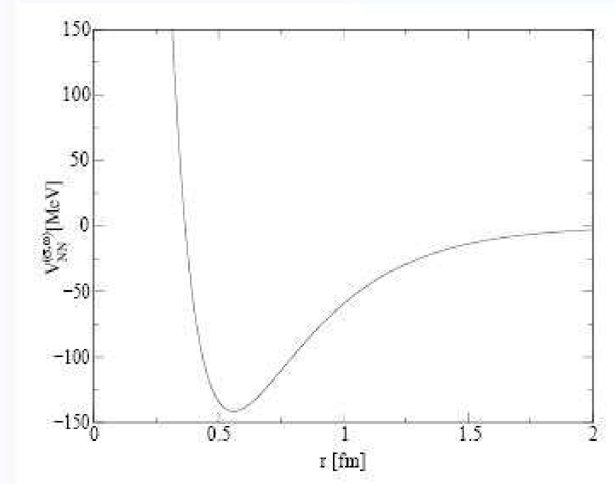
Meson exchange model

example: scalar (σ) meson

$$
\begin{aligned}
(-\triangle + m_\sigma^2)\sigma(\vec{r}) &= -g_\sigma \delta(\vec{r}) \\
\Rightarrow \quad \sigma(r) &= -\frac{g_\sigma}{4\pi}\frac{e^{-m_\sigma r}}{r} \\
V_{NN}^{(\sigma)}(r) &= g_\sigma \sigma(r) = -\frac{g_\sigma^2}{4\pi}\frac{e^{-m_\sigma r}}{r}
\end{aligned}
$$



Meson	I^π	T	S	M[MeV]
$\pi^0, \pi^\pm$	0^-	1	0	140
σ	0^+	0	0	≈ 500
$K^0, K^\pm$	0^-	1/2	± 1	495
η	0^-	0	0	550
$\rho^0, \rho^\pm$	1^-	1	0	770
ω	1^-	0	0	780
δ	0^+	1	0	900



Credit: T. Klahn

Relativistic Field-Theoretical Interactions

$$
\begin{aligned}
\mathcal{H} &= \sum_t \int \frac{d^3p}{(2\pi)^3} f_t \sqrt{p^2 + M^{*2}} + V(\sigma) \\
&+ \frac{1}{2}\left[(\nabla\sigma)^2 + m_\sigma^2\sigma^2 - (\nabla\omega)^2 - m_\omega^2\omega^2 - (\nabla b)^2 - m_\rho^2 b^2 + g_\omega \omega n + \frac{g_\rho}{2} b(n_p - n_n)\right] \\
V(\sigma) &= \frac{\kappa}{6}(g_\sigma\sigma)^3 + \frac{\lambda}{24}(g_\sigma\sigma)^4\,, \\
M^* &= M - g_\sigma\sigma\,, \\
\rho_s &= \sum_t \int \frac{d^3p}{(2\pi)^3} f_t \frac{M^*}{\sqrt{p^2 + M^{*2}}} = \frac{m_\sigma^2}{g_\sigma}\sigma + \frac{dV(\sigma)}{d\sigma}\,, \\
n &= \sum_t \int \frac{d^3p}{(2\pi)^3} f_t\,, \\
f_t^{-1} &= 1 + \exp\left[\frac{1}{T}\left(\sqrt{p^2 + M^{*2}} + g_\omega\omega \mp \frac{g_\rho}{2} n - \mu_t\right)\right]\,, \\
\nabla^2\sigma &= m_\sigma^2\sigma - g_\sigma\rho_s + g_\sigma\frac{dV(\sigma)}{d\sigma}\,, \\
\nabla^2\omega &= m_\omega^2\omega - g_\omega n\,, \\
\nabla^2 b &= m_\rho^2 b - \frac{g_\rho}{2}(n_p - n_n)\,.
\end{aligned}
$$

Finite-Range Interactions

$$\mathcal{H} = \sum_t \left[\frac{\tau_t}{2M} - n_t(\alpha_L \widetilde{n_t} + \alpha_U \widetilde{n_{t'}}) + \beta'_L(\tau_t \widetilde{n_t} + n_t \widetilde{\tau_t}) + \beta'_U(n_t \widetilde{\tau_{t'}} + \tau_t \widetilde{n_{t'}})\right]$$

$$+ \sum_t \left[\sigma'_L n_t \left(\widetilde{n_t} n_t^{2/3} + \widetilde{n_t^{5/3}}\right) + \sigma'_U n_t \left(n_t^{2/3} \widetilde{n_{t'}} + \widetilde{n_{t'}^{5/3}}\right)\right] ,$$

$$\widetilde{g(r_1)} \equiv \int d^3 r_2 f(r_{12}/a) g(r_2) , \qquad f(r_{12}/a) = \frac{1}{4\pi r_{12} a^2} e^{-r_{12}/a} ,$$

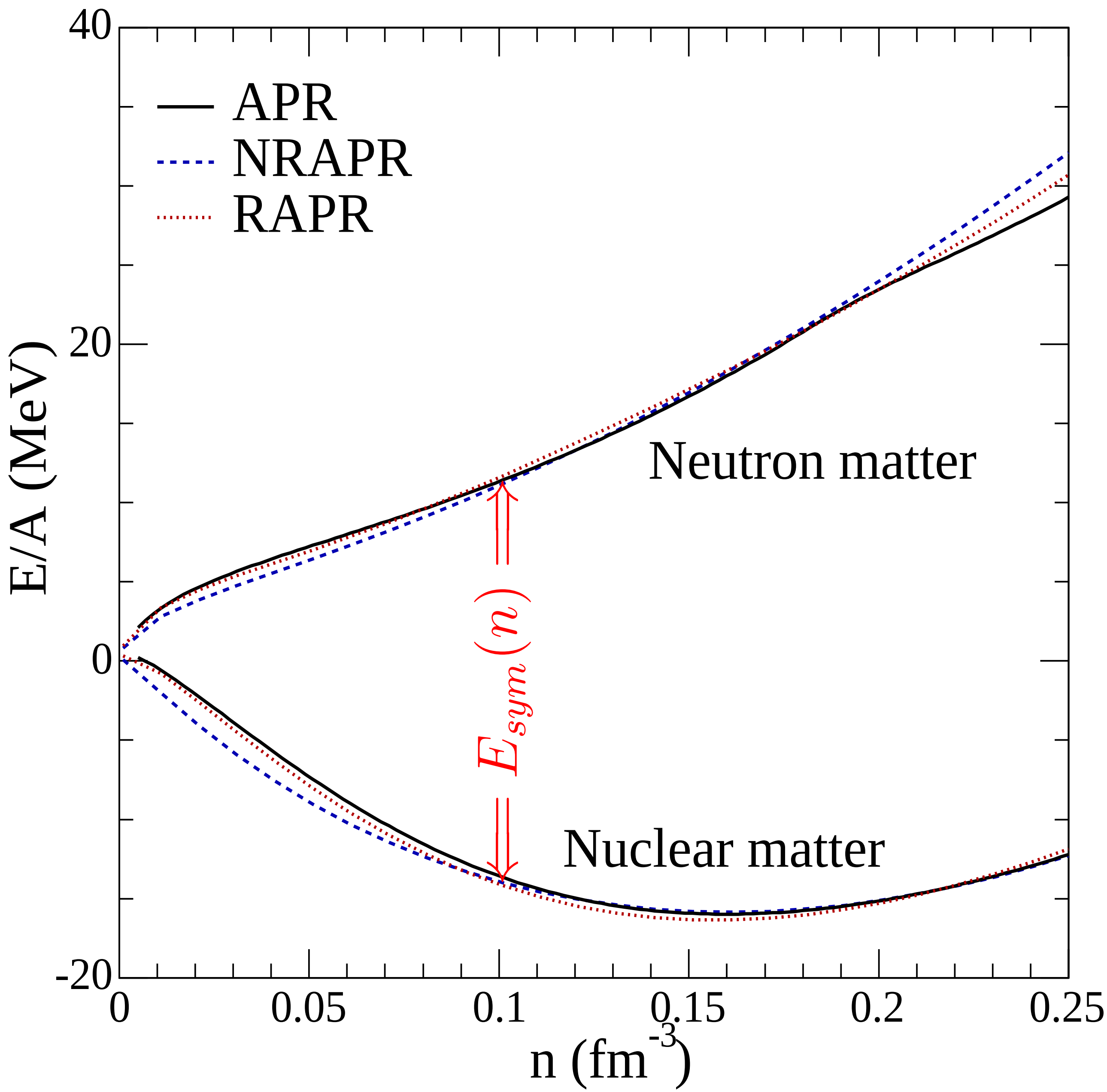
$$n = \sum_t n_t = \sum_t \int \frac{d^3p}{(2\pi)^3} f_t , \qquad \tau_t = \int \frac{d^3p}{(2\pi)^3} p^2 f_t ,$$

$$f_t^{-1} = 1 + \exp\left[\frac{1}{T}\left(\frac{p^2}{2m_t^*} + V_t - \mu_t\right)\right] ,$$

$$\frac{1}{2m_t^*} = \frac{\delta \mathcal{H}}{\delta \tau_t} = \frac{1}{2M} + 2(\beta'_L \widetilde{n_t} + \beta'_U \widetilde{n_{t'}}) ,$$

$$V_t = \frac{\delta \mathcal{H}}{\delta n_t} = -2(\alpha_L \widetilde{n_t} + \alpha_U \widetilde{n_{t'}}) + 2(\beta'_L \widetilde{\tau_t} + \beta'_U \widetilde{\tau_{t'}})$$

$$+ 2\sigma'_L \left(\frac{5}{3} n_t^{2/3} \widetilde{n_t} + \widetilde{n_t^{5/3}}\right) + 2\sigma'_U \left(\frac{5}{3} n_t^{2/3} \widetilde{n_{t'}} + \widetilde{n_{t'}^{5/3}}\right) ,$$



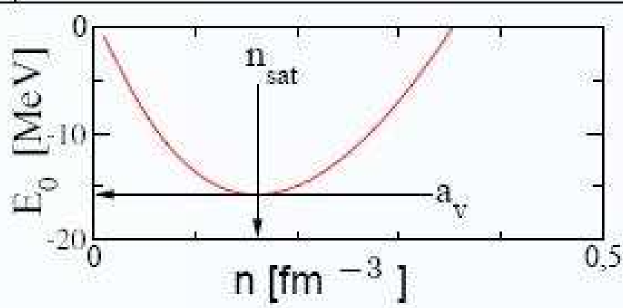
Steinier, Prakash, Lattimer & Ellis 2005

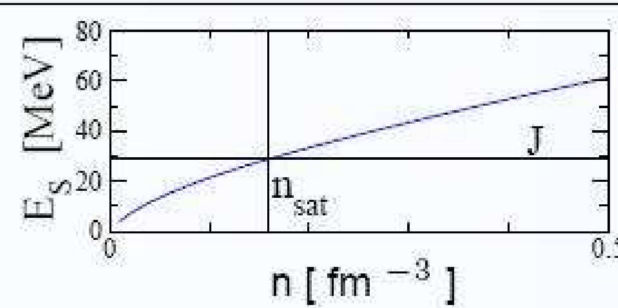
Exploring the Limits - The EoS beyond saturation

$$E(n,\beta) = E_0(n) + \beta^2 E_S(n) \approx a_V + \frac{K}{18}\epsilon^2 - \frac{K'}{162}\epsilon^3 + \ldots + \beta^2\left(J + \frac{L}{3}\epsilon + \ldots\right) + \ldots$$

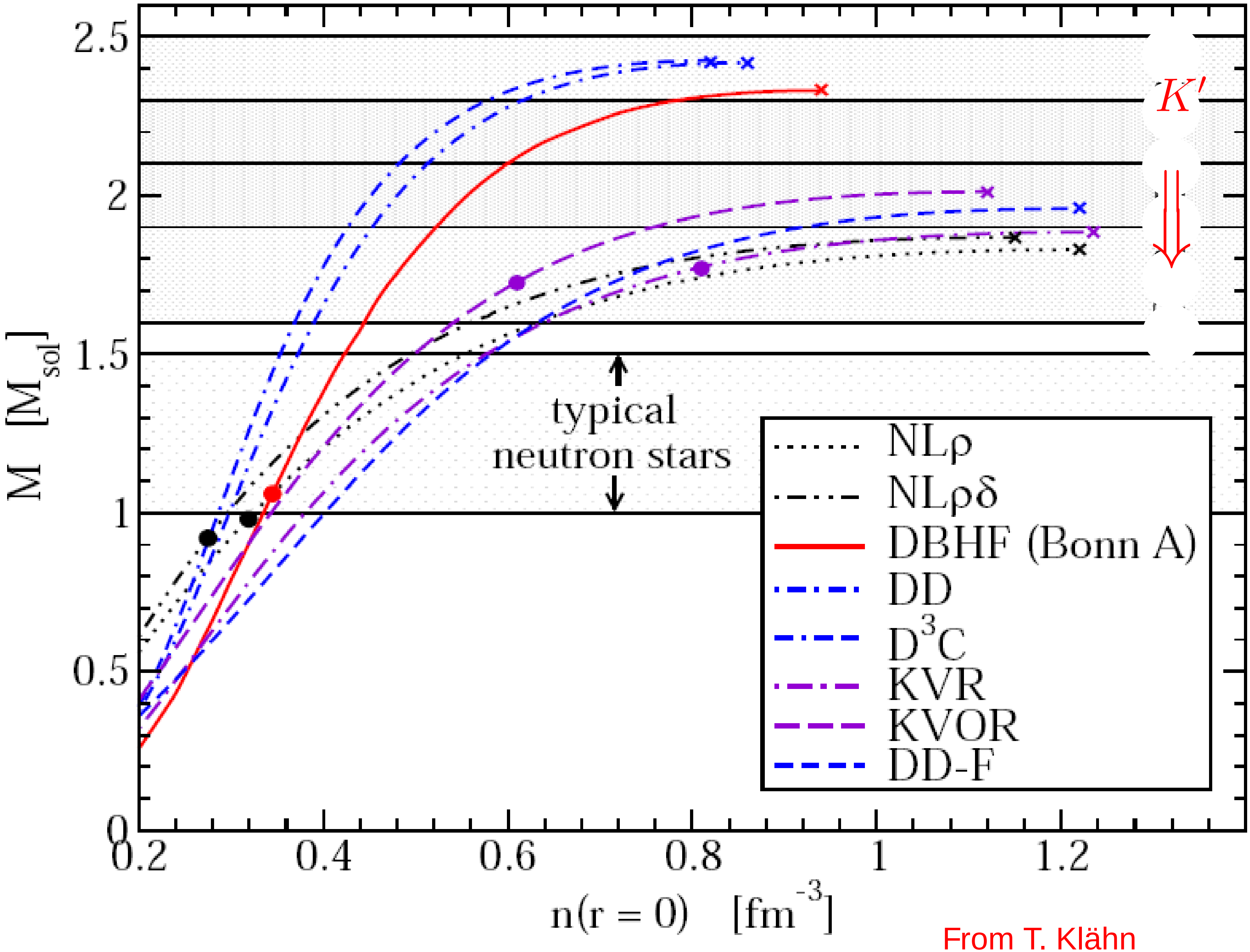
$$\epsilon = (n - n_{sat})/n \qquad \beta = (n_n - n_p)/(n_n + n_p)$$

	$n_{\rm sat}$	a_V	K	K'	J	L	m_D/m
	[fm^{-3}]	[MeV]	[MeV]	[MeV]	[MeV]	[MeV]	
exp.	0.16 ± 0.01	-16 ± 1	$200 - 300$		$25 - 35$		$0.6 - 1.0$
NLρ	0.1459	−16.062	203.3	576.5	30.8	83.1	0.603
NL$\rho\delta$	0.1459	−16.062	203.3	576.5	31.0	92.3	0.603
DBHF	0.1779	−16.160	201.6	507.9	33.7	69.4	0.684
DD	0.1487	−16.021	240.0	−134.6	32.0	56.0	0.565
D^{3}C	0.1510	−15.981	232.5	−716.8	31.9	59.3	0.541
KVR	0.1600	−15.800	250.0	528.8	28.8	55.8	0.800
KVOR	0.1600	−16.000	275.0	422.8	32.9	73.6	0.800
DD-F	0.1469	−16.024	223.1	757.8	31.6	56.0	0.556

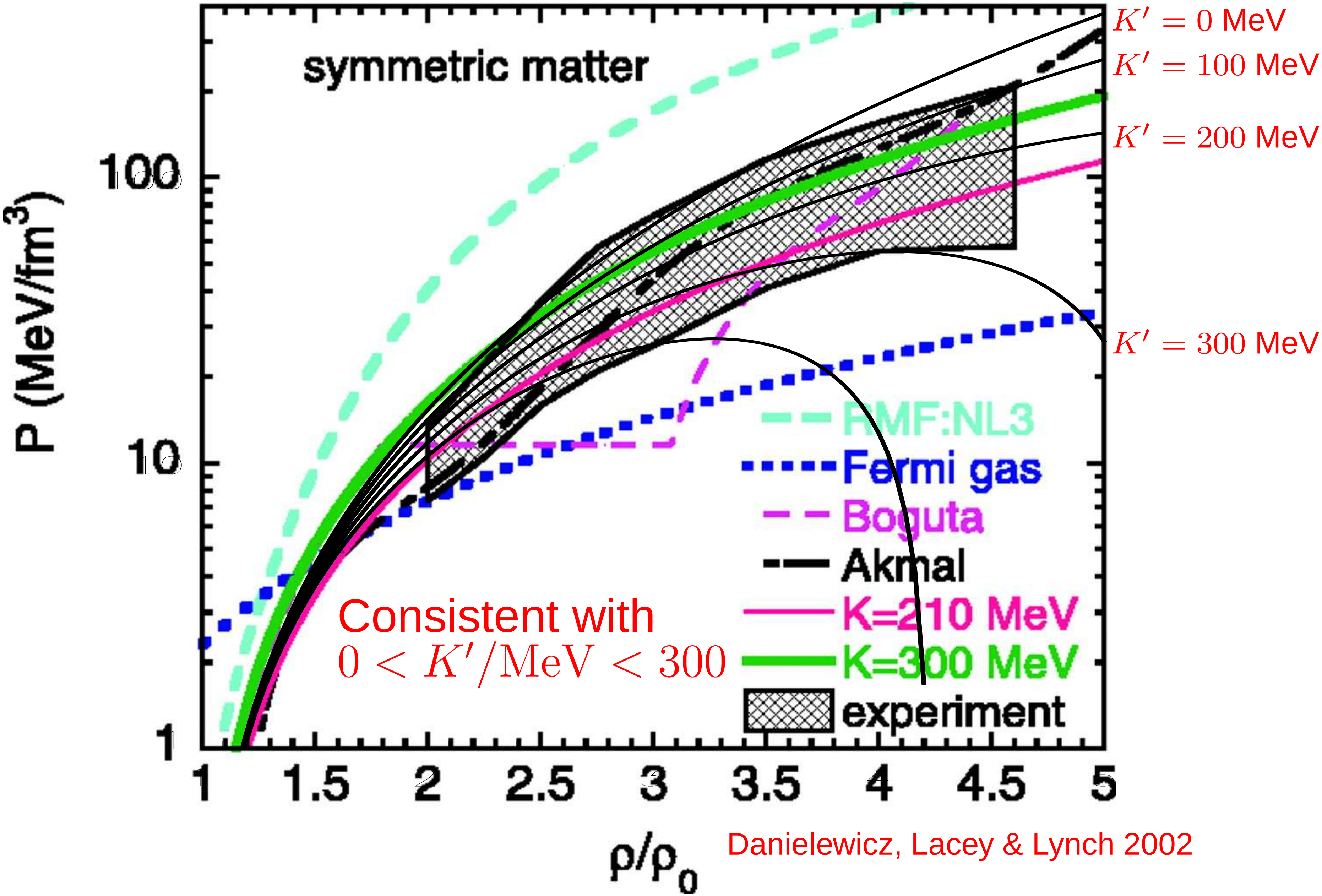




T. Klähn et al., Phys.Rev.C **74**, 035802 (2006)


M [M_{sol}]
n(r = 0) [fm^{-3}]
typical
neutron stars
K'
NLρ
NLρδ
DBHF (Bonn A)
DD
D^3C
KVR
KVOR
DD-F
From T. Klähn

Flow Constraint From Heavy Ions



Danielewicz, Lacey & Lynch 2002

Exotic Components

- Normal Stars
  - Hyperons

    Negatively-charged and lowest-mass hyperons favored; Λ and Σ^- have attractive nucleon interactions. Supported by binding and energy levels of double-Λ hypernuclei; $\Sigma^- - N$ interaction uncertain.
  - Bose condensates
    - Pions

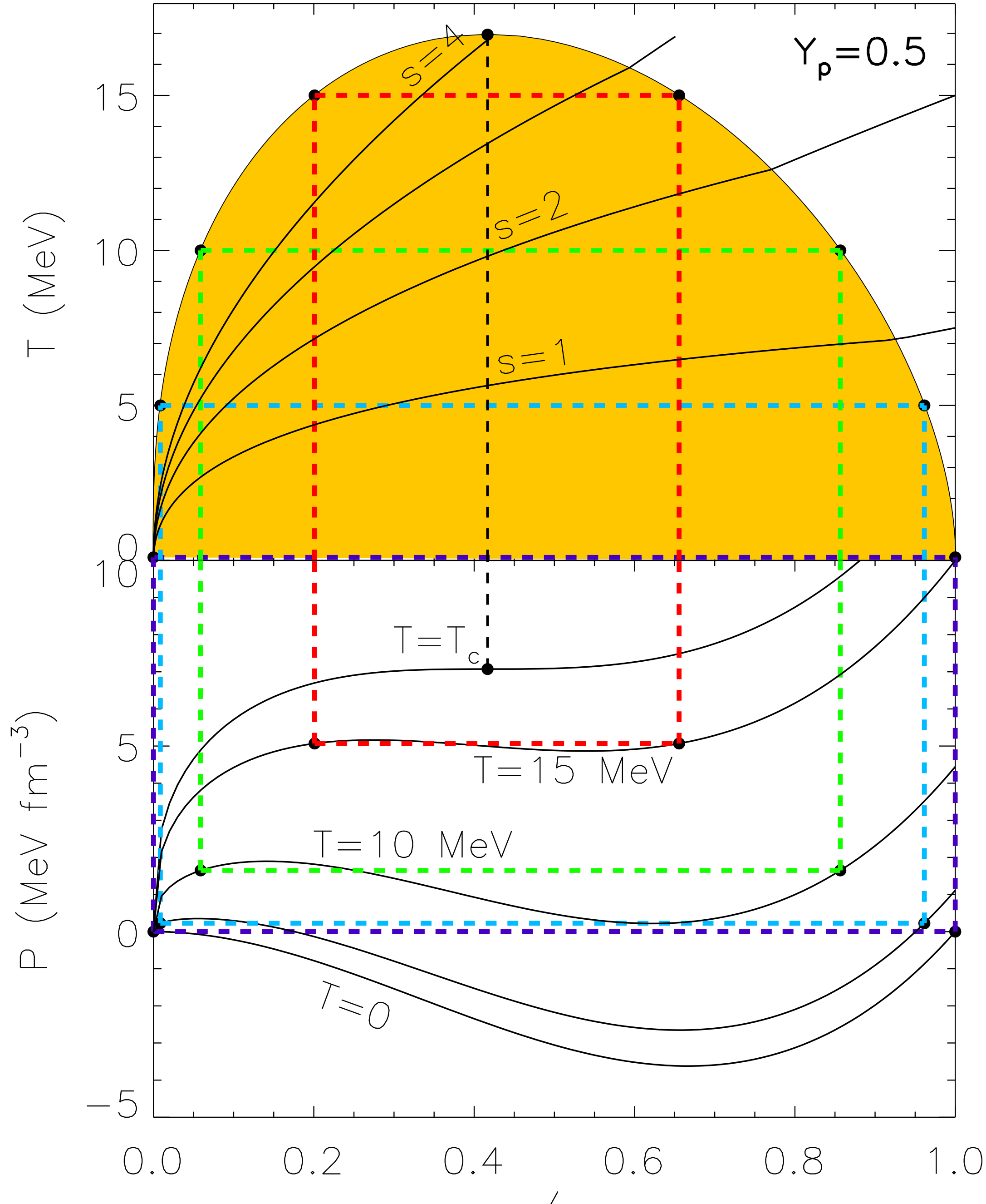
      Inhibited by a repulsive s-wave interaction with nucleons.
    - Kaons

      Possibly enhanced by attractive $K^- - N$ interaction, resulting in kaon mass decreasing with density. $K^+ - N$ interaction weakly repulsive. Supported by enhanced K^- production in heavy ion collisions. Maximum masses ≤ 1.6 M$_\odot$.
  - Deconfined quark matter (hybrid stars)

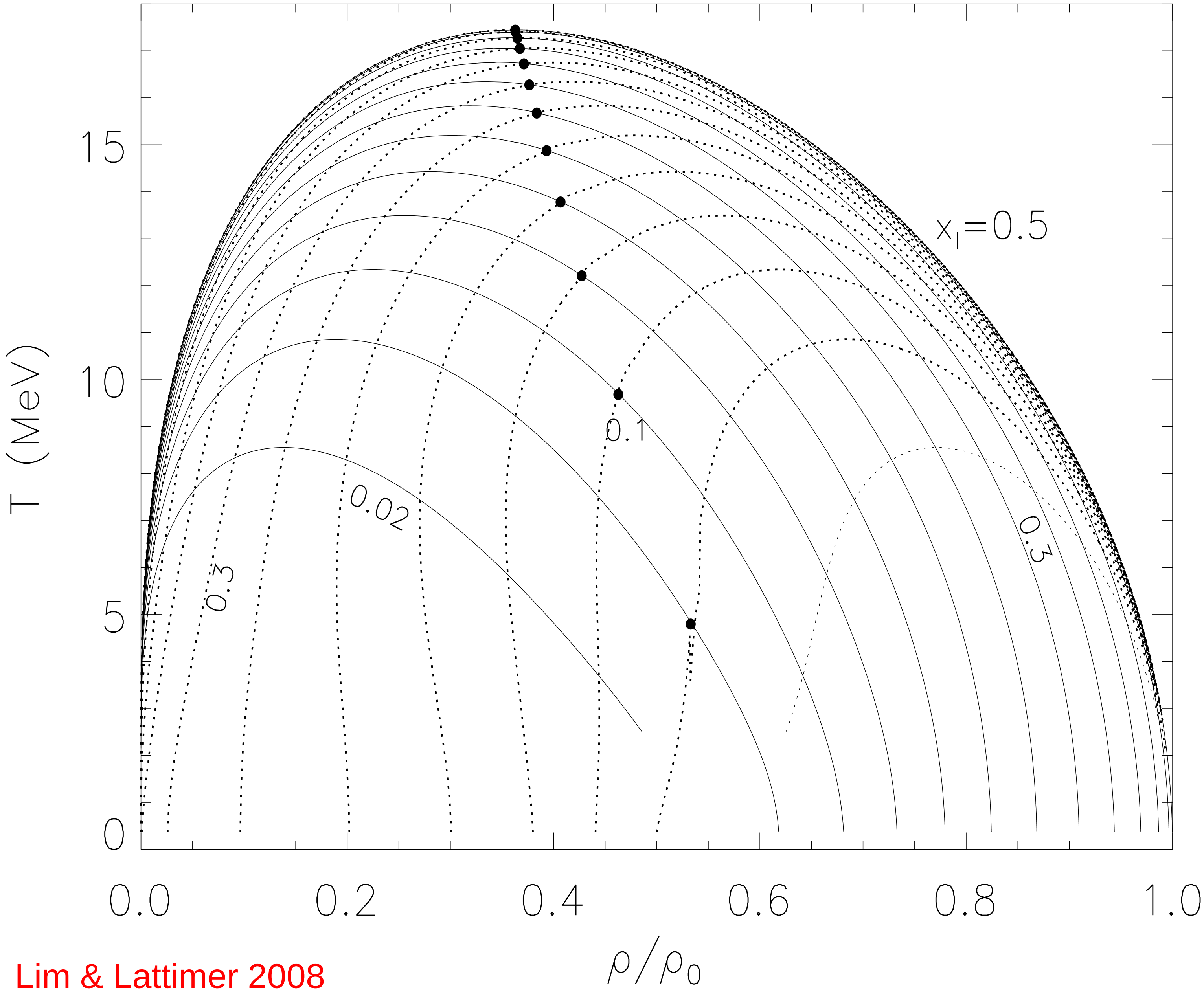
    Maximum masses ≤ 2.1 M$_\odot$.
- Self-Bound Stars
  - Strange quark matter (u, d, s)

    Maximum masses ≤ 2.1 M$_\odot$. But inconsistent with pulsars: Strong, significant, crust needed to anchor magnetic field; without crust, fields decay in $< 10^3$ yr. Glitches imply crust with $I_{crust}/I > 0.01$; $I_{crust}/I_{SQS} \sim 10^{-5}$.

Matter at Subsaturation Densities



Coexistence Curves



Lim & Lattimer 2008

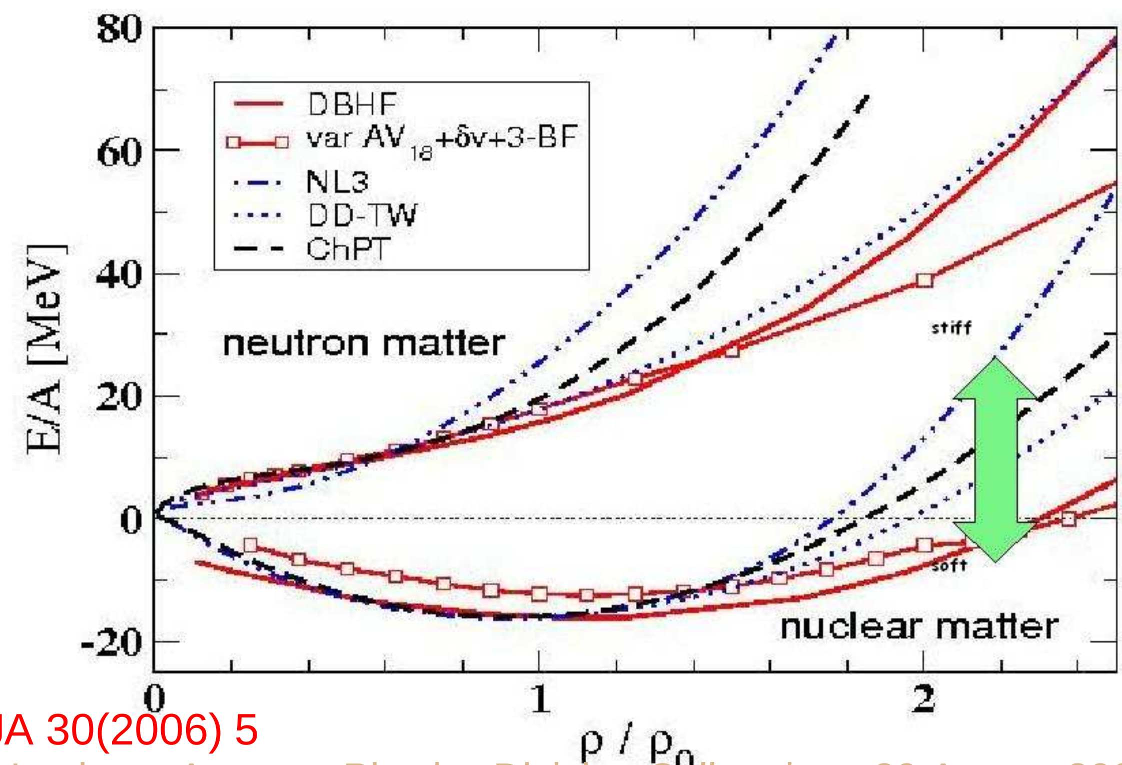
Nuclear Symmetry Energy

The density dependence of $E_{sym}(n)$ is crucial. Some information is available from nuclei (for $n < n_s$). Heavy ion collisions have potential for constraining it for $n > n_s$. It is common to expand $E_{sym}(n)$ as

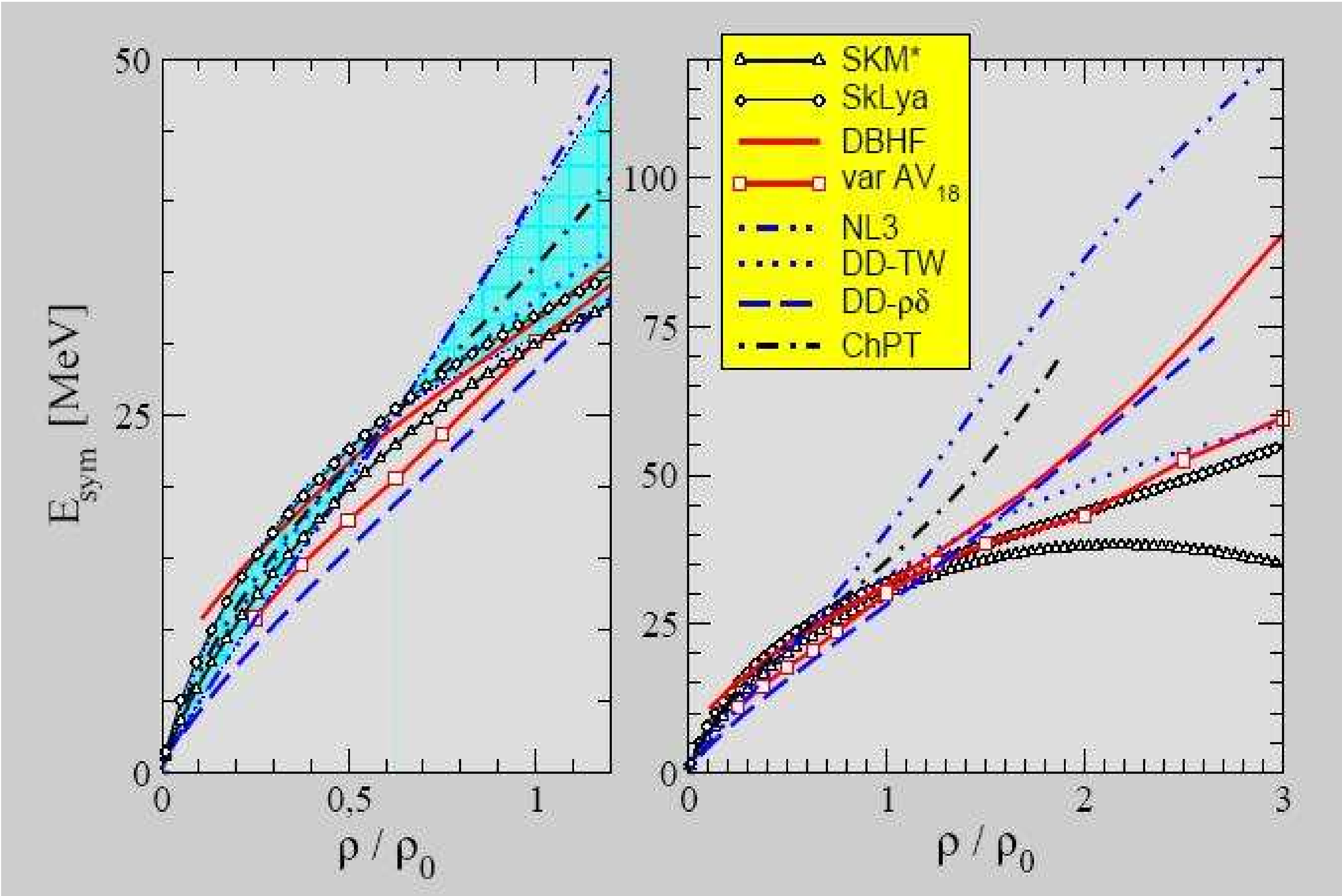
$$E_{sym}(n) \simeq J + \frac{L}{3}\left(\frac{n}{n_s} - 1\right) + \frac{K_{sym}}{18}\left(\frac{n}{n_s} - 1\right)^2 + \cdots$$

$$J = E_{sym}(n_s), \qquad L = 3n_s\left(\frac{\partial E_{sym}}{\partial n}\right)_{n_s}, \qquad K_{sym} = 9n_s^2\left(\frac{\partial^2 E_{sym}}{\partial n^2}\right)_{n_s}$$

Almost no information is available for K_{sym}.



C. Fuchs, H.H. Wolter, EPJA 30(2006) 5



C. Fuchs, H.H. Wolter, EPJA 30(2006) 5

Nuclear Mass Formula

Bethe-Weizsäcker (neglecting pairing and shell effects)

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2.$$

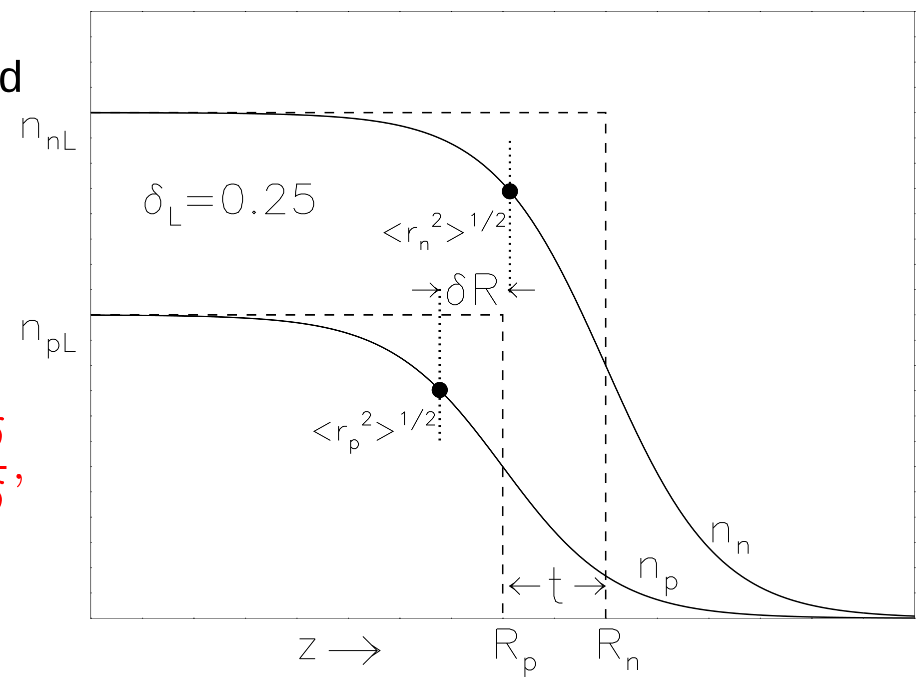
Myers & Swiatecki introduced the surface asymmetry term:

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2 - S_s (N-Z)^2/A^{4/3}.$$

Droplet extension: consider the neutron/proton asymmetry of the nuclear surface.

$$E(A,Z) = (-a_v + S_v \delta^2)(A - N_s) + a_s A^{2/3} + a_C Z^2/A^{1/3} + \mu_n N_s.$$

N_s is the number of excess neutrons associated with the surface, $I = (N-Z)/(N+Z)$, $\delta = 1 - 2x = (A - N_s - 2Z)/(A - N_s)$ is the asymmetry of the nuclear bulk fluid, and μ_n is the neutron chemical potential. From thermodynamics,



$$N_s = -\frac{\partial a_s A^{2/3}}{\partial \mu_n} = \frac{S_s}{S_v} \frac{\delta}{1-\delta} = A \frac{I - \delta}{1 - \delta},$$

$$\delta = I \left(1 + \frac{S_s}{S_v A^{1/3}}\right)^{-1},$$

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2 \left(1 + \frac{S_s}{S_v A^{1/3}}\right)^{-1}.$$

Nuclear Structure Considerations

Information about E_{sym} can be extracted from nuclear binding energies and models for nuclei. For example, consider the schematic liquid droplet model (Myers & Swiatecki):

$$E(A,Z) \simeq -a_v A + a_s A^{2/3} + \frac{S_v}{1+(S_s/S_v)A^{-1/3}} A + a_c Z^2 A^{-1/3}$$

Fitting binding energies results in a strong correlation between S_v and S_s, but not definite values.

Blue: $\Delta E < 0.01$ MeV/b

Green: $\Delta E < 0.02$ MeV/b

Gray: $\Delta E < 0.03$ MeV/b

Circle: Moeller et al. (1995)

Crosses: Best fits

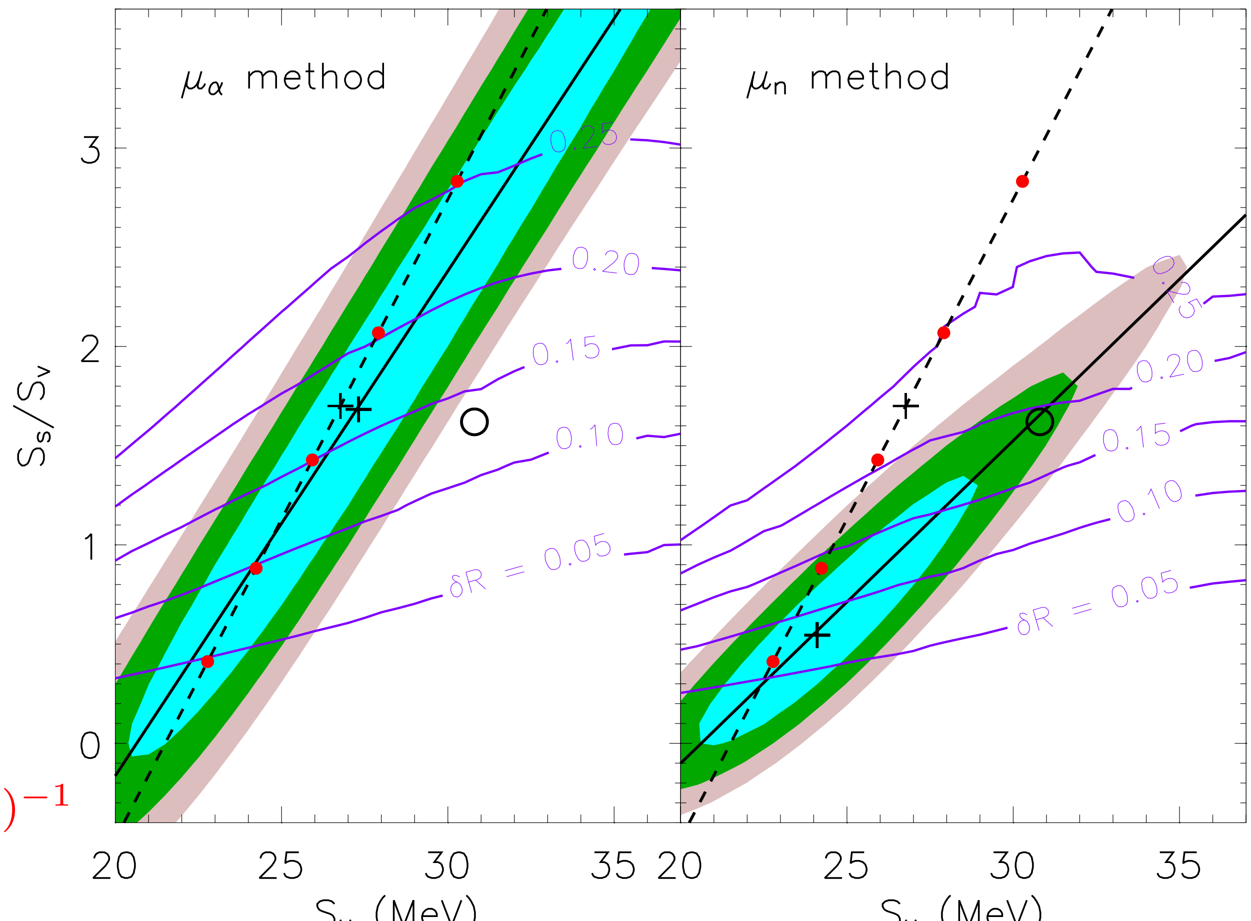
Dashed: Danielewicz (2004)

Solid: Steiner et al. (2005)

$a_C = \frac{3e^2}{5r_0} f$

$\mu_\alpha : f = 1 + \frac{A}{6Z}(1 + \frac{S_v}{S_s} A^{1/3})^{-1}$

$\mu_n : f = 1$



Schematic Dependence

Nuclear Hamiltonian:

$$H = H_B + \frac{Q}{2} n'^2 \,, \quad H_B \simeq n \left[-B + \frac{K}{18}\left(1 - \frac{n}{n_s}\right)^2 \right] + E_{sym}(1-2x)^2$$

Lagrangian minimization of energy with respect to n (symmetric matter):

$$H_B - \mu_0 n = \frac{Q}{2} n'^2 = \frac{K}{18} n \left(1 - \frac{n}{n_s}\right)^2 \,, \qquad \mu_0 = -a_v$$

Liquid Droplet surface parameters: $a_s = 4\pi r_0^2 \sigma_0,\qquad S_s = 4\pi r_0^2 \sigma_\delta$

$$\sigma_0 \quad = \quad \int_{-\infty}^{+\infty} [H - \mu_0 n] dz = 2 \int_0^{n_s} (H_B - \mu_0 n) \frac{dn}{n'} = \frac{4}{45}\sqrt{QKn_s^3}$$

$$t_{90-10} \quad = \quad \int_{0.1 n_s}^{0.9 n_s} \frac{dn}{n'} = 3\sqrt{\frac{Qn_s}{K}} \int_{0.1}^{0.9} \frac{du}{\sqrt{u}(1-u)} \simeq 9\sqrt{\frac{Qn_s}{K}}$$

$$\sigma_\delta \quad = \quad S_v \sqrt{\frac{Q}{2}} \int_0^{n_s} n \left(\frac{S_v}{E_{sym}} - 1 \right) (H_B - \mu_0 n)^{-1/2} dn$$

$$\frac{S_s}{S_v} \quad = \quad \frac{t_{90-10}}{r_0} \int_0^1 \frac{\sqrt{u}}{1-u} \left(\frac{S_v}{E_{sym}} - 1 \right) du, \qquad \delta R = \sqrt{\frac{3}{5}} r_0 \frac{S_s}{S_v} \frac{N-Z}{3Z}$$

$$E_{sym} \quad \simeq \quad S_v \left(\frac{n}{n_s} \right)^p \Longrightarrow \int \rightarrow 0.28 \; (p = \frac{1}{2}) \,, \; 0.93 \; (p = \frac{2}{3}) \,, \; 2.0 \; (p = 1)$$

$$E_{sym} \quad \simeq \quad S_v + \frac{L}{3} \left(\frac{n}{n_s} - 1 \right) \Longrightarrow \int \rightarrow 2 - 2\sqrt{\frac{3S_v}{L} - 1} \tan^{-1} \sqrt{\left(1 + \frac{S_v}{3L}\right)^{-1}} \simeq \frac{2L}{3S_v}$$

Nuclei in Dense Matter

- Baym, Bethe & Pethick (1971): Zero-temperature compressible liquid drop model, external neutron gas (modifies surface energy), Coulomb lattice effects via the Wigner-Seitz approximation. Surface energy does not decrease fast enough with density.
- Ravenhall, Bennett & Pethick : Surface energies from a semi-infinite interface using Hartree-Fock.
- Negele & Vautherin : Zero-temperature Hartree-Fock computation of density profiles in high-density matter using a Skyrme interaction.
- Bonche & Vautherin : Finite-temperature Hartree-Fock.
- Lattimer, Pethick, Ravenhall & Lamb (1985): Finite-temperature compressible liquid droplet model using a Skyrme force; consistent surface energy from a Thomas-Fermi semi-infinite interface; Wigner-Seitz.
- Lattimer & Swesty (1991): Table generation with LPRL-like EOS with adjustable nuclear force.
- Shen, Toki, Oyamatsu & Sumiyoshi (1998): Table generation with relativistic mean field model using Thomas-Fermi approximation, simplified gradient terms, and parametrized density profiles; Wigner-Seitz.

The Astrophysical EOS: The Next Generation

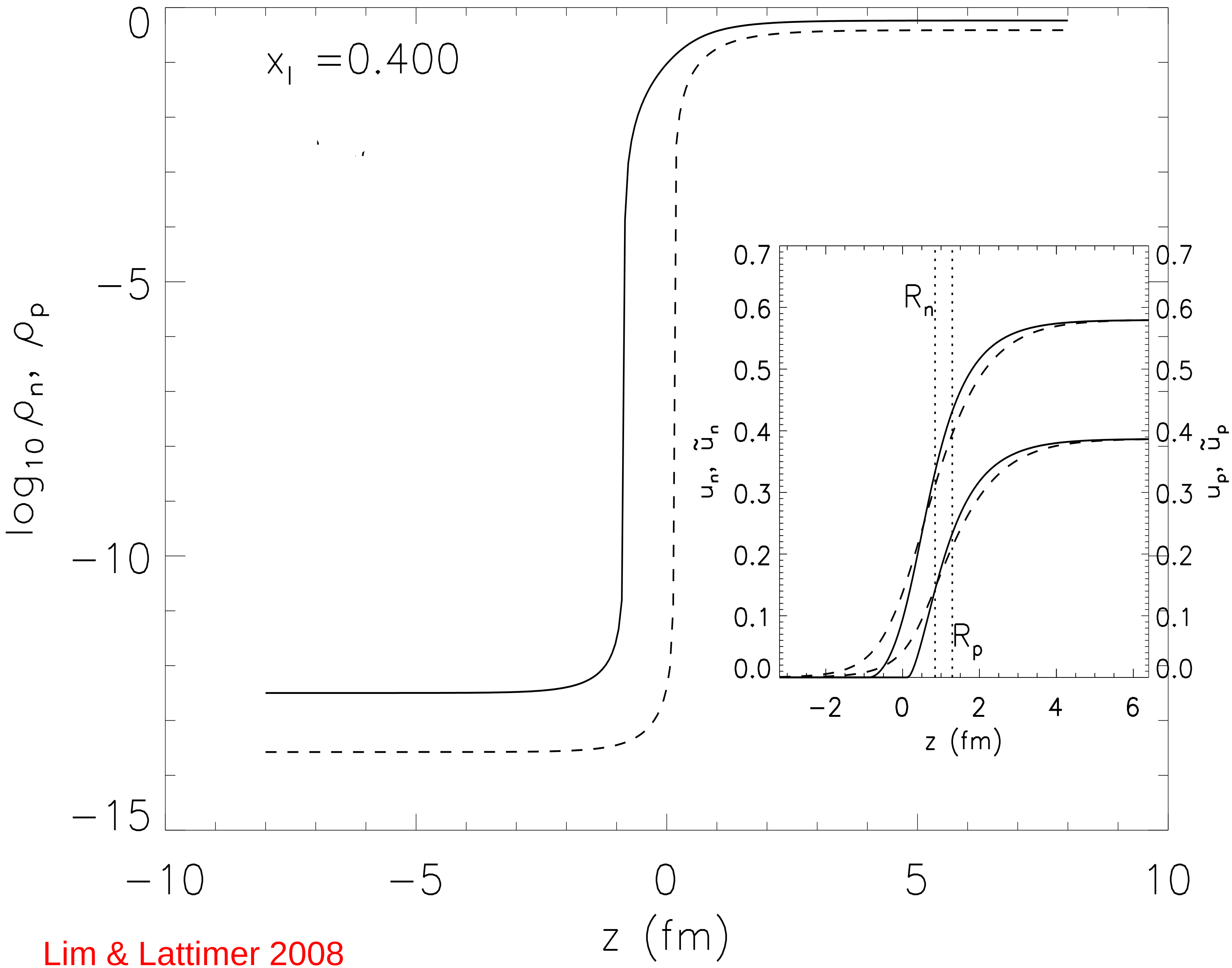
- Consistent Thomas-Fermi model with density profiles determined from energy minimization.
- Non-relativisitic potential and relativistic mean field interactions yield coupled, second-order differential Euler equations for nucleon densities or fields.
  - Example: Skyrme interaction

$$\mathcal{H} = \mathcal{H}_B = \frac{1}{2}\sum_{tt'} Q_{tt'} \nabla n_t \cdot \nabla n_{t'} \Longrightarrow \sum_{t'} Q_{tt'} \nabla^2 n_{t'} = \frac{\partial H_B}{\partial n_t} - \mu_t^0$$

  - Such equations are difficult to solve.
- Finite-range forces have Euler equations from energy minimization that are more stable than differential equations. Essentially, they are integral equations:

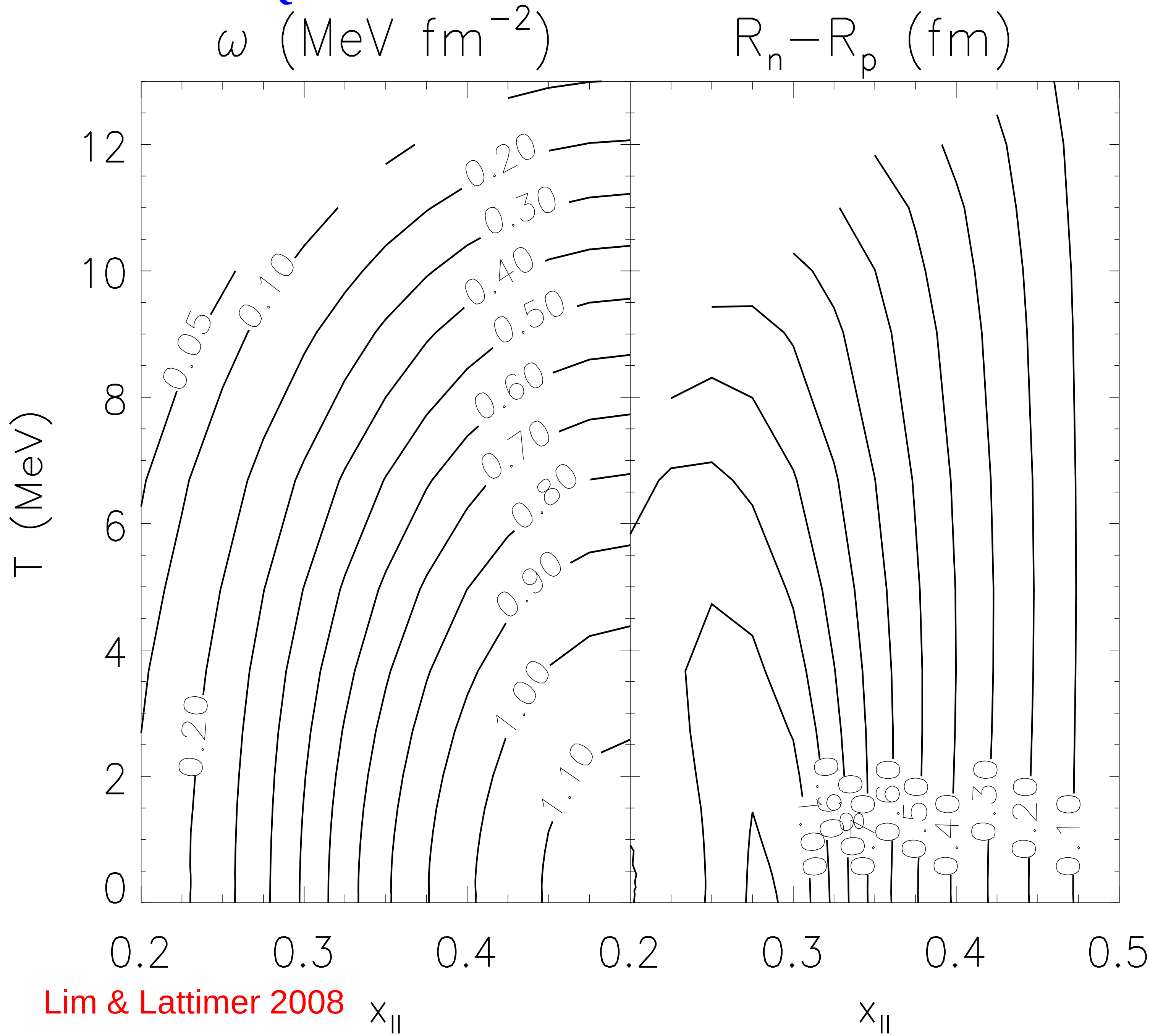
$$\mu_t(r) = \text{ constant}$$

Semi-Infinite Interface



Lim & Lattimer 2008

Surface Quantities From Semi-Infinite Interfaces



Lim & Lattimer 2008

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m+4\pi pr^3)(\epsilon+p)}{r(r-2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\epsilon}{c^2}r^2$$

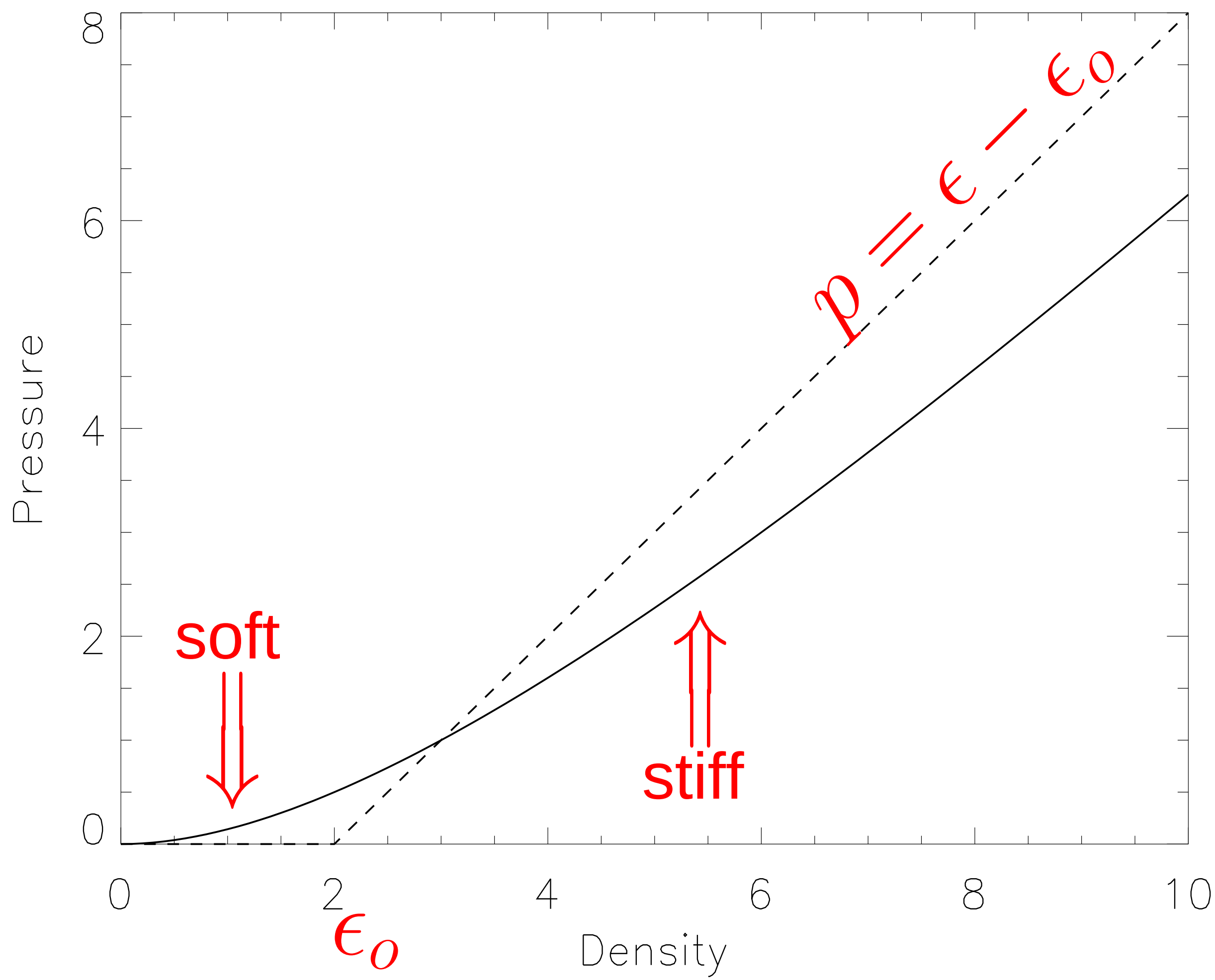
p is pressure, ϵ is mass-energy density

Useful analytic solutions exist:

- Uniform density $\epsilon =$ constant
- Tolman VII $\epsilon = \epsilon_c[1-(r/R)^2]$
- Buchdahl $\epsilon = \sqrt{pp_*} - 5p$

Extreme Properties of Neutron Stars

- The most compact configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".



Maximally Compact Equation of State

Koranda, Stergioulas & Friedman (1997)

$$\begin{aligned} p(\epsilon) = 0, \qquad \epsilon &\leq \epsilon_o \\ p(\epsilon) = \epsilon - \epsilon_o, \qquad \epsilon &\geq \epsilon_o \end{aligned}$$

This EOS has a parameter ϵ_o, which corresponds to the surface energy density. The structure equations then contain only this one parameter, and can be rendered into dimensionless form using

$$y = m\epsilon_o^{1/2}, \qquad x = r\epsilon_o^{1/2}, \qquad q = p\epsilon_o^{-1}.$$

$$\begin{aligned} \frac{dy}{dx} &= 4\pi x^2(1+q) \\ \frac{dq}{dx} &= -\frac{(y+4\pi q x^3)(1+2q)}{x(x-2y)} \end{aligned}$$

The solution with the maximum central pressure and mass and the minimum radius:

$$\begin{aligned} q_{max} &= 2.026, \qquad y_{max} = 0.0851, \qquad x_{min}/y_{max} = 2.825 \\ p_{max} &= 307\left(\frac{\epsilon_o}{\epsilon_s}\right) \text{ MeV fm}^{-3}, \quad M_{max} = 4.2\left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ M}_\odot, \quad R_{min} = 2.825\frac{GM_{max}}{c^2}. \end{aligned}$$

Moreover, the scaling extends to the axially-symmetric case, yielding

$$P_{min} \propto \left(\frac{M_{max}}{R_{min}^3}\right)^{1/2} \propto \epsilon_o^{-1/2}, \qquad P_{min} = 0.82\left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ ms}$$

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

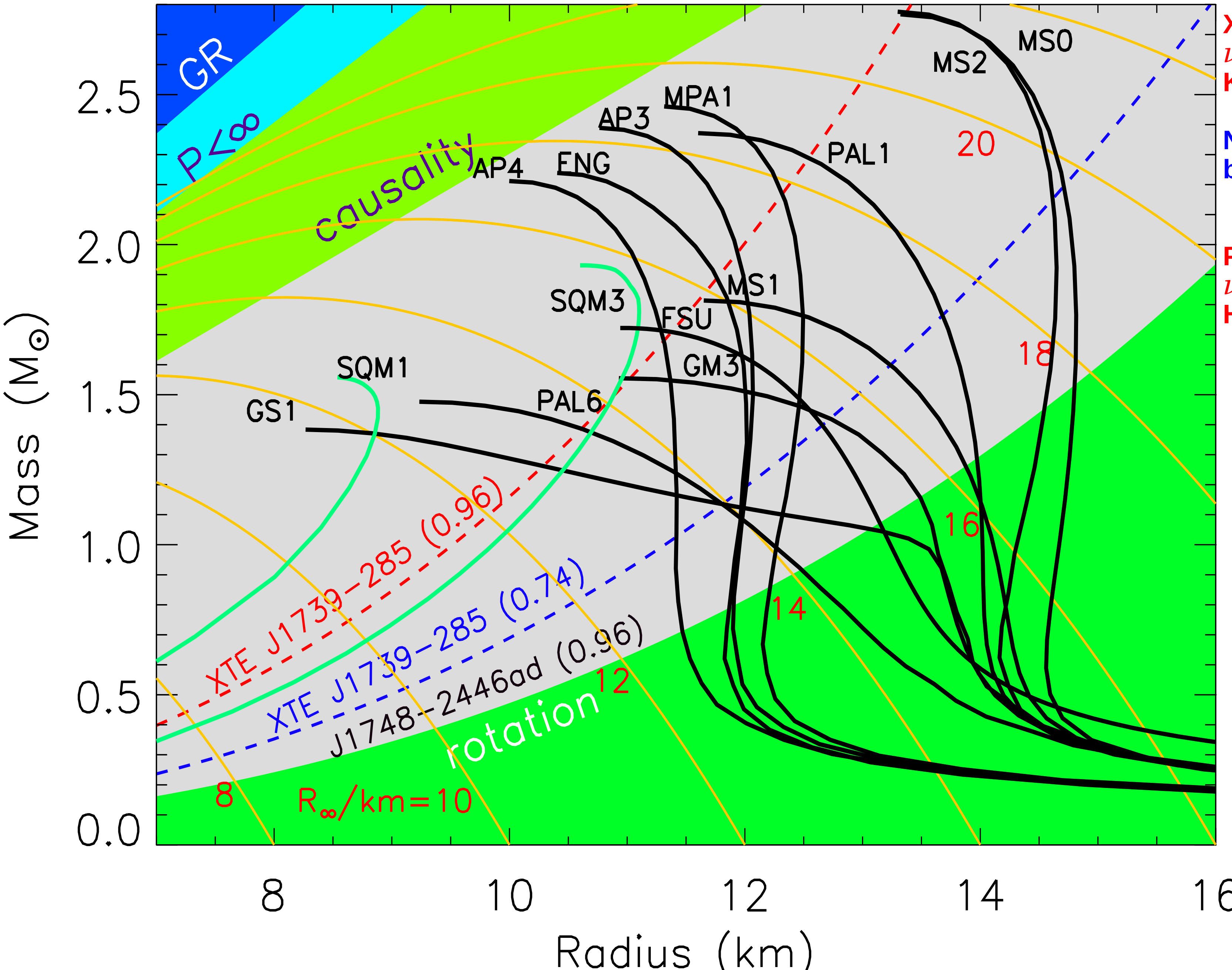
- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ **Rhoades & Ruffini (1974), Hartle (1978)**
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)$ km

  Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)
- $\epsilon_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2$ g cm^{-3} **Lattimer & Prakash (2005)**
- $P_{min} \simeq (0.74 \pm 0.03)(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

  Koranda, Stergioulas & Friedman (1997)
- $P_{min} \simeq 0.96(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms (empirical)

  Lattimer & Prakash (2004)
- $\epsilon_c > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2$ g cm^{-3} (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

Constraints from Pulsar Spins



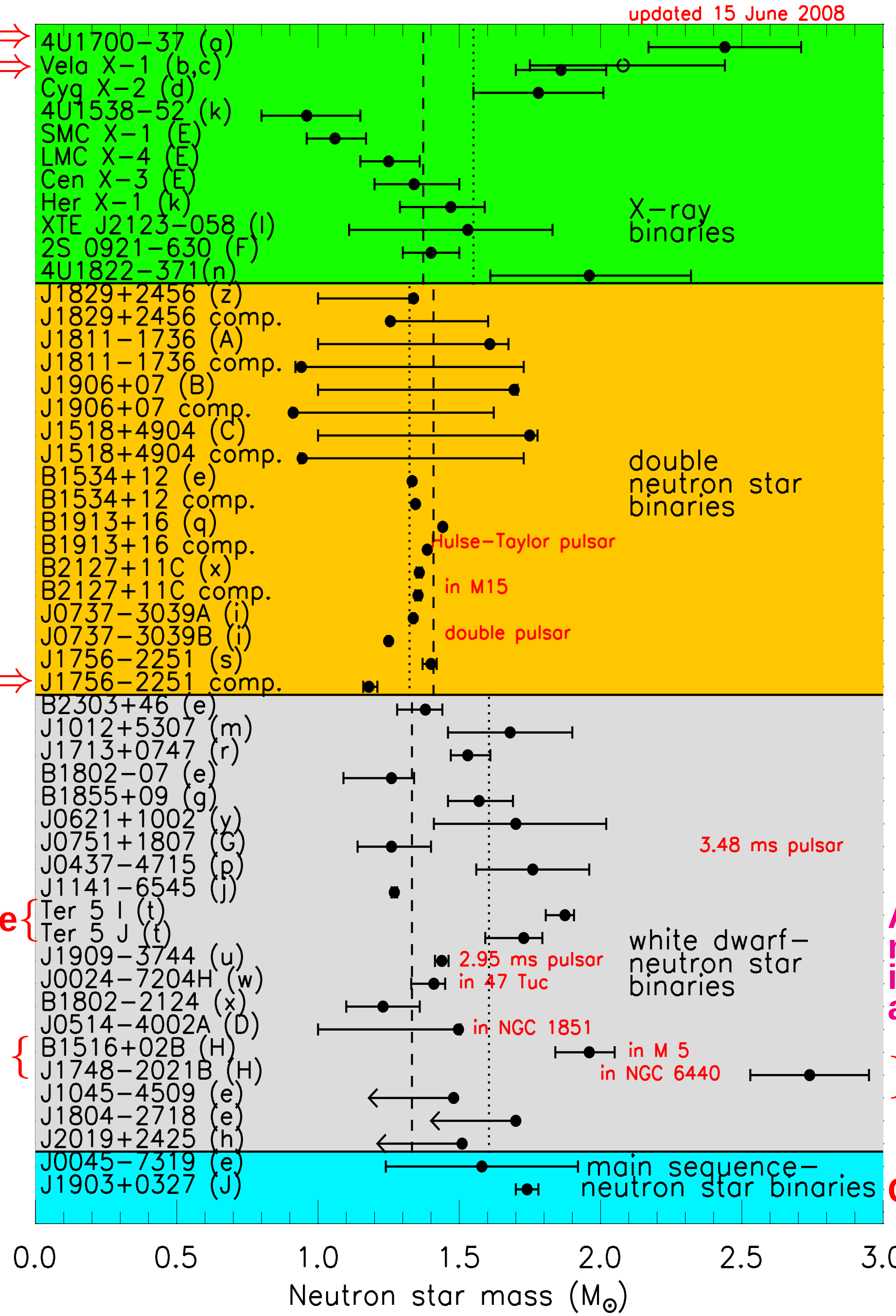
XTE J1739-285
$\nu = 1122$ **Hz**
Kaaret et al. 2006

Not confirmed to be a rotation rate

PSR J1748-2446ad
$\nu = 716$ **Hz**
Hessels et al. 2006

Observed Masses

Black hole? ⇒
Firm lower mass limit? ⇒



$M \simeq 1.18$ M$_\odot$ ⇒

$M > 1.68$ M$_\odot$, **95% confidence** {

Freire et al. 2007 {

Although simple average mass of w.d. companions is 0.27 M$_\odot$ larger, weighted average is 0.015 M$_\odot$ larger

} **w.d. companion? statistics?**

Champion et al. 2008

Neutron Star Matter Pressure and the Radius

$$p \simeq K\epsilon^{1+1/n}$$

$$n^{-1} = d\ln p/d\ln\epsilon - 1 \sim 1$$

$$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$$

$$R \propto p_*^{1/2}\epsilon_*^{-1}M^0$$

$$(1 < \epsilon_*/\epsilon_0 < 2)$$

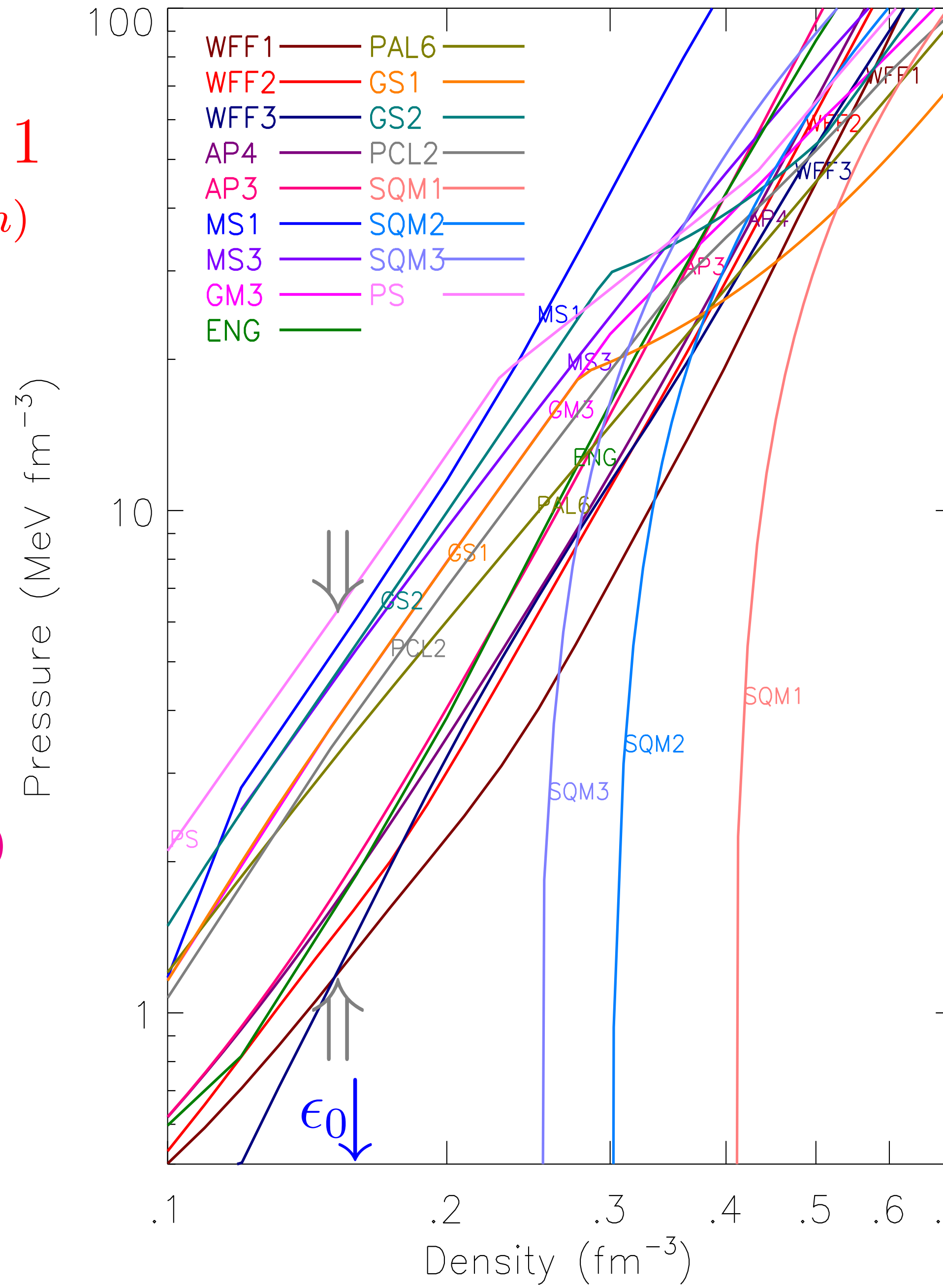
Wide variation:

$$1.2 < \frac{p(\epsilon_0)}{\text{MeV fm}^{-3}} < 7$$

GR phenomenological result (Lattimer & Prakash 2001)

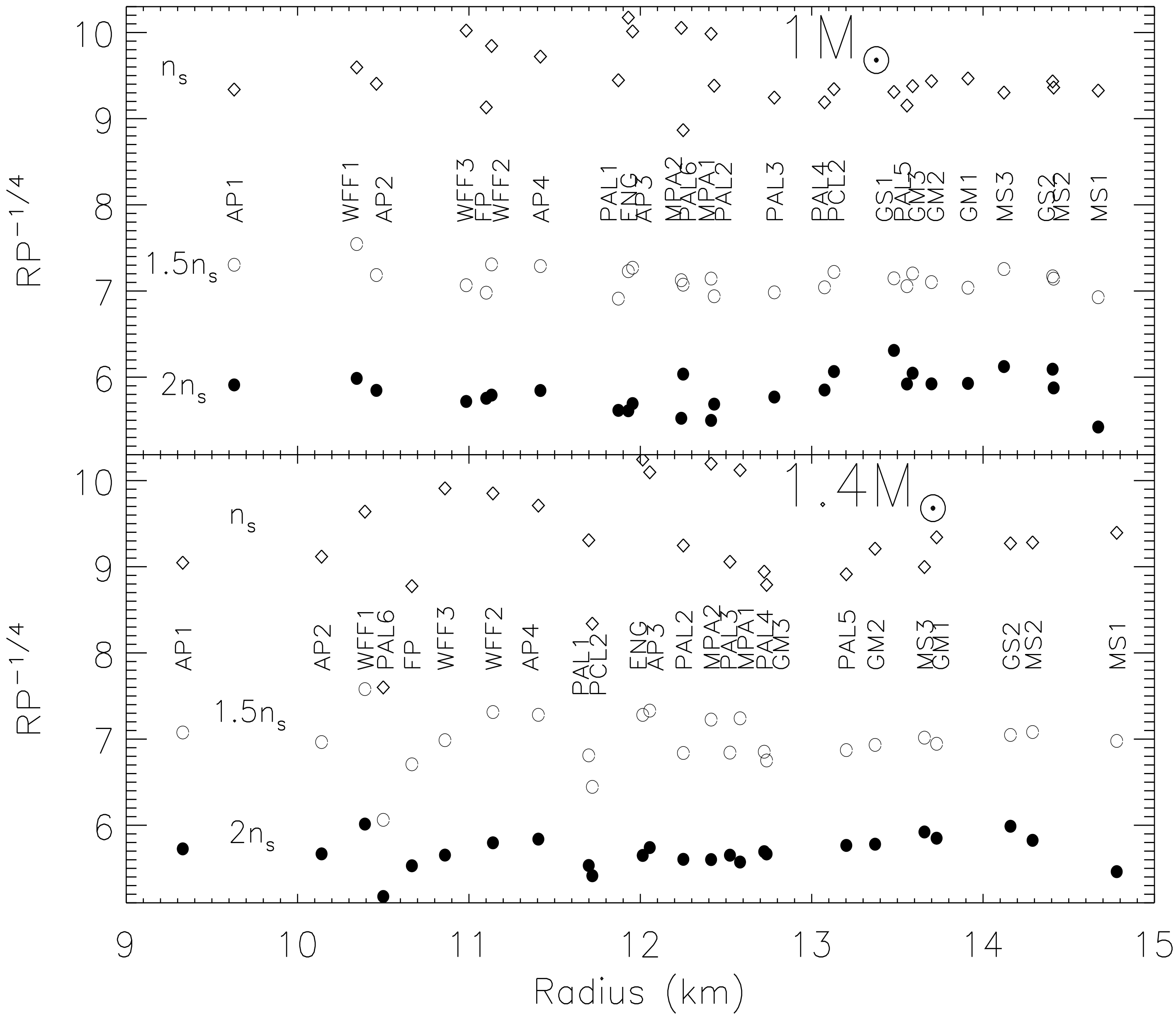
$$R \propto p_*^{1/4}\epsilon_*^{-1/2}$$

$$p_* = n^2\frac{dE_{sym}}{dn} = \frac{n^2 L}{3n_s}$$



The Radius – Pressure Correlation

$R \propto p^{1/4}$

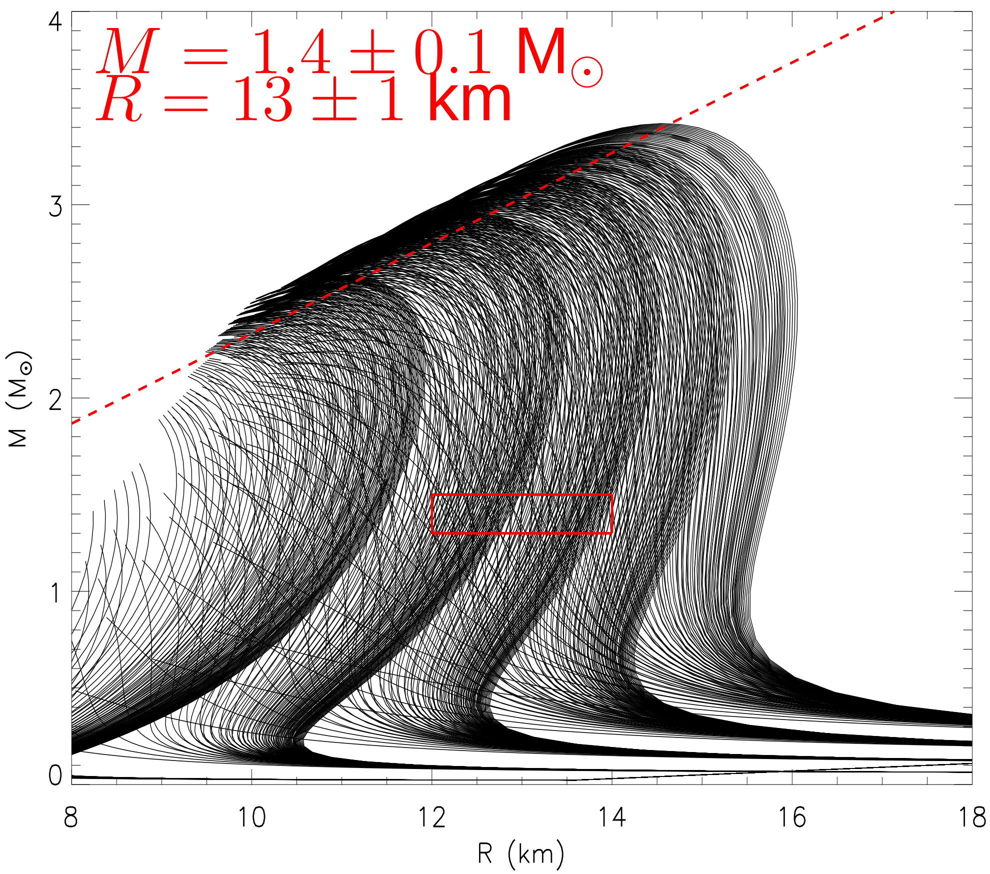


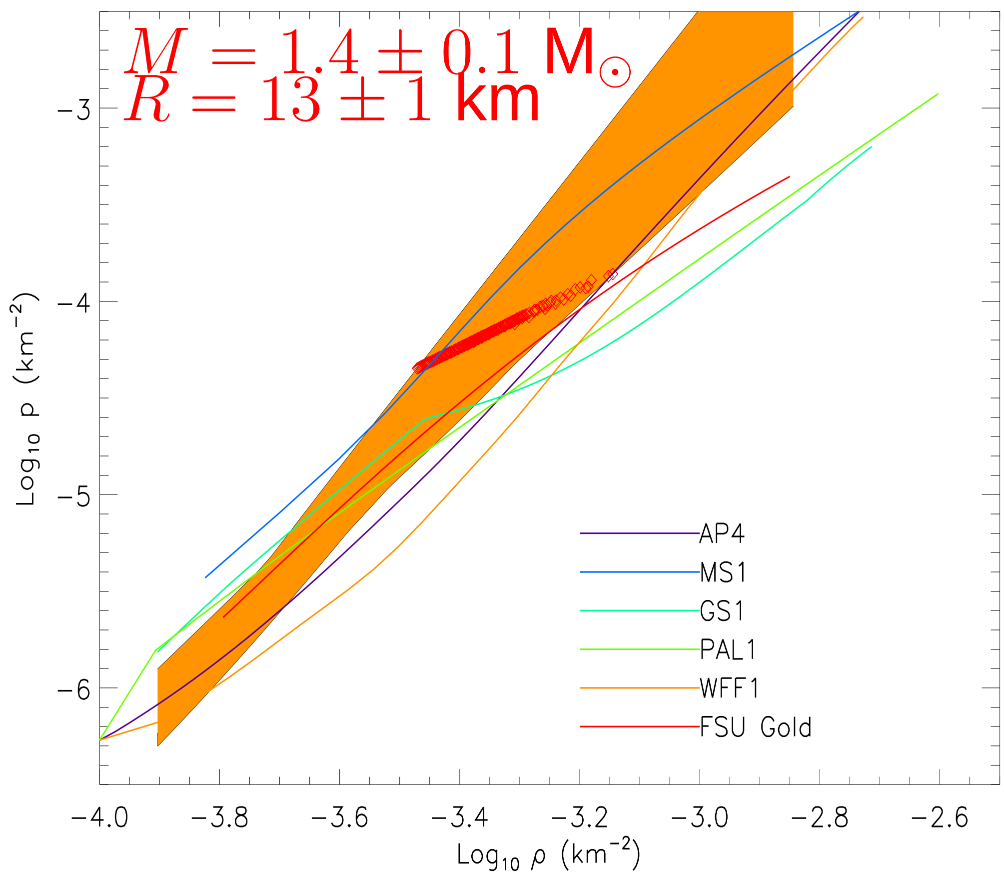
Lattimer & Prakash (2001)

TOV Inversion

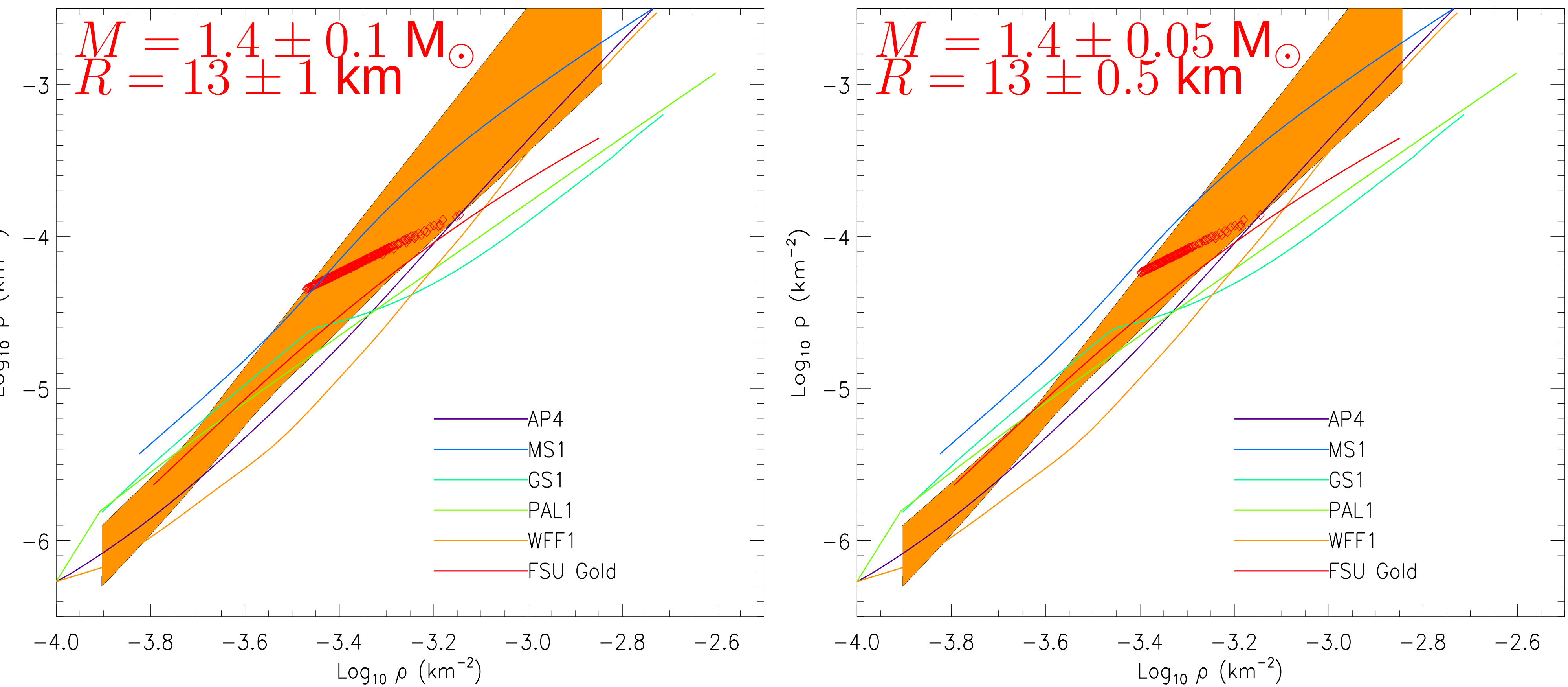
How would a simultaneous $M - R$ determination constrain the EOS? Each M-R curve specifies a unique $p - \rho$ relation.

- Generate physically reasonable $M - R$ curves and the $p - \rho$ relations that they specify.
- Generate physical $p - \rho$ relations and compute $M - R$ curves from them; select those $M - R$ curves passing within the error box.





TOV Inversion (cont.)



Observational Constraints for Neutron Stars

- Maximum and Minimum Masses (binary pulsars)
- Minimum Rotational Period*
- Radiation Radius or Redshift*
- Core Cooling Timescale (URCA or not)*
- Crustal Cooling Timescale*
- Seismology*
- Moment of Inertia*
- Proto-Neutron Star Neutrinos (Binding Energy, Opacities, Radii)*
- Pulse Shape Modulation*
- Gravitational Radiation* (Masses, Radii from tidal effects)

* Significant dependence on symmetry energy

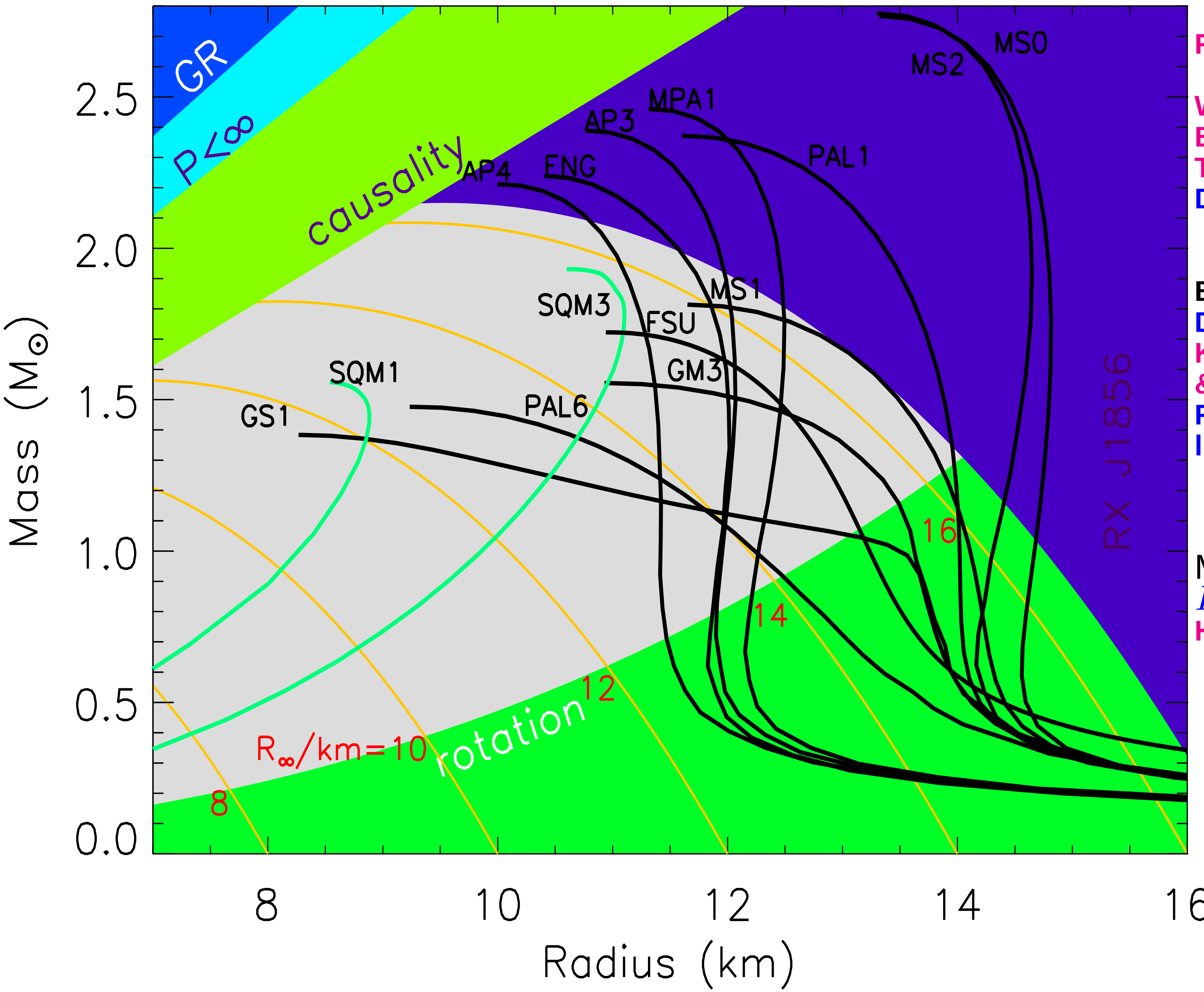
Radiation Radius

- Combination of flux and temperature measurements yields apparent angular diameter (pseudo-BB):

$$\frac{R_\infty}{d} = \frac{R}{d}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii are from
  - Nearby isolated neutron stars (parallax measurable)
  - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)
  - X-ray pulsars in systems of known distance
    - CXOU J010043.1-721134 in the SMC: $R_\infty \geq 10.8$ km (Esposito & Mereghetti 2008)

Radiation Radius: Nearby Neutron Star



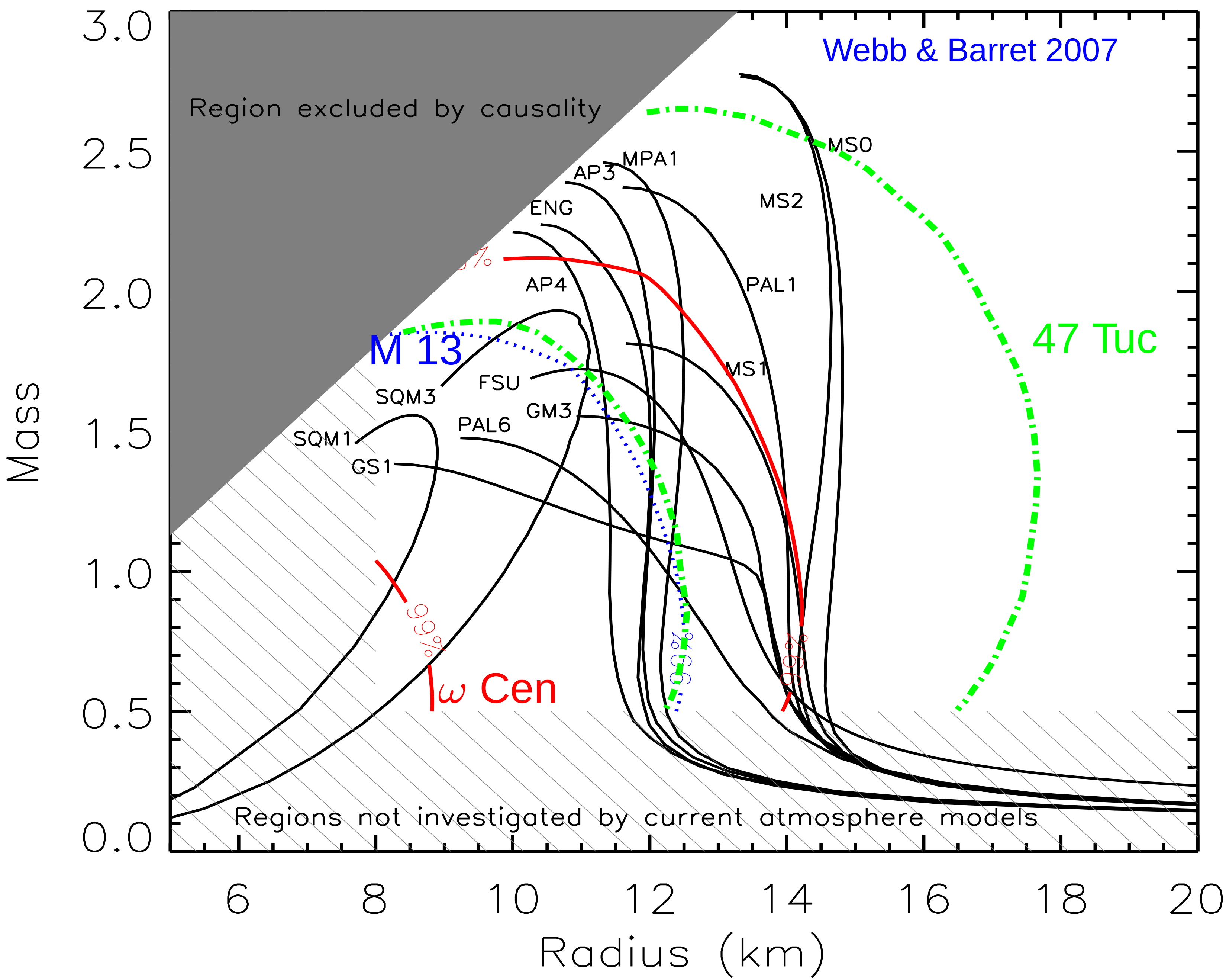
RX J1856-3754:

Walter & Lattimer 2002
Braje & Romani 2002
Truemper 2005
D=120 pc

BUT
D=140 pc
Kaplan, van Kerkwijk & Anderson 2002
Raises R_∞ limit to 19.5 km

Magnetic H atmosphe
$R_\infty \approx 17$ **km**
Ho et al. 2007

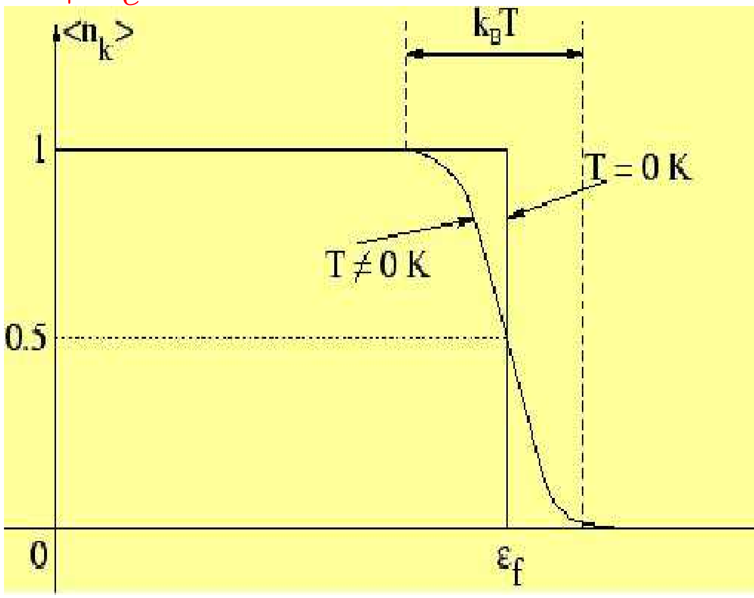
Radiation Radius: Globular Cluster Sources



Neutron Star Cooling

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay.

$$n \to p + e^- + \nu_e\,, \qquad p \to n + e^+ + \bar{\nu}_e$$



Energy conservation guaranteed by beta equilibrium
$\mu_n - \mu_p = \mu_e$
Momentum conservation requires $|k_{Fn}| \le |k_{Fp}| + |k_{Fe}|$.
Charge neutrality requires $k_{Fp} = k_{Fe}$,
therefore $|k_{Fp}| \ge 2|k_{Fn}|$.
Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \ge x_{DU} = 1/9$.
With muons ($n > 2n_s$), $x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$
If $x < x_{DU}$, bystander nucleons needed:
modified Urca process is then dominant.

$$(n,p) + n \to (n,p) + p + e^- + \nu_e, \qquad (n,p) + p \to (n,p) + n + e^+ + \bar{\nu}_e$$

Neutrino emissivities:

$$\dot{\epsilon}_{MURCA} \simeq \left(\frac{T}{\mu_n}\right)^2 \dot{\epsilon}_{DURCA}\,.$$

Beta equilibrium composition:

$$x_\beta \simeq (3\pi^2 n)^{-1}\left(\frac{4E_{sym}}{\hbar c}\right)^3 \simeq 0.04\left(\frac{n}{n_s}\right)^{0.5-2}\,.$$

Neutron Star Cooling