Constraints on The Equation of State from Astrophysical Observations

Madappa Prakash
Ohio University, Athens, OH

The Equation of State at Nonzero Density & Temperature, and its Application in Astrophysics

ANL Workshop, Aug 25 - Aug 29, 2008 @ Argonne, IL

Physics & Astrophysics of Neutron Stars

- *Can* observations of $M, \ R$ & B.E etc., (structure & composition) & $P, \ \dot{P}, \ T_s$ & B etc., (evolution) uniquely pin down the dense matter eqution of state?
- Neutron stars implicated in x-ray & γ-ray bursters, mergers with other neutron stars & black holes, etc.
- **Observational Programs :**

SK, SNO, LVD's, AMANDA ... (ν's)
HST, CHANDRA, XMM, RXTE ... (γ's)
LIGO, VIRGO, GEO600, TAMA ... (Gravity Waves)

Connections:
Atomic, Cond. Matter, Nucl. & Part., Grav. Physics

- Theory : Many-body theory of strongly interacting systems, Dynamical response (ν- & γ- propagation & emissivities)
- Experiment : h, e^-, γ, and ν- scattering experiments on nuclei, masses of neutron-rich nuclei, heavy-ion reactions, etc.

Observational Constraints

- Maximum Mass
- Minimum Rotational Period
- Radius (or Radiation Radius or Surface Redshift)
- Moment of Inertia
- Proto-Neutron Star Neutrinos (Binding Energy & $\nu-$ Opacities)
- Surface Temperatures; Core Cooling Timescale (Urca or not)
- Crustal Cooling Timescale
- Seismology
- Pulse Shape Modulation
- Gravitational Radiation

Lattimer & Prakash , Phys. Rep. **442**, 109 (2007).

Equations of Stellar Structure-I

• In hydrostatic equilibrium, the structure of a spherically symmetric neutron star from the Tolman-Oppenheimer-Volkov (TOV) equations:

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r)$$

$$\frac{dP(r)}{dr} = -\frac{GM(r)\epsilon(r)}{c^2 r^2} \frac{\left[1 + \frac{P(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right]}{\left[1 - \frac{2GM(r)}{c^2 r}\right]}$$

• G := Gravitational constant
• P := Pressure
• ϵ := Energy density
• $M(r)$:= Enclosed gravitational mass
• $R_s = 2GM/c^2$:= Schwarzschild radius

Equations of Stellar Structure-II

- The gravitational and baryon masses of the star:

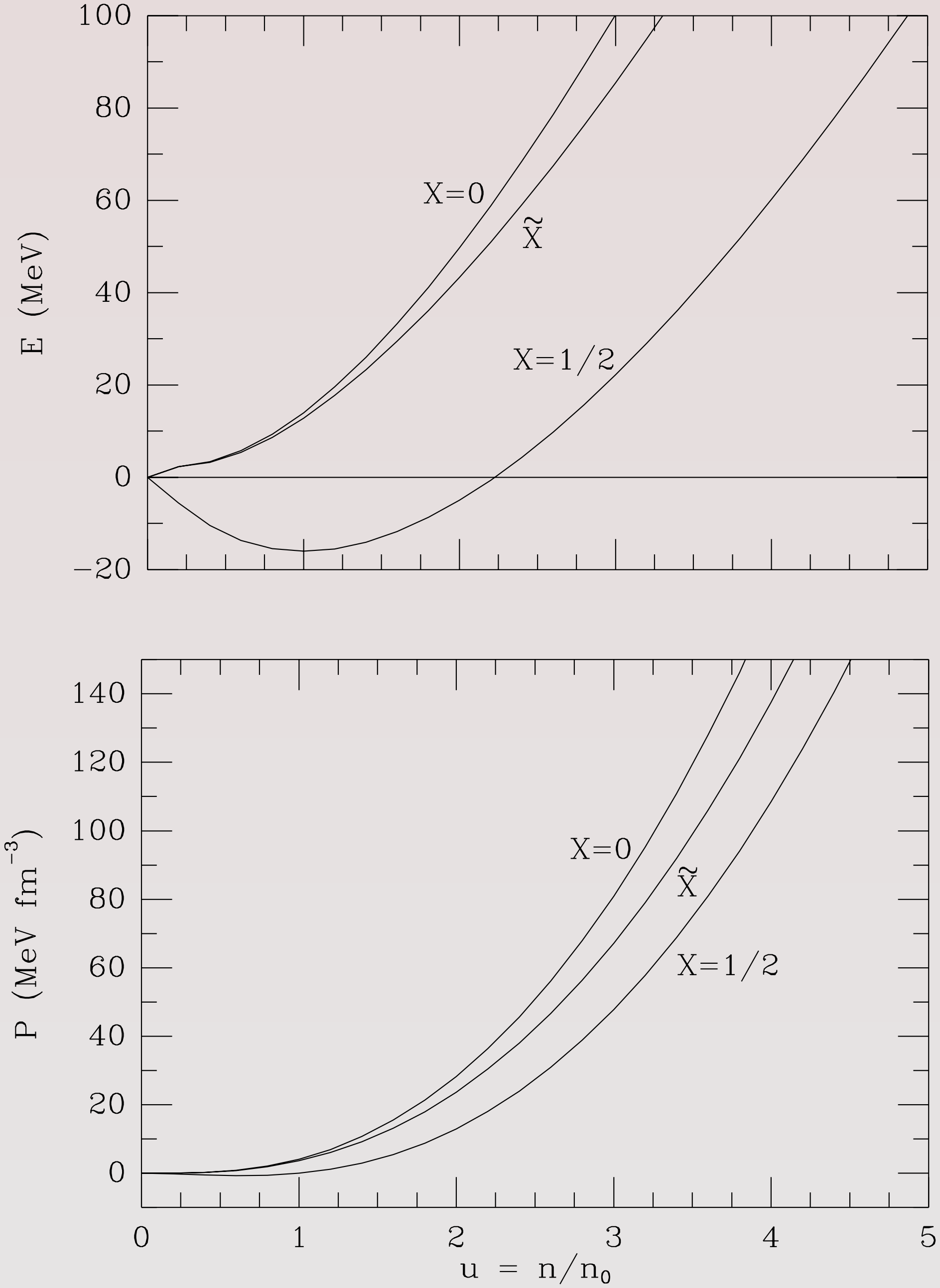
$$
\begin{aligned}
M_G c^2 &= \int_0^R dr\, 4\pi r^2\, \epsilon(r) \\
M_A c^2 &= m_A \int_0^R dr\, 4\pi r^2\, \frac{n(r)}{\left[1 - \dfrac{2GM(r)}{c^2 r}\right]^{1/2}}
\end{aligned}
$$

- m_A := Baryonic mass
- $n(r)$:= Baryon number density
- The binding energy of the star $B.E. = (M_A - M_G)c^2$.

To determine star structure :

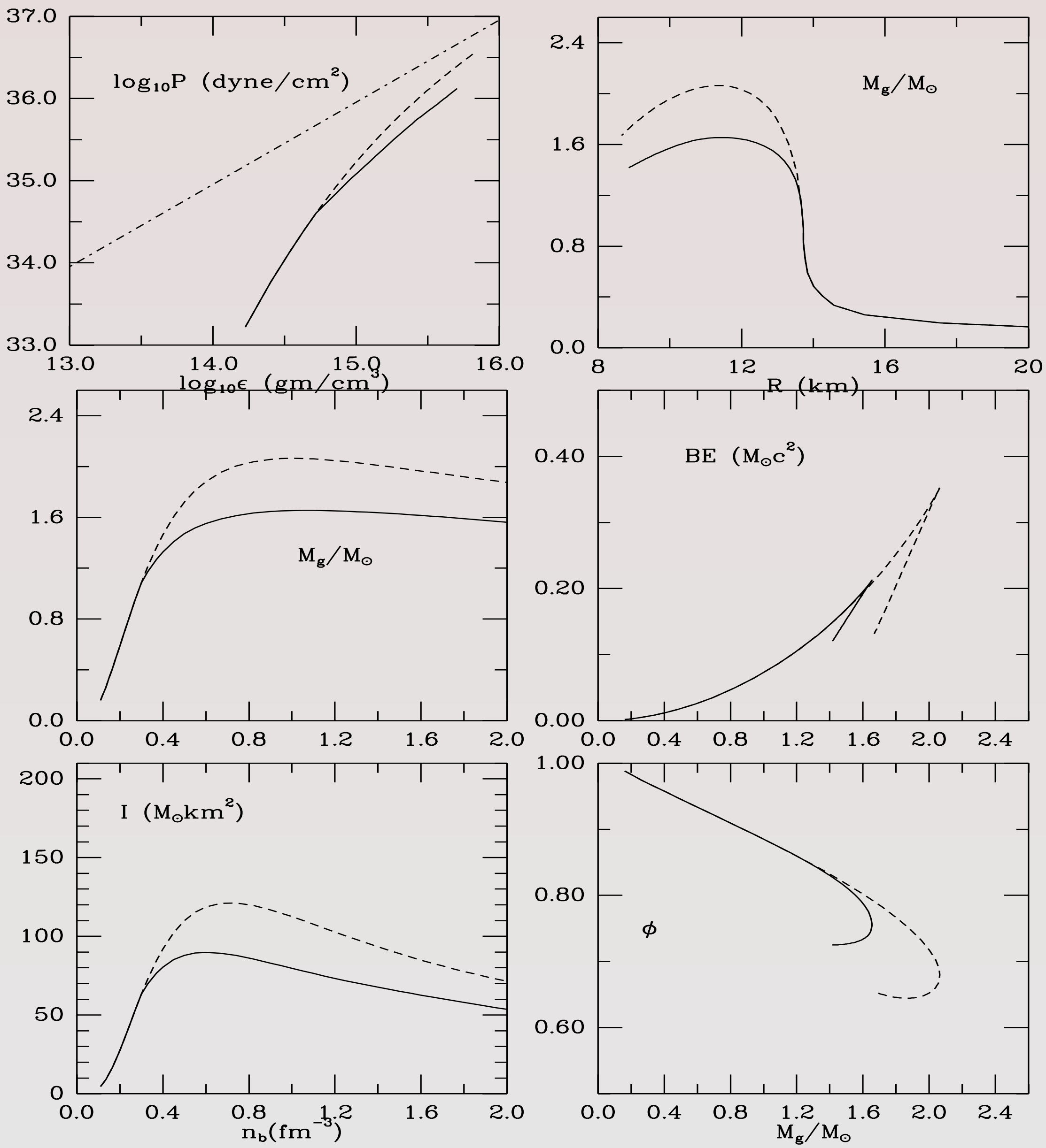
- Specify equation of state, $P = P(\epsilon)$
- Choose a central pressure $P_c = P(\epsilon_c)$ at $r = 0$
- Integrate the 2 DE's out to surface $r = R$, where $P(r = R) = 0$.

Nucleonic Equation of State



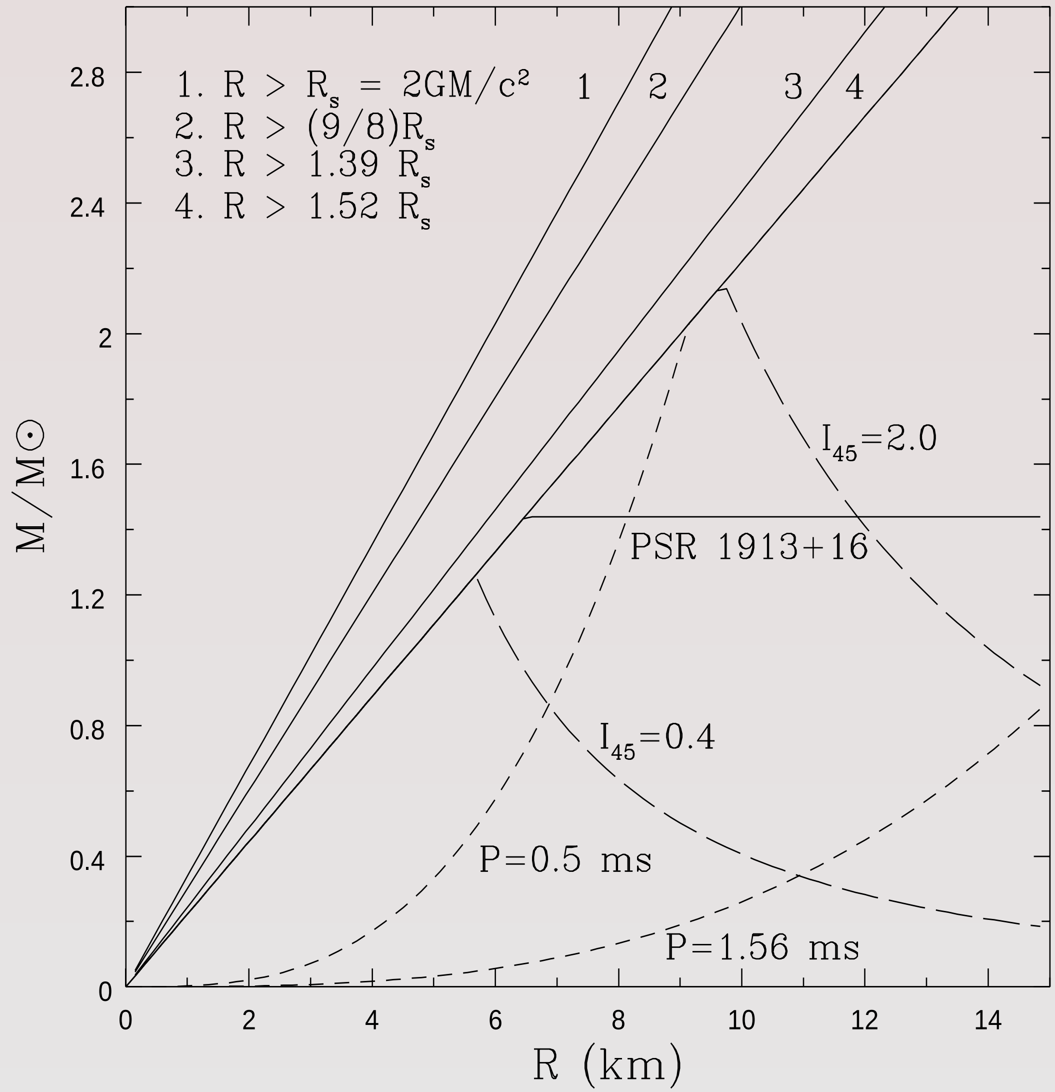
- Energy (E) & Pressure (P) vs. scaled density ($u = n/n_0$).
- Nuclear matter equilibrium density $n_0 = 0.16 \text{ fm}^{-3}$.
- Proton fraction $x = n_p/(n_p + n_n)$.
- Nuclear matter : $x = 1/2$.
- Neutron matter : $x = 0$.
- Stellar matter in $\beta-$ equilibrium : $x = \tilde{x}$.

Results of Star Structure



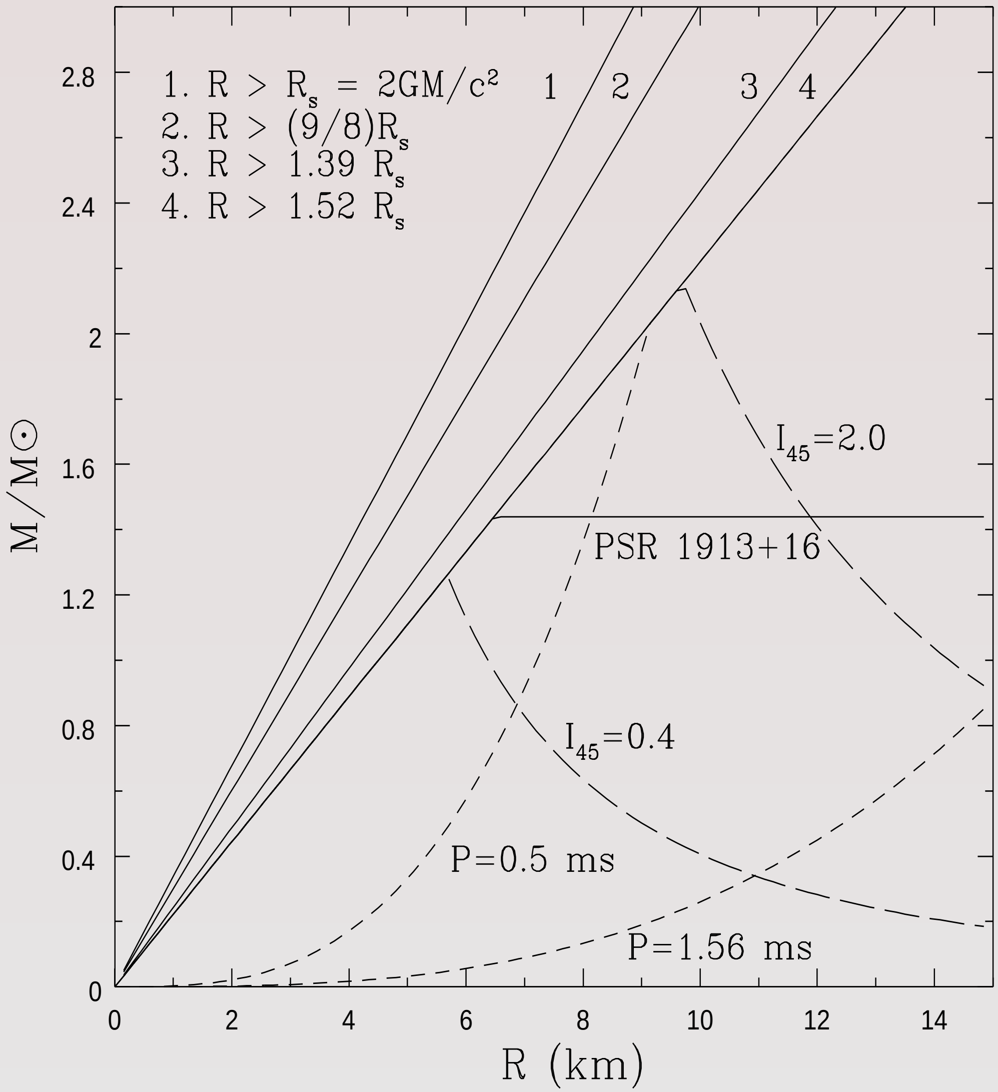
- Stellar properties for soft & stiff (by comparison) EOS's.
- Causal limit : $P = \epsilon$.
- M_g : Gravitational mass
- R : Radius
- BE : Binding energy
- n_b : Central density
- I : Moment of inertia
- ϕ : Surface red shift , $e^{\phi/c^2} = (1 - 2GM/c^2R)^{-1/2}$.

Constraints on the EOS-I



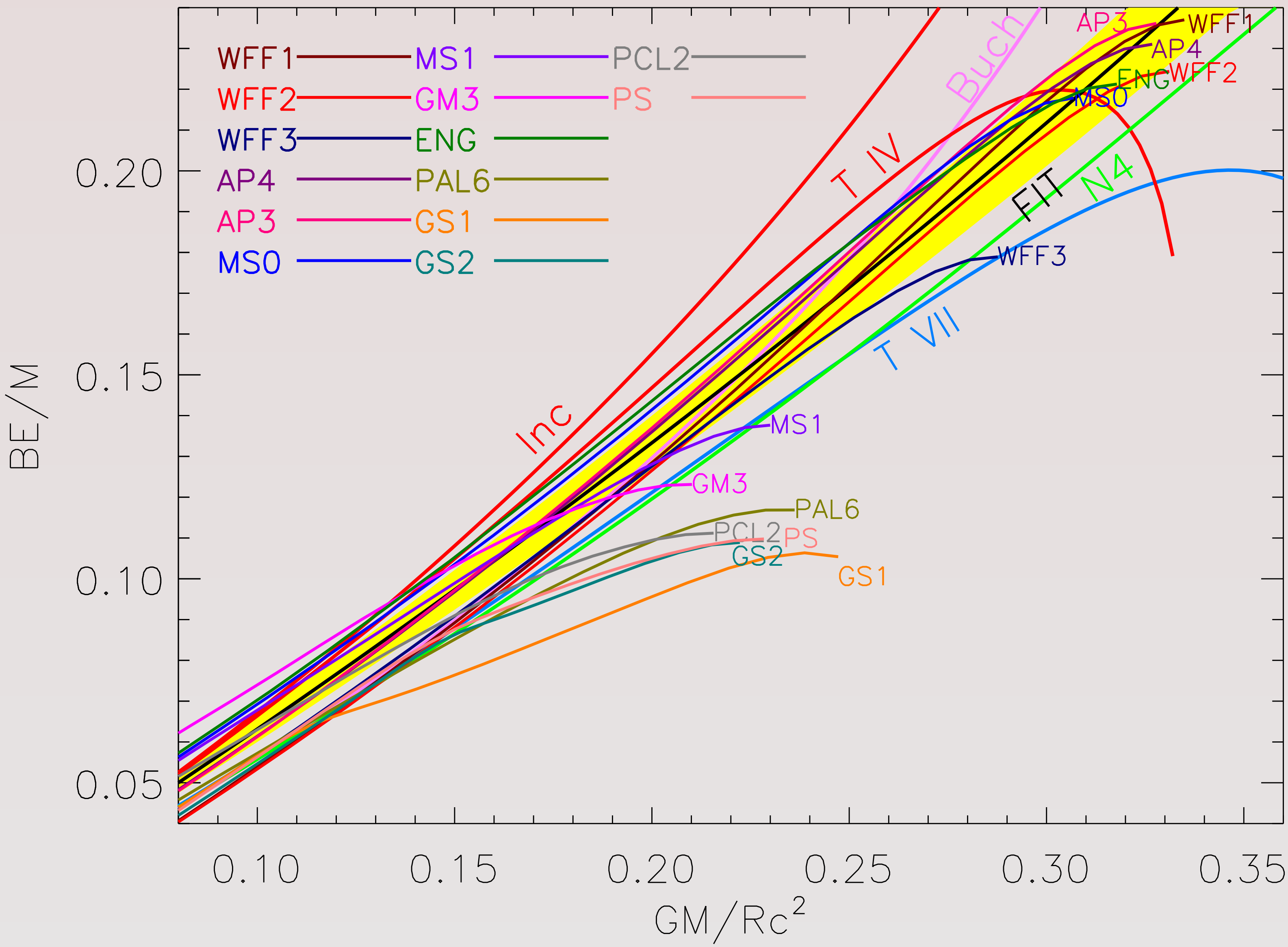
- $R > R_s = 2GM/c^2 \Rightarrow$ $M/M_\odot \geq R/R_{s\odot}$; $R_{s\odot} = 2GM_\odot/c^2$ $\simeq 2.95$ km .
- $P_c < \infty$ $\Rightarrow R > (9/8)R_s$ $\Rightarrow M/M_\odot \geq$ $(8/9)R/R_{s\odot}$.
- Sound speed c_s : $c_s = (dP/d\epsilon)^{1/2} \leq c$ $\Rightarrow R > 1.39R_s$ $\Rightarrow M/M_\odot \geq$ $R/(1.39R_{S\odot})$.
- If $P = \epsilon$ above $n_t \simeq 2n_0$, $R > 1.52R_s \Rightarrow$ $M/M_\odot R/(1.52R_{s\odot})$.

Constraints on the EOS-II



- $M_{max} \geq M_{obs}$; In PSR 1913+16, $M_{obs} = 1.44\ \mathrm{M}_\odot$.
- In PSR 1957+20, $P_K = 1.56$ ms : $\Omega_K \simeq 7.7 \times 10^3 \left(\frac{M_{max}}{\mathrm{M}_\odot}\right)^{1/2} \left(\frac{R_{max}}{10\ \mathrm{km}}\right)^{-3/2}\ \mathrm{s}^{-1}$
- Mom. of Inertia I : $I_{max} = 0.6 \times 10^{45}\ \mathrm{g\ cm}^2 \left(\frac{M_{max}}{\mathrm{M}_\odot}\right) \left(\frac{R_{max}}{10\ \mathrm{km}}\right)^2 f(M_{max}, R_{max})$
- In SN 1987A $B.E. \simeq (1-2) \times 10^{53}$ ergs.

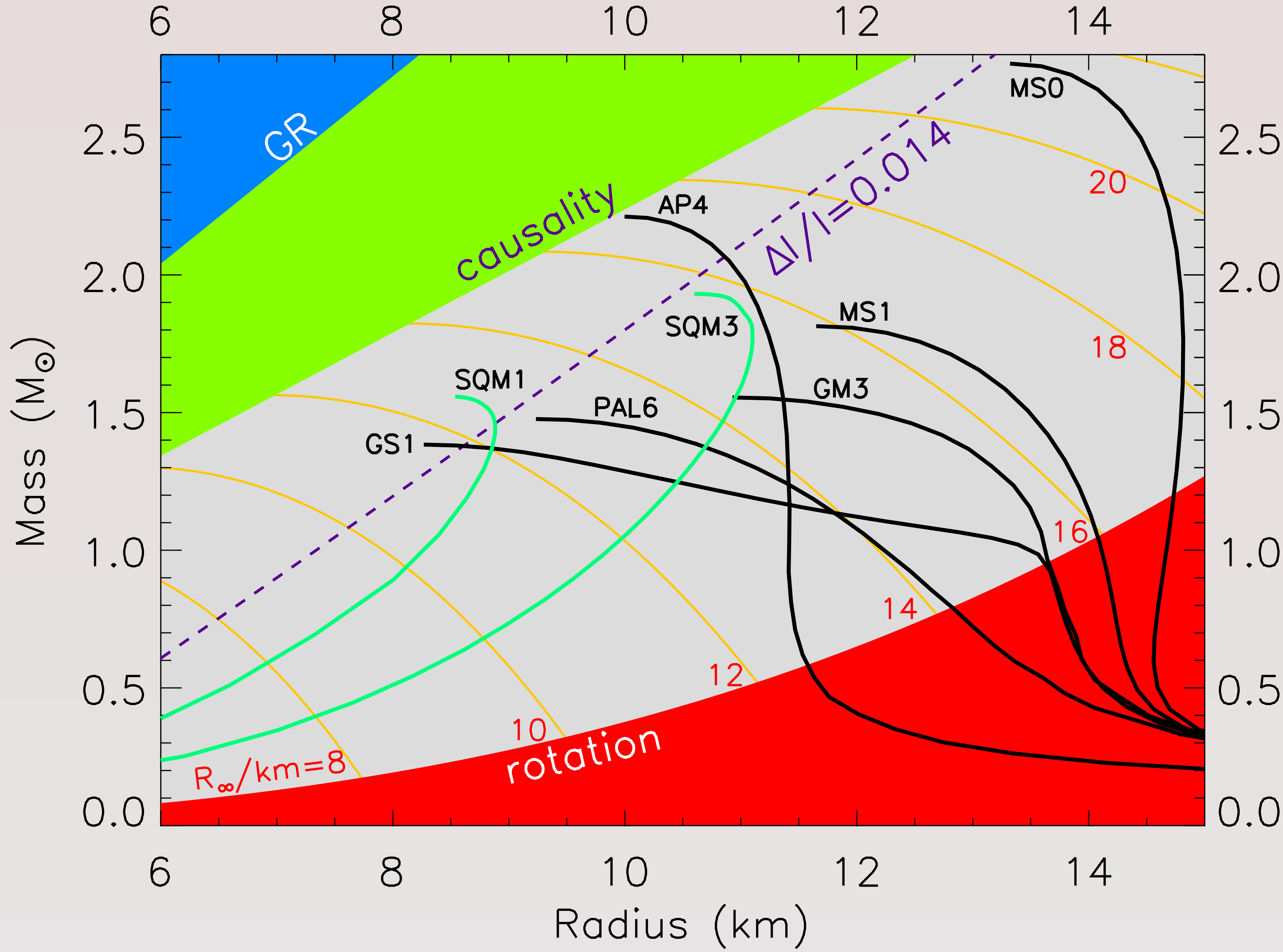
Binding Energies



$$BE/M \simeq (0.60 \pm 0.5)\ (GM/Rc^2)\ (1 - GM/2Rc^2)^{-1}$$

Lattimer & Prakash , Phys. Rep. **442**, 109 (2007).

Mass Radius Relationship



$$R_\infty = R/\sqrt{1-(2GM/c^2R)}$$

Lattimer & Prakash , Phys. Rep. **442**, 109 (2007).

General Constraints

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ Rhoades & Ruffini (1974), Hartle (1978)
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)$ km

Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)

- $\rho_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2$ g cm^{-3} Lattimer & Prakash (2005)
- $P_{min} \simeq (0.74 \pm 0.03)(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

Koranda, Stergioulas & Friedman (1997)

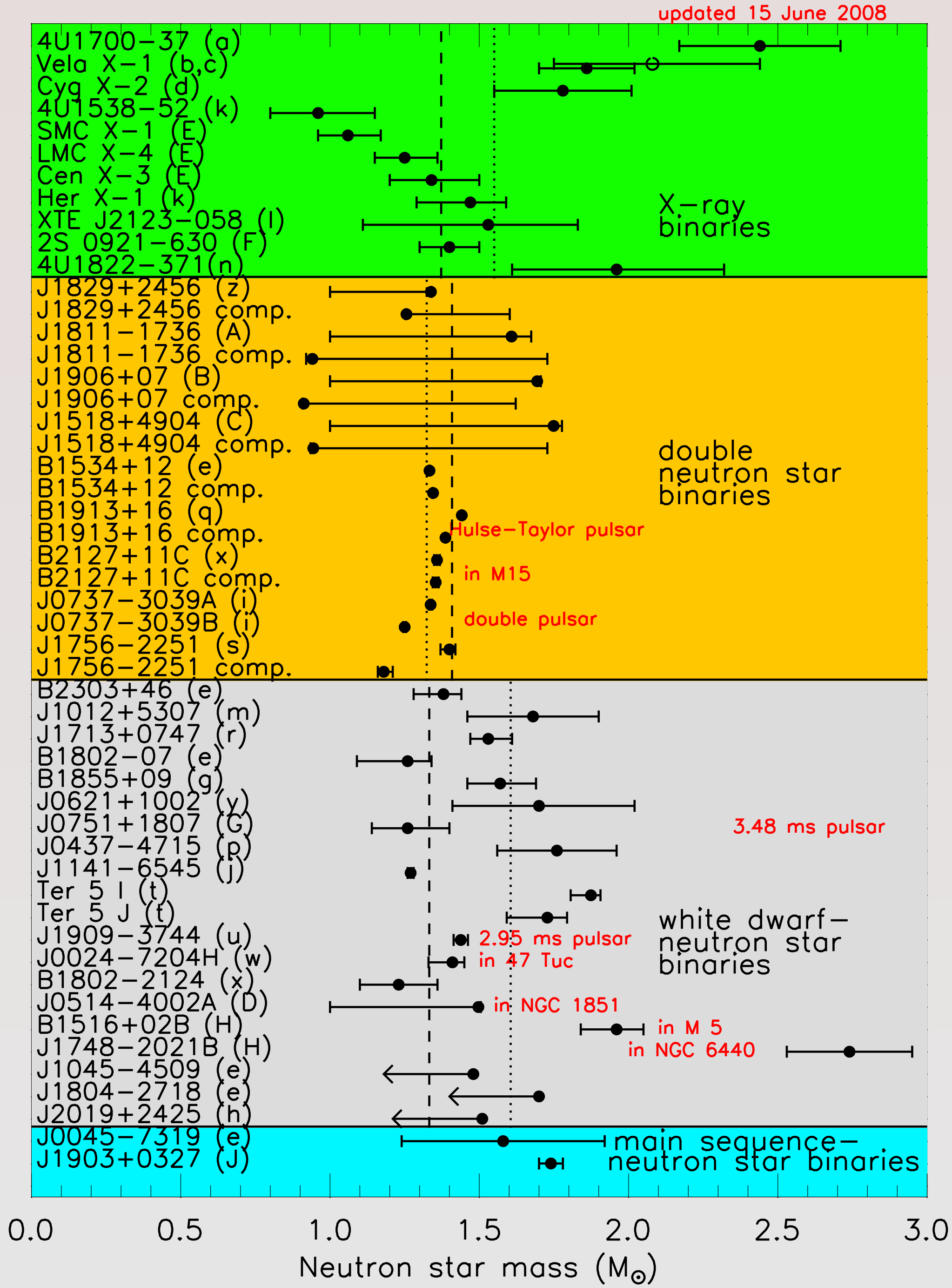
- $P_{min} \simeq 0.96(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms (empirical)

Lattimer & Prakash (2004)

- $\rho_c > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2$ g cm^{-3} (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

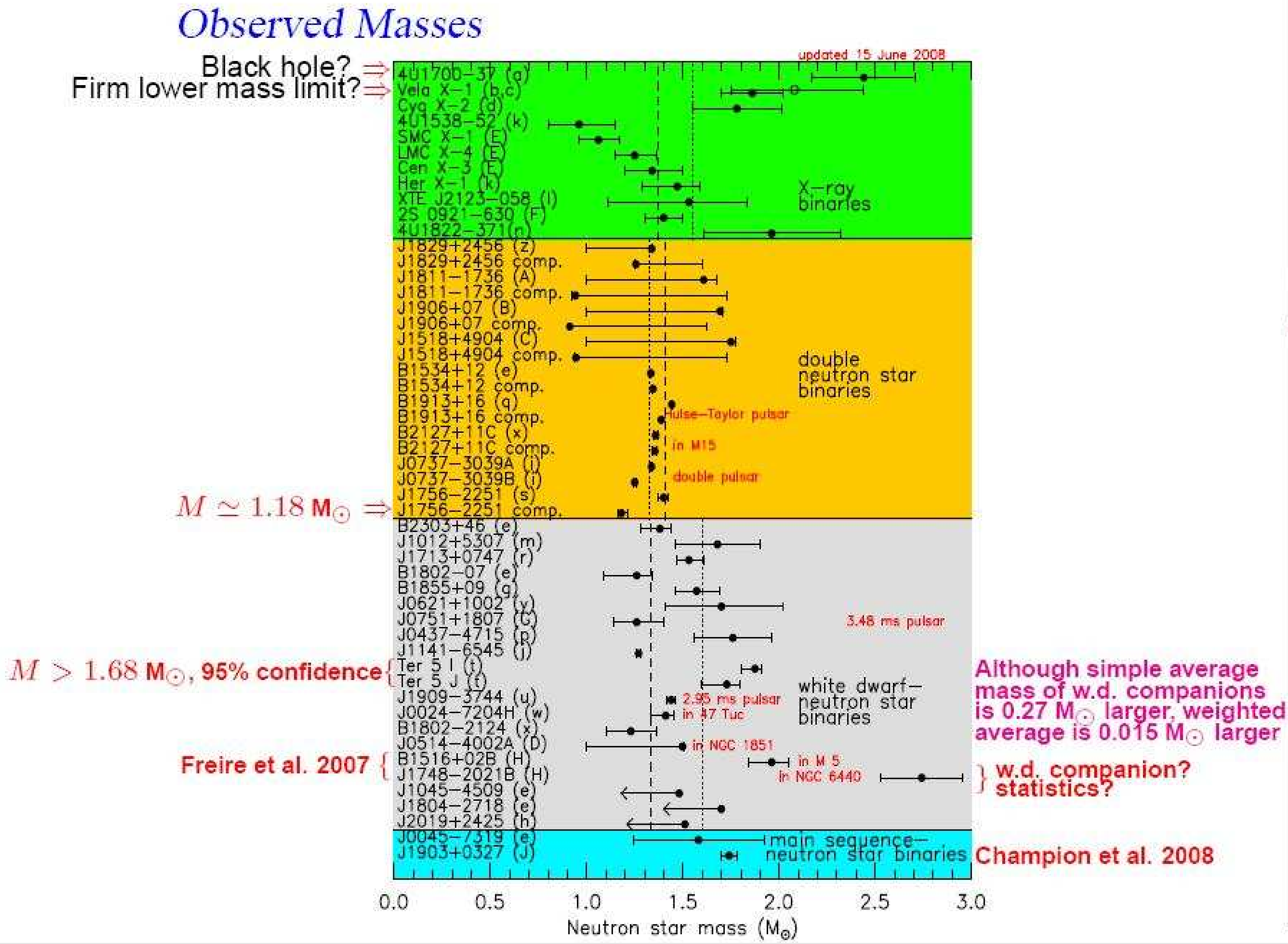
Lattimer & Prakash , Phys. Rep. **442**, 109 (2007).

Measured Neutron Star Masses

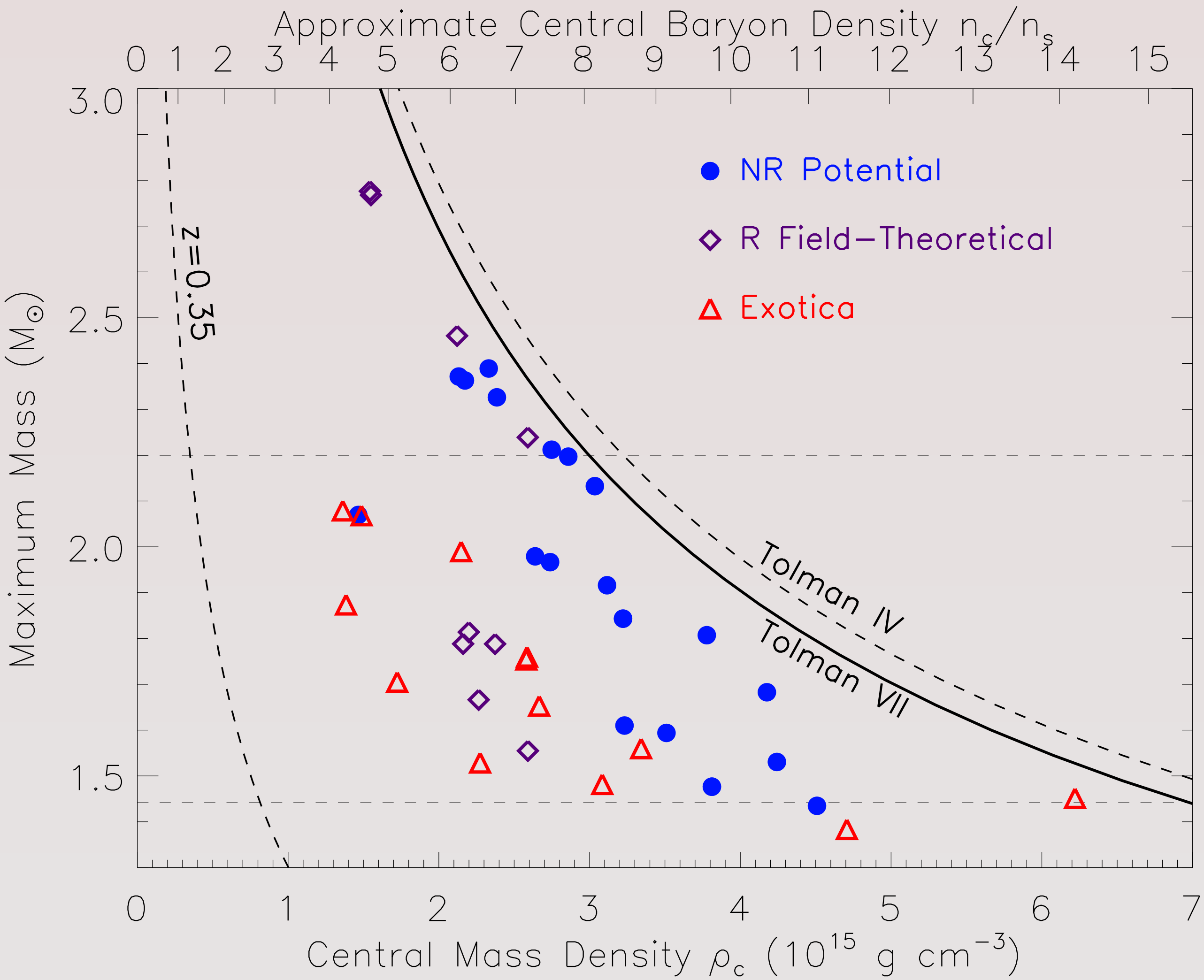


- Mean & error weighted mean in $M_\odot$
- X-ray binaries: 1.55 & 1.37
- Double NS binaries: 1.32 & 1.41
- WD & NS binaries: 1.60 & 1.33
- Lattimer & Prakash, PRL, 94 (2005) 111101; updated

Measured Neutron Star Masses



Ultimate Energy Density of Cold Matter



- Tolman VII: $\epsilon = \epsilon_c(1 - (r/R)^2)$
- $\epsilon_c \propto (M_\odot/M)^2$
- A measured red-shift provides a lower limit.
- **Crucial to establish an upper limit to M_{max} .**

Lattimer & Prakash, PRL, **94** (2005) 111101.

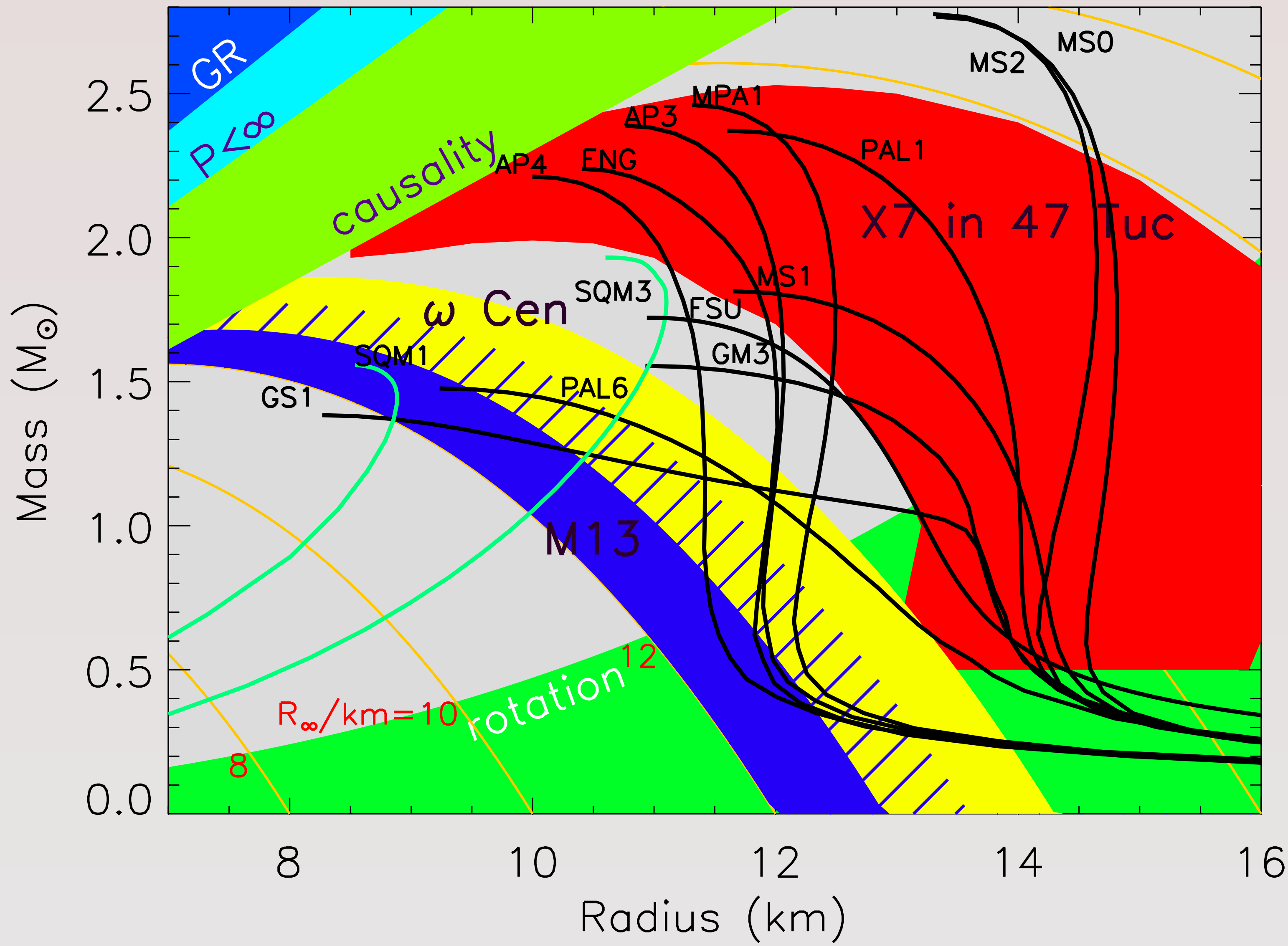
Neutron star radius measurements

Object	R_∞ (km)	D (kpc)	$kT_{eff,\infty}$ (eV)	Ref.
Omega Cen (Chandra)	13.5 ± 2.1	$5.36 \pm 6\%$	66^{+4}_{-5}	Rutledge et al. ('02)
Omega Cen (XMM)	13.6 ± 0.3	$5.36 \pm 6\%$	67 ± 2	Gendre et al. ('02)
M13 (XMM)	12.6 ± 0.4	$7.80 \pm 2\%$	76 ± 3	Gendre et al. ('02)
47 Tuc X7 (Chandra)	$14.5^{+1.6}_{-1.4}$ $(1.4\ M_\odot)$	$5.13 \pm 4\%$		Rybicki et al. ('05)
M28 (Chandra)	$14.5^{+6.9}_{-3.8}$	$5.5 \pm 10\%$	90^{+30}_{-10}	Becker et al. ('03)
EXO 0748-676 (Chandra)	13.8 ± 1.8 $(2.10 \pm 0.28\ M_\odot)$	9.2 ± 1.0		Ozel ('06)

$R_\infty = R/\sqrt{1-(2GM/c^2R)}\ ; \qquad F = 4\pi T_{eff}^4 (R_\infty/D)^2$

Atmospheric (sometimes magnetic) modeling required.

Constraints from Radiation Radii



Lattimer & Prakash , Phys. Rep. **442**, 109 (2007).

Pressure of NS Matter

Neutron Star Matter Pressure and the Radius

$p \simeq K\epsilon^{1+1/n}$

$n^{-1} = d\ln p/d\ln\epsilon - 1 \sim 1$

$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$

$R \propto p_*^{1/2} \epsilon_*^{-1} M^0$

$(1 < \epsilon_*/\epsilon_0 < 2)$

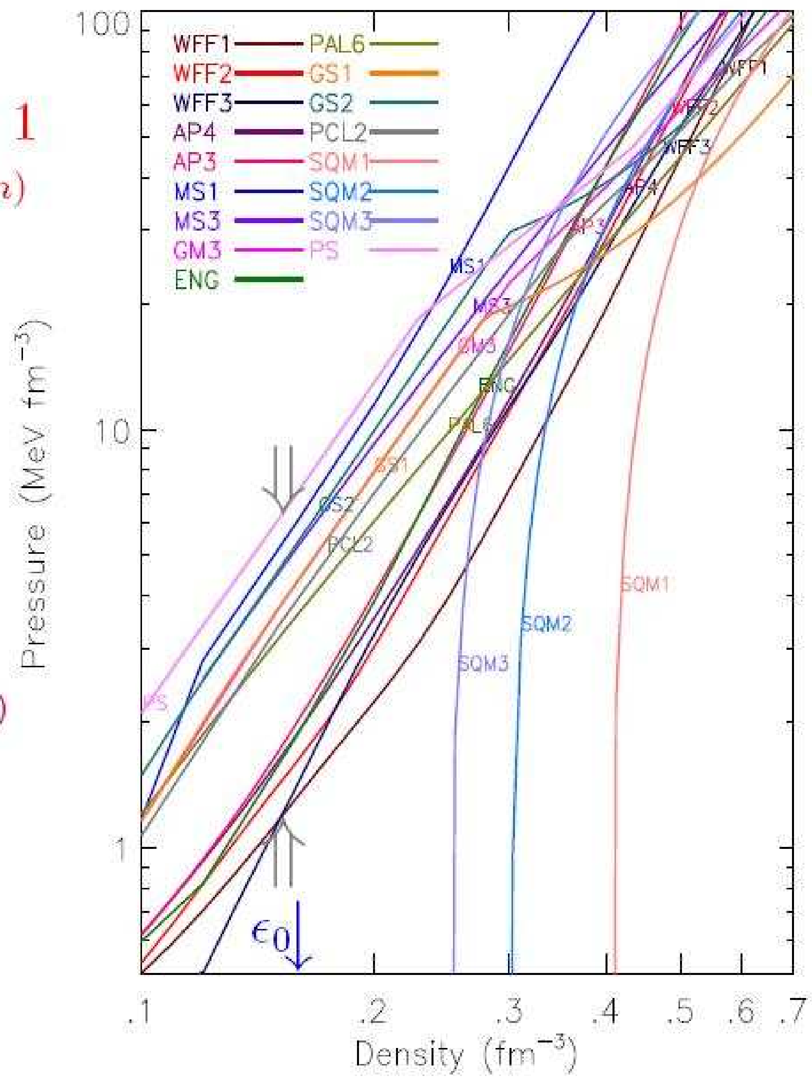
Wide variation:

$1.2 < \frac{p(\epsilon_0)}{\mathrm{MeV\ fm^{-3}}} < 7$

GR phenomenological result (Lattimer & Prakash 2001)

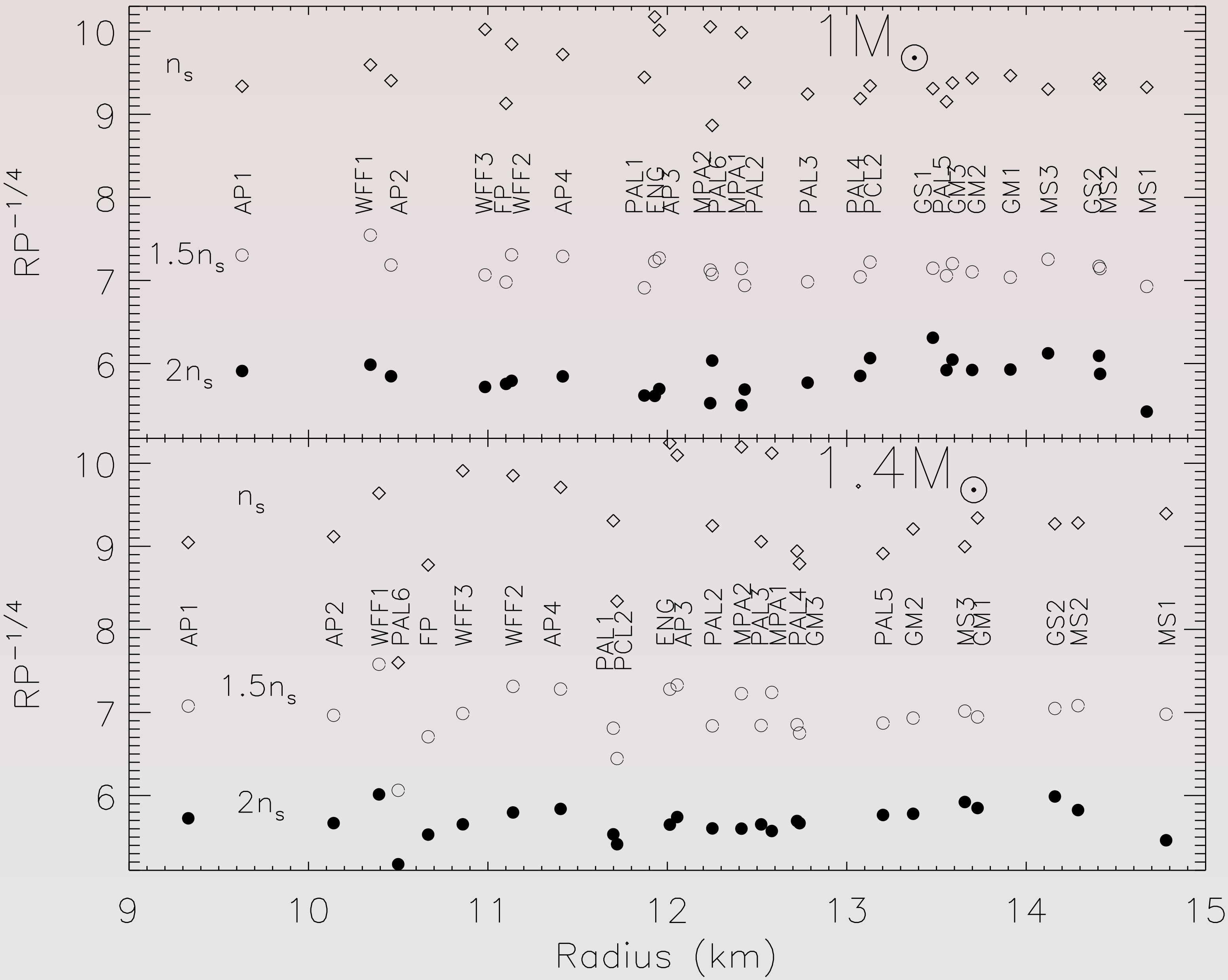
$R \propto p_*^{1/4} \epsilon_*^{-1/2}$

$p_* = n^2 dE_{sym}/dn$



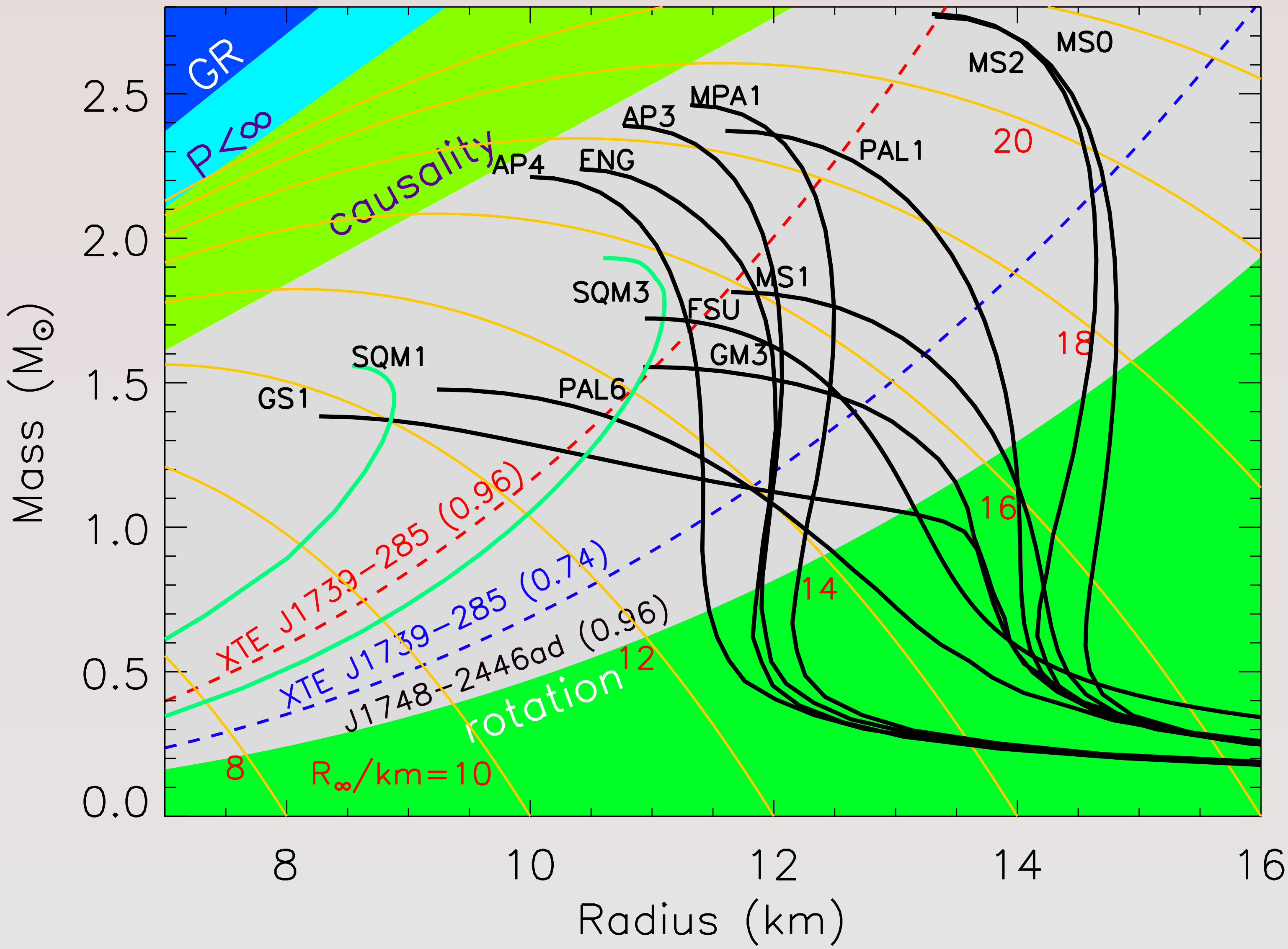
Lattimer & Prakash , ApJ **550**, 426 (2001).

Pressure-Radius Correlation



Lattimer & Prakash , ApJ **550**, 426 (2001).

Rotational Constraints



PSR J1748-2446ad: 716 Hz ; XTE J1739-285: 1122 Hz

Lattimer & Prakash , Phys. Rep. **442**, 109 (2007).

Moment of inertia (I) measurements

Spin precession periods:

$$P_{p,i} = \frac{2c^2 a P M (1 - e^2)}{G M_{-i} (4 M_i + 3 M_{-i})} \, .$$

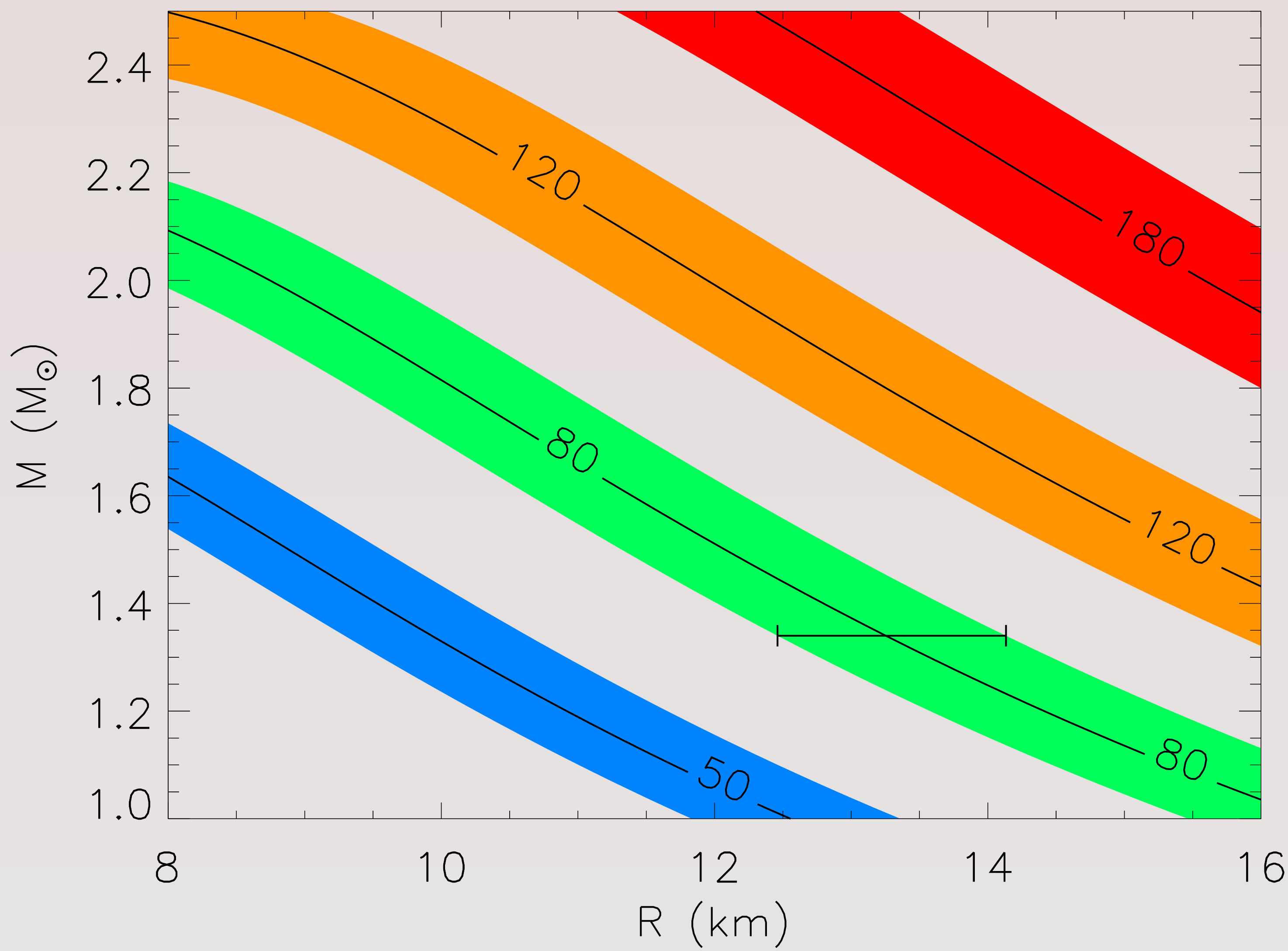
Spin-orbit coupling causes a periodic departure from the expected time-of-arrival of pulses from pulsar A of amplitude

$$\delta t_A = \frac{M_B}{M} \frac{a}{c} \delta_i \cos i = \frac{a}{c} \frac{I_A}{M_A a^2} \frac{P}{P_A} \sin \theta_A \cos i$$

P: Orbital period a: Orbital separation e: Eccentricity
$M = M_1 + M_2$: Total mass
i: Orbital inclination angle θ_A: Angle between $\mathbf{S}_A$ and $\mathbf{L}$.
I_A: Moment of Inertia of A

For PSR 0707-3039, $\delta t_A \simeq (0.17 \pm 0.16) I_{A,80}\ \mu\mathrm{s}$;
Needs improved technology & is being pursued.

Limits on R from M & I measurements



- 10% error bands on I in $\mathrm{M}_\odot$ km^2
- Horizontal error bar for $M = 1.34\ \mathrm{M}_\odot$ & $I = 80 \pm 8\ \mathrm{M}_\odot\ \mathrm{km}^2$

J. M. Lattimer & B. F. Schutz, Astrophys. Jl. **629** (2005)

Thermal Evolution of a Neutron Star

(Spherical, non-rotating & non-magnetic)

$$\frac{dM}{dr} = 4\pi r^2 \epsilon \, ; \qquad \frac{dP}{dr} = -\frac{GM\epsilon}{c^2 r^2}\left[1+\frac{P}{\epsilon}\right]\left[1+\frac{4\pi r^3 P}{Mc^2}\right] e^{2\Lambda}$$

$$\frac{d}{dr}\left(Te^{\Phi/c^2}\right) = -\frac{3}{16\sigma}\frac{\kappa\rho}{T^3}\frac{L_d}{4\pi r^2} e^{\Phi/c^2} e^{\Lambda}$$

$$\frac{d\Phi}{dr} = \frac{G\left(M + 4\pi r^3 P/c^2\right)}{r^2} e^{2\Lambda}$$

$$\frac{d}{dr}\left(L_\nu e^{2\Phi/c^2}\right) = \epsilon_\nu e^{2\Phi/c^2} 4\pi r^2 e^{\Lambda}$$

$$\frac{d}{dr}\left(Le^{2\Phi/c^2}\right) = -c_v \frac{dT}{dt} e^{\Phi/c^2} 4\pi r^2 e^{\Lambda} \, , \quad \text{with} \quad \Lambda = \exp(1 - 2GM/c^2 r)^{-1/2} \, .$$

(P, ϵ) : (Pressure, energy density) M : Enclosed mass

κ : Opacity of matter Φ : Gravitational potential

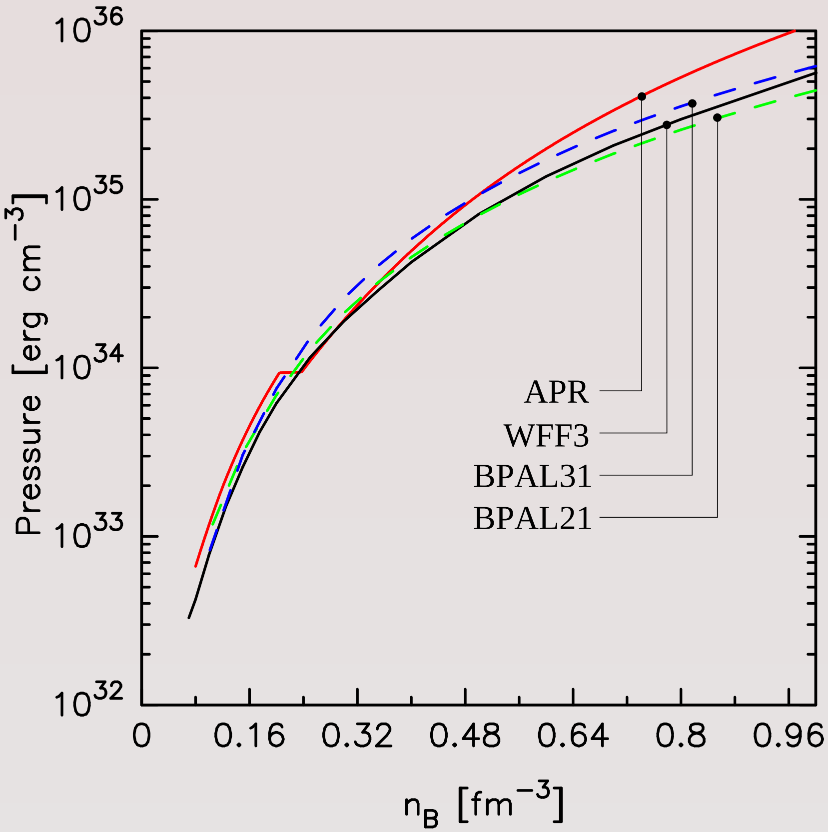
L_d : Luminosity (thermal conductivity & radiation)

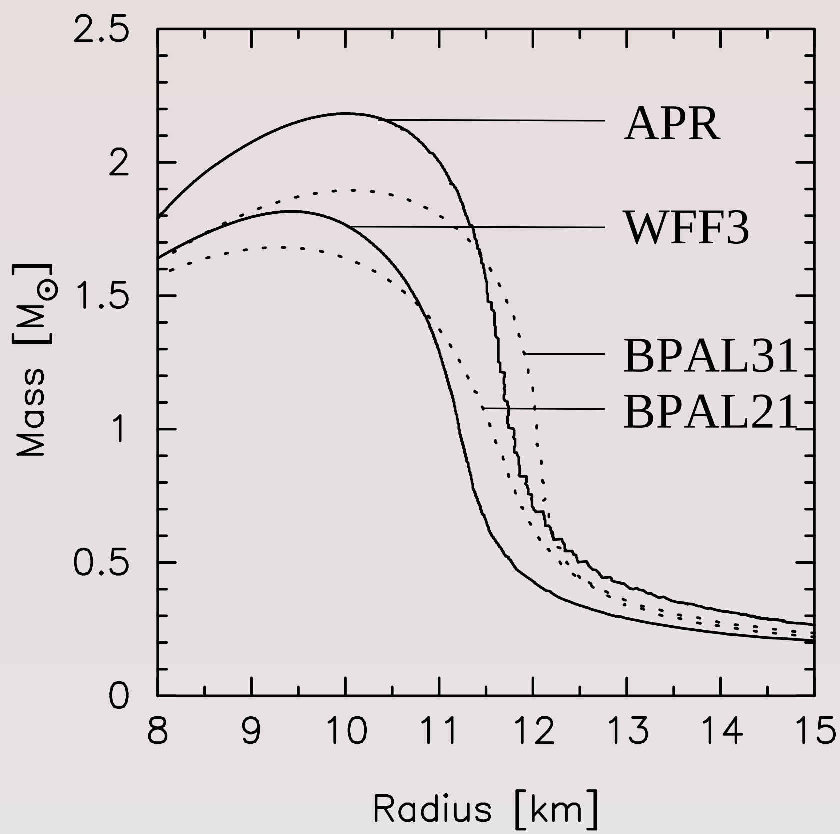
(L_ν, ϵ_ν) : Neutrino (luminosity, emissivity)

$L = L_d + L_\nu$; Net luminosity

c_v : Specific heat/volume, Time t measured at $r = \infty$.

Equation of State





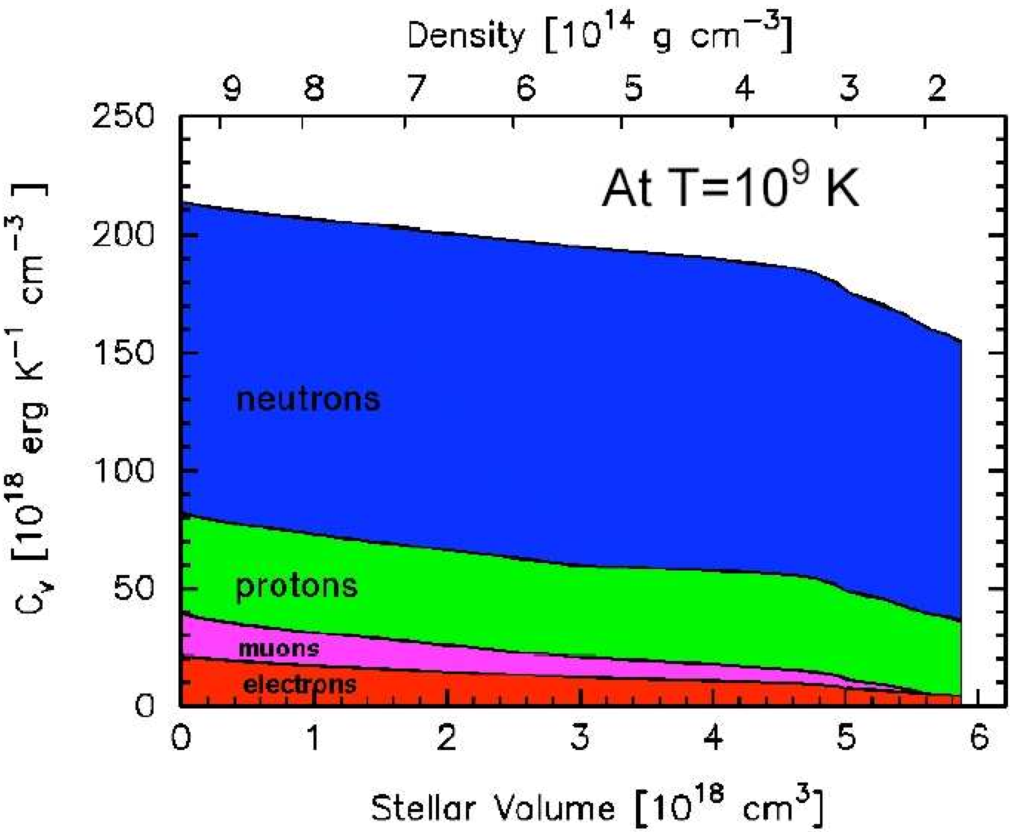
Moderate variation with nucleons-only matter.

Page, Lattimer, Prakash & Steiner, ApJS **155**, 623 (2004).

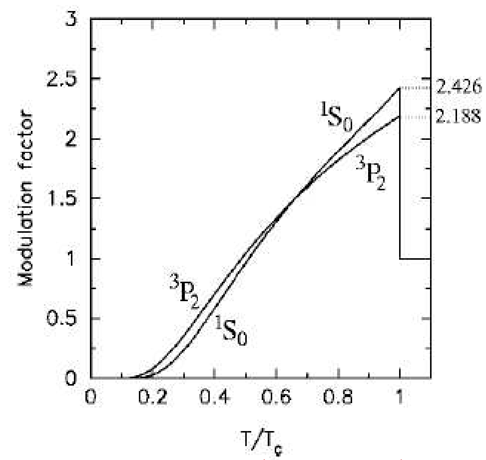
Specific Heat

Distribution of C_v in the core among constituents

$$C_V = N(0)\frac{\pi^2}{3}k_B^2 T \qquad N(0) = \frac{m^* p_F}{\pi^2 \hbar^3}$$



$$C_V^{paired} = C_V^{normal} \times M(T/T_c) \approx C_V^{normal} \times e^{-\Delta(T)/kT}$$



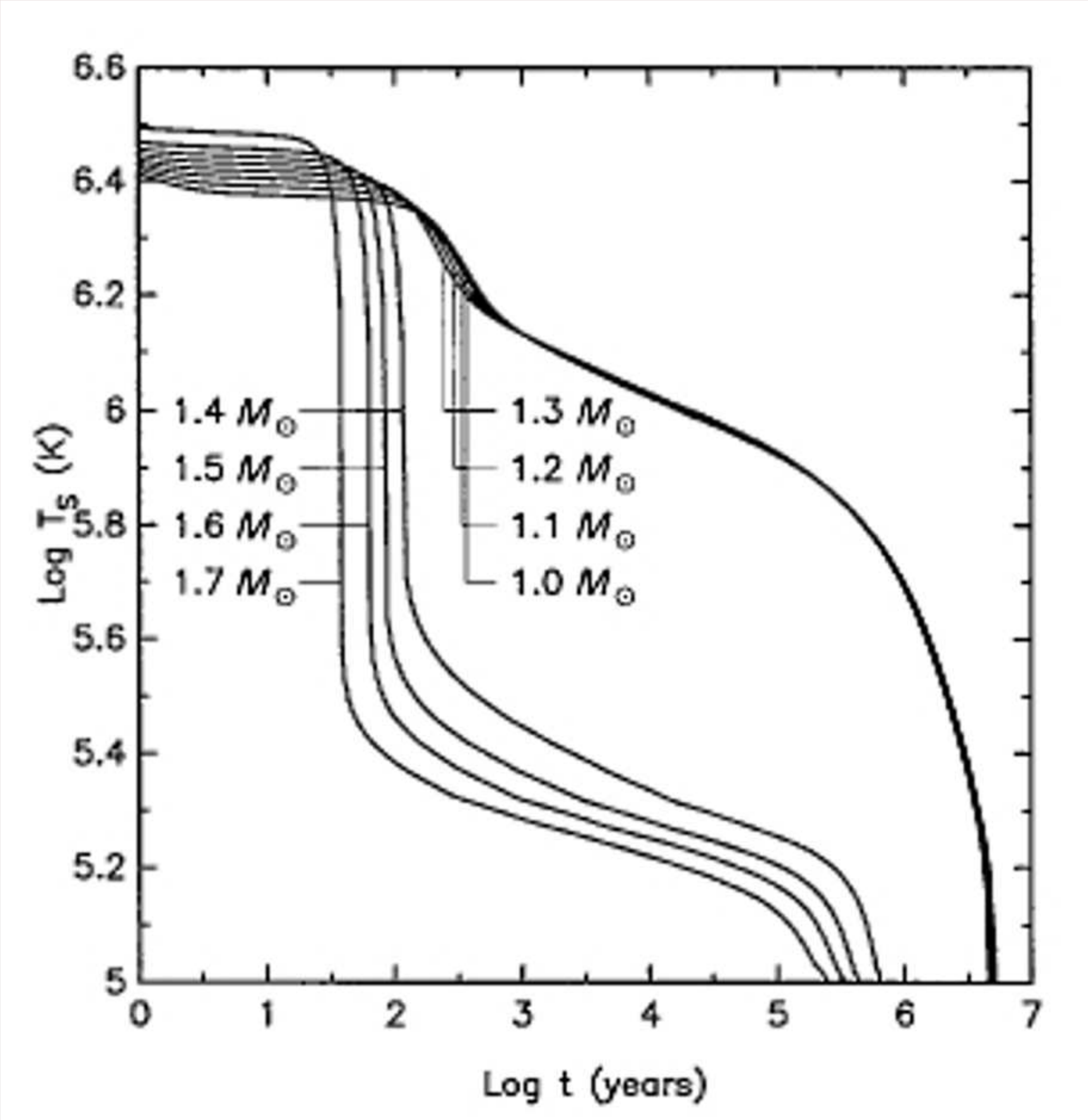
Page, Lattimer, Prakash & Steiner, ApJS **155**, 623 (2004).

Neutrino Emissivities

Name	Process	Emissivity (erg s^{-1} cm^{-3})	References
Modified Urca	$n + n' \to n + p + e^- + \bar{\nu}_e$ $n' + p + e^- \to n' + n + \nu_e$	$\sim 10^{20} T_9^8$	Friman & Maxwell 1979
Kaon Condensate	$n + K^- \to n + e^- + \bar{\nu}_e$ $n + e^- \to n + K^- + \nu_e$	$\sim 10^{24} T_9^6$	Brown et al., 1988
Pion Condensate	$n + \pi^- \to n + e^- + \bar{\nu}_e$ $n + e^- \to n + \pi^- + \nu_e$	$\sim 10^{26} T_9^6$	Maxwell et al., 1977
Direct Urca	$n \to p + e^- + \bar{\nu}_e$ $p + e^- \to n + \nu_e$	$\sim 10^{27} T_9^6$	Lattimer et al., 1991
Hyperon Urca	$B_1 \to B_2 + l + \bar{\nu}_l$ $B_2 + l \to B_1 + \nu_l$	$\sim 10^{26} T_9^6$	Prakash et al., 1992
Quark Urca	$d \to u + e^- + \bar{\nu}_e$ $u + e^- \to d + \nu_e$	$\sim 10^{26} \alpha_c T_9^6$	Iwamoto 1980

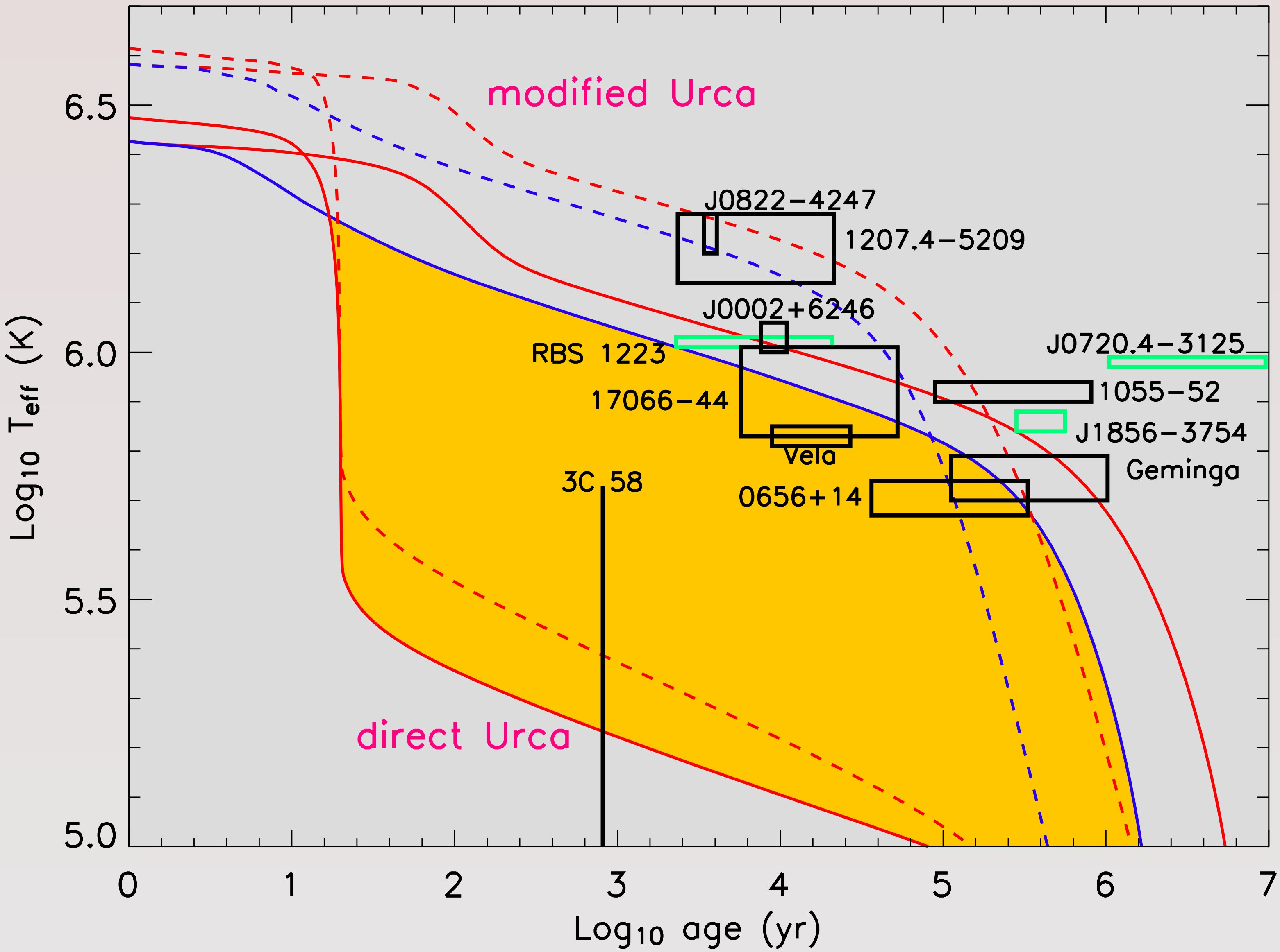
T_9: Temperature in units of 10^9 K.

Direct versus Modified Urca



- Unlike MUrca, Durca exhibits threshold effects.
- Superfluidity abates DUrca cooling.
- Page & Applegate, ApJ **394**, L17 (1992).
- Cooper pair breaking & reformation affects both DUrca & MUrca.

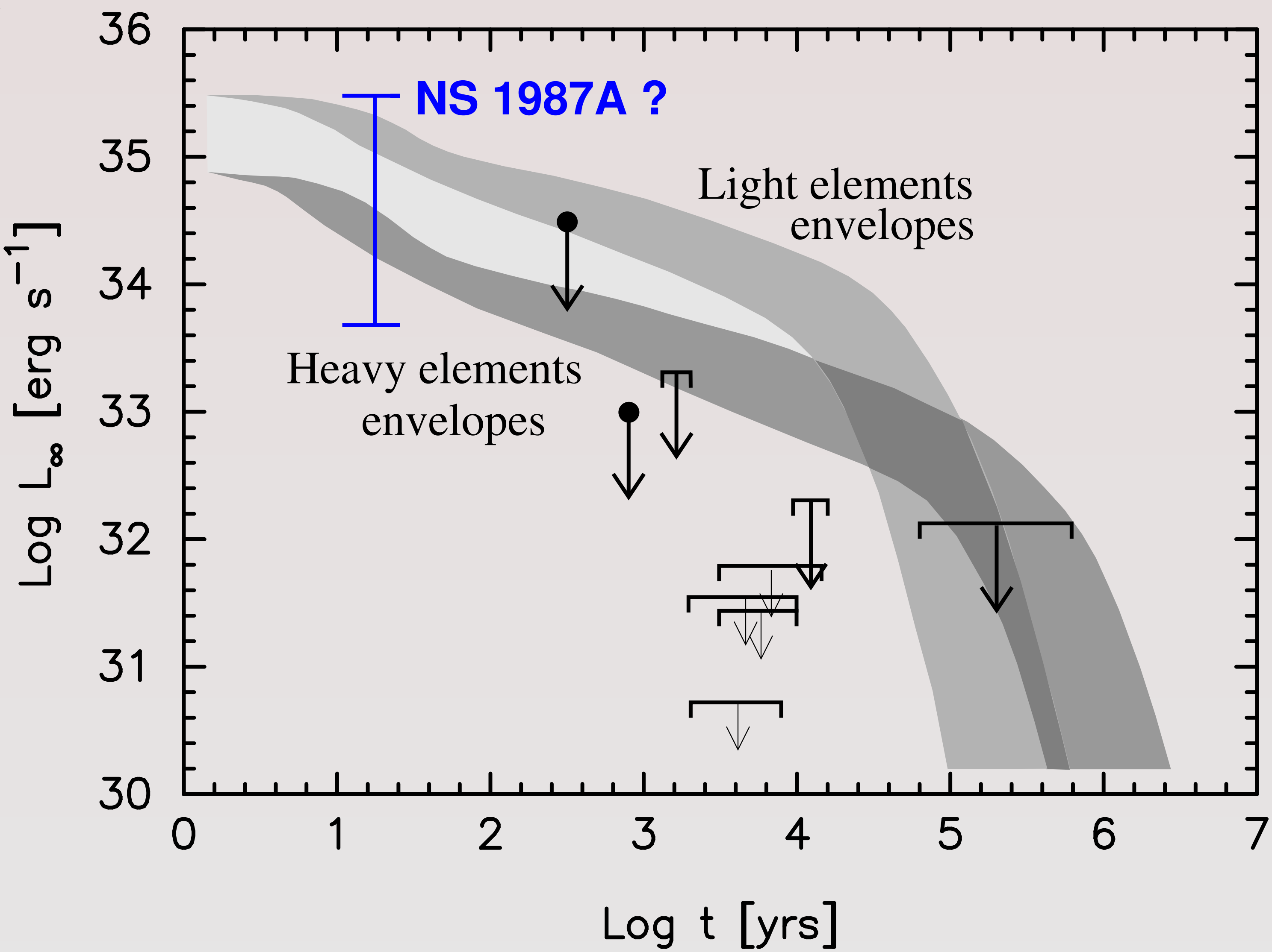
Inferred Surface Temperatures



Lattimer & Prakash , Science **304**, 536 (2004).

New Cold Objects

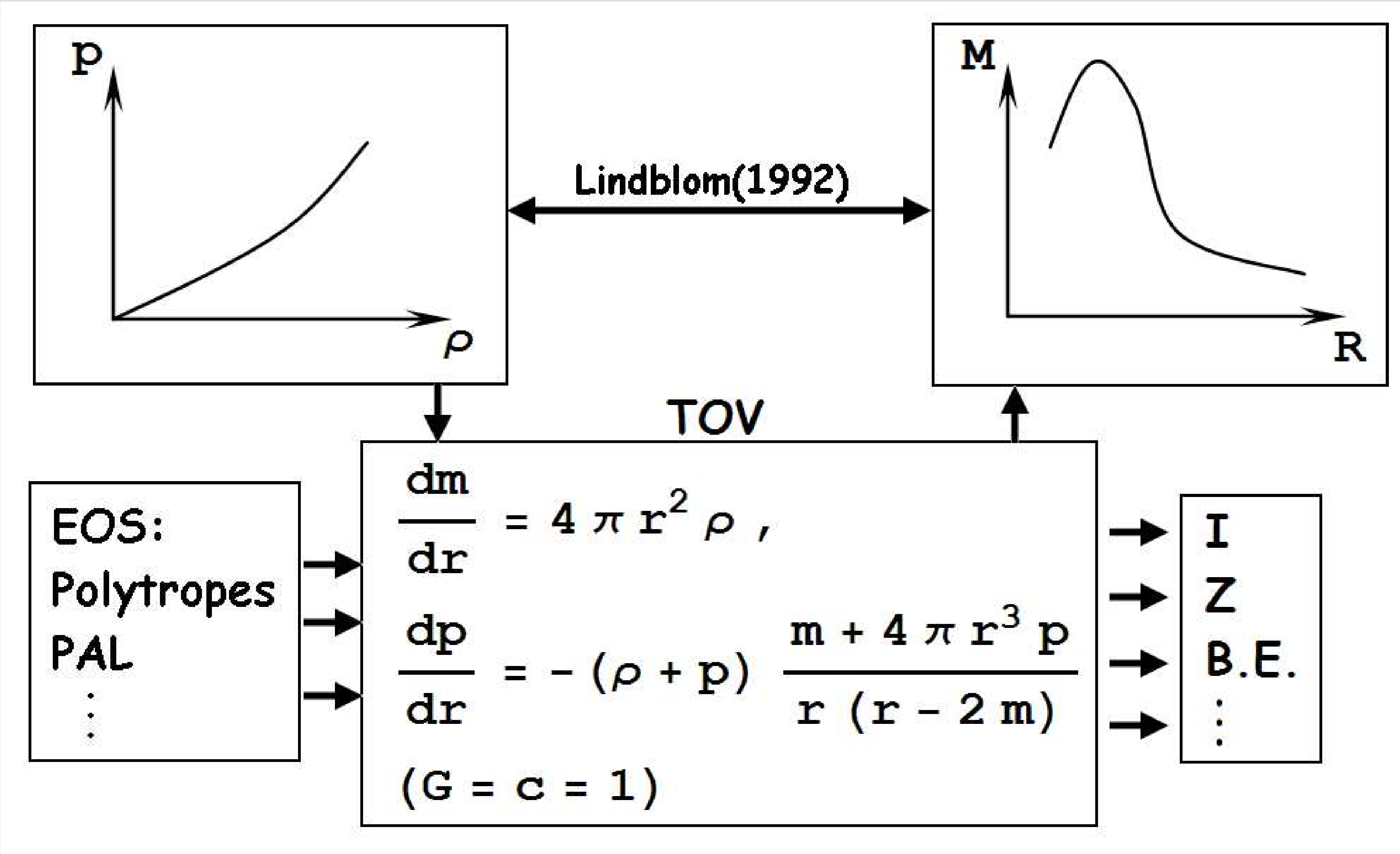
Several cases fall below the "Minimal Cooling" paradigm & point to enhanced cooling, if these objects correspond to neutron stars.



Page, Lattimer, Prakash & Steiner, ApJS **155**, 623 (2004).

Deconstructing a Neutron Star

- Constructing the EOS from several (M & R)'s of the same stars.



Inversion of the TOV equations

Recast TOV using new variable h

Advantages:
- finite at surface & center
- m & r depenent variables

$$dh = \frac{dp}{p + \rho\,(p)}$$

$$\frac{dr^2}{dh} = -2\,r^2\,\frac{r - 2\,m}{m + 4\,\pi\,r^3\,p}\,,$$

$$\frac{dm}{dh} = -4\,\pi\,r^3\,\rho\,\frac{r - 2\,m}{m + 4\,\pi\,r^3\,p}$$

With EOS

$p\,(h)$ and $\rho\,(h)$

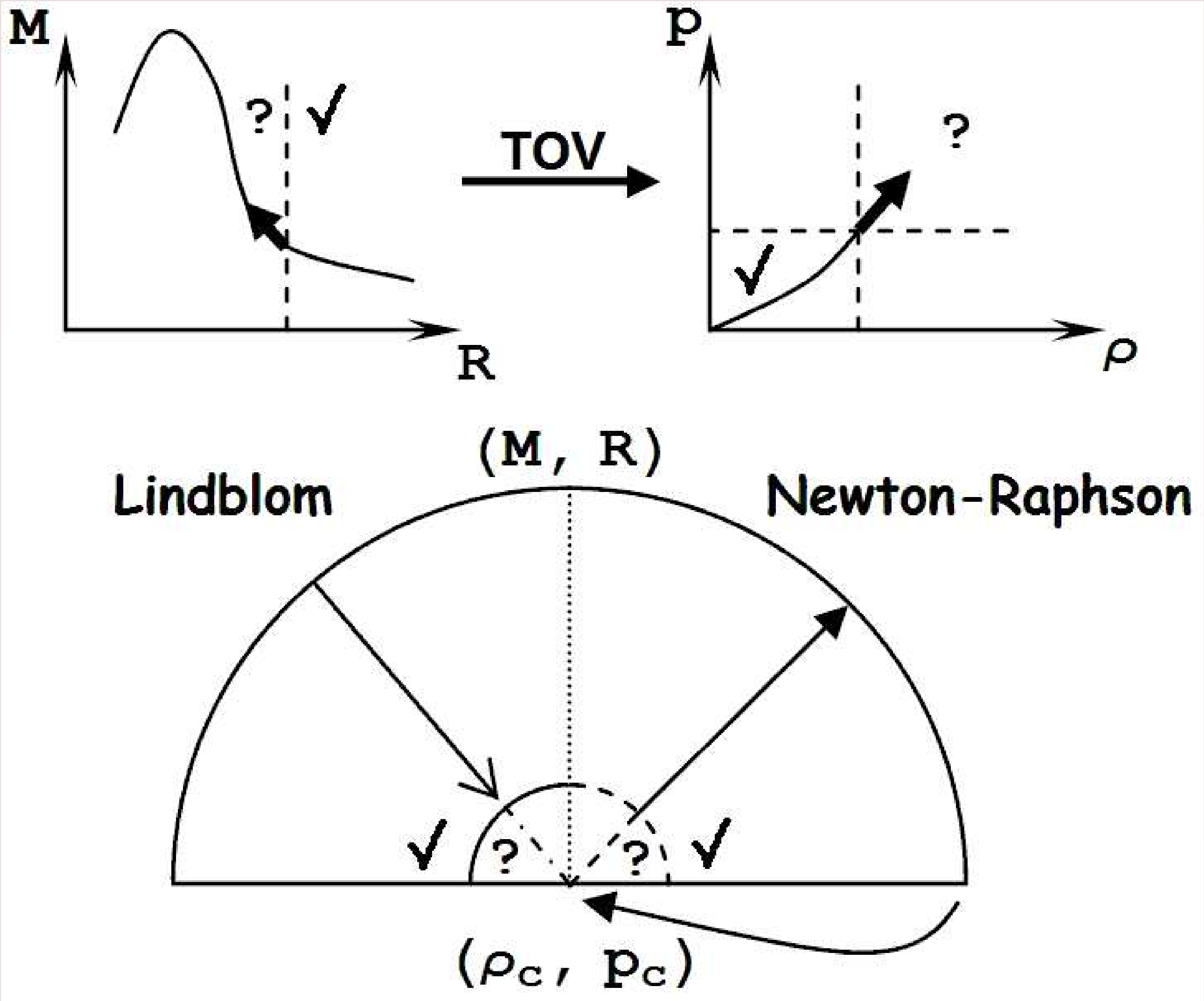
At star's center

$$r^2\,(h) = \frac{3\,(h_c - h)}{2\,\pi\,(3\,p_c + \rho_c)}\left(1 + \frac{3\,(d\rho\,/\,dh)_c + 15\,p_c - 5\,\rho_c}{10\,(3\,p_c + \rho_c)}\,(h_c - h) + \ldots\right),$$

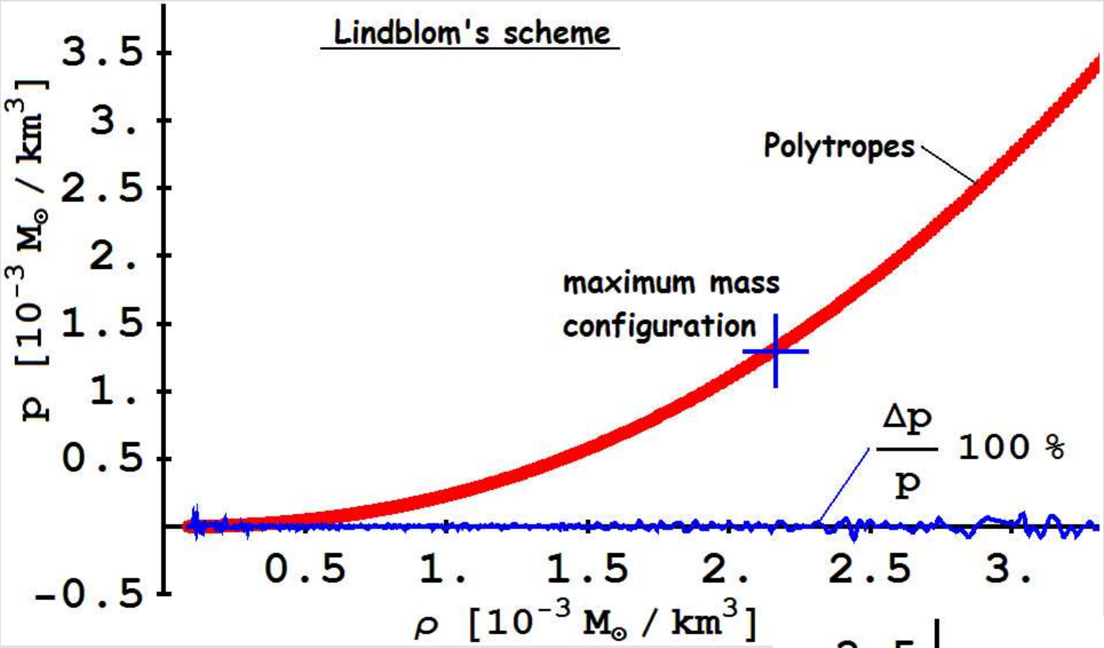
$$m\,(h) = \frac{4\,\pi}{3}\,r^3\,(h)\,\rho_c\left(1 - \frac{3\,(d\rho\,/\,dh)_c}{5\,\rho_c}\,(h_c - h) + \ldots\right)$$

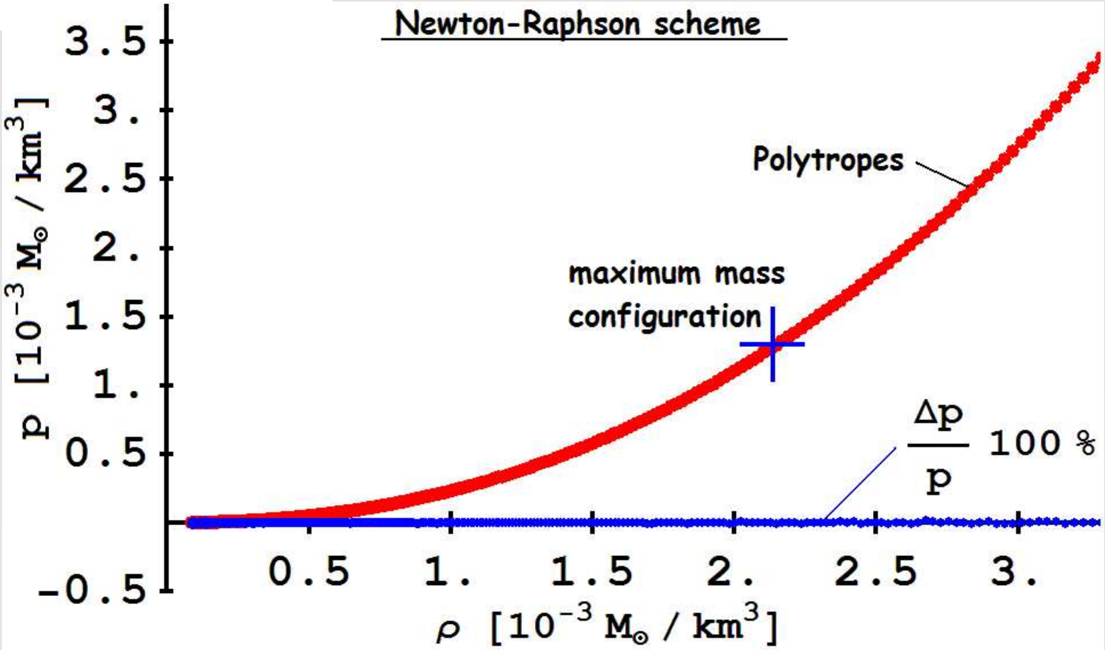
Ongoing work : Postnikov, Prakash & Lattimer (2008)

Two ways to delimit the EOS



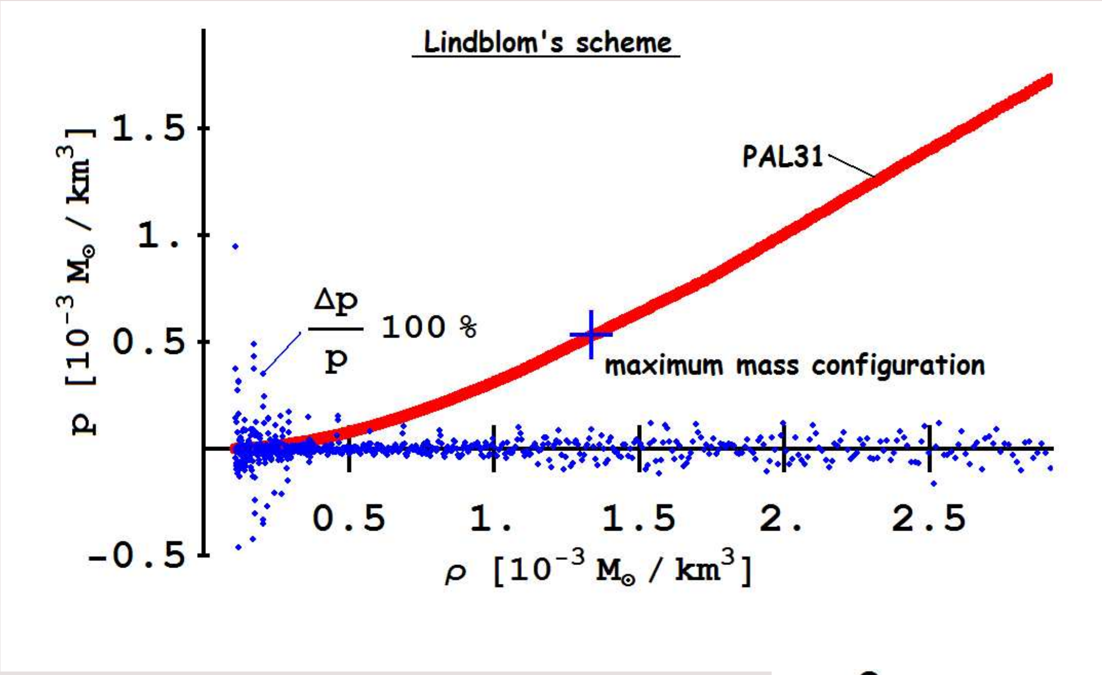
Tests with a two-power polytrope

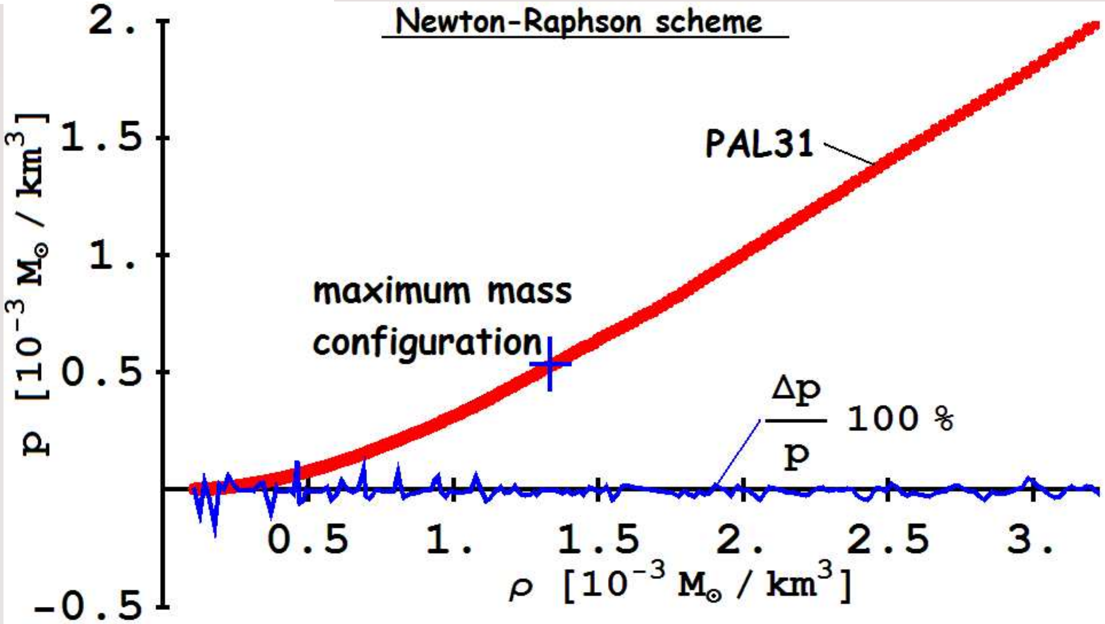




: For low-density, polytropic index $n = 0.8$, for high-density $n = 2$.

Tests with a model EOS





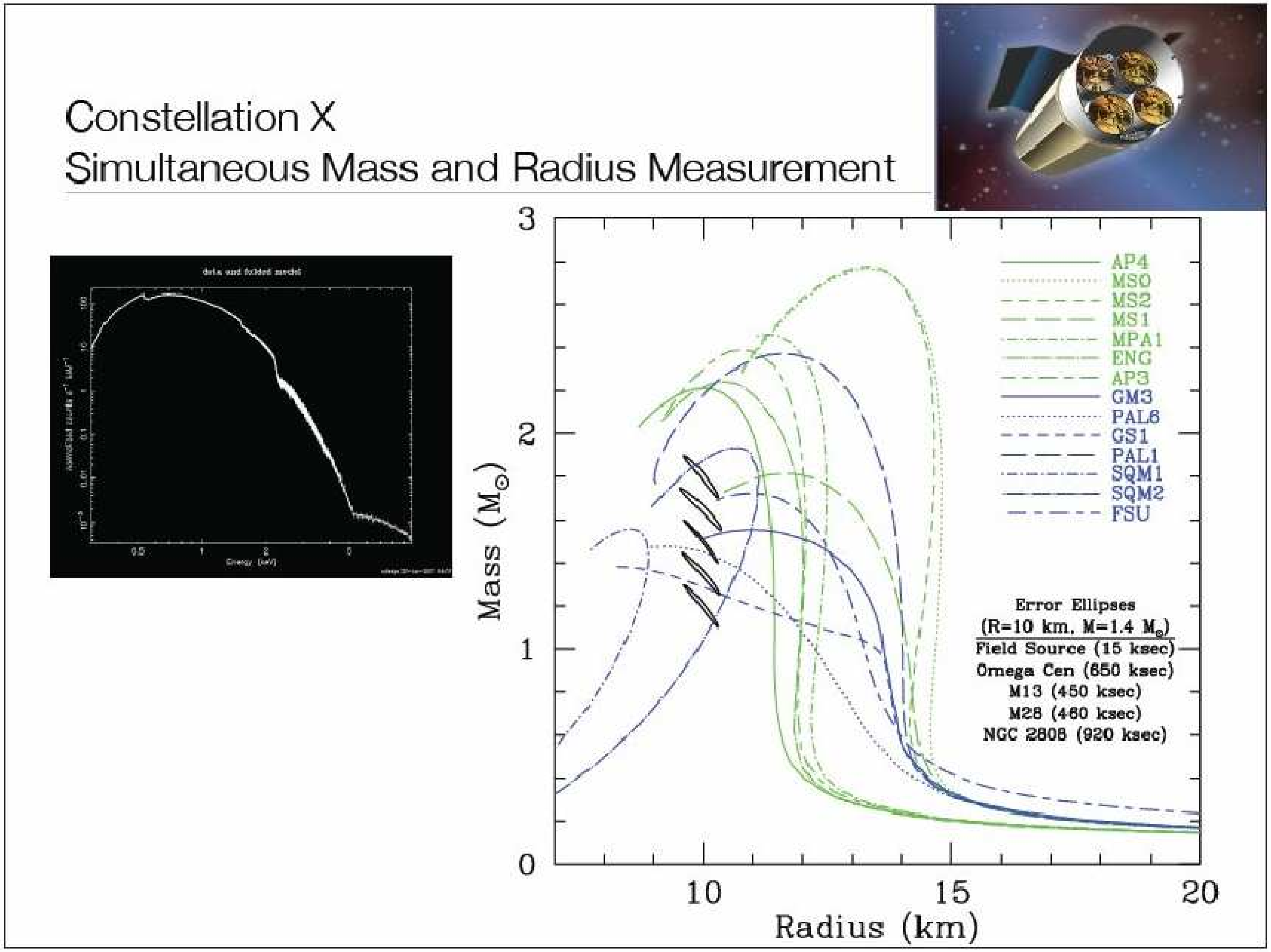
PAL31: $K_0 = 240$ MeV & $F(u) = u$ with n/n_0.

Requirements

- Masses and radii of several (say 5 to 7) same neutron stars can pin down (through deconstruction) the equation of state of neutron-star matter and shed light on the dense matter equation of state (strong many-body interactions).

- Precise laboratory experiments, particularly those involving neutron-rich nuclei, are sorely needed to pin down the near-nuclear aspects of the symmetry energy (masses, neutron skin thicknesses, collective excitations, etc.)

Future Prospects



Courtesey Bob Rutledge