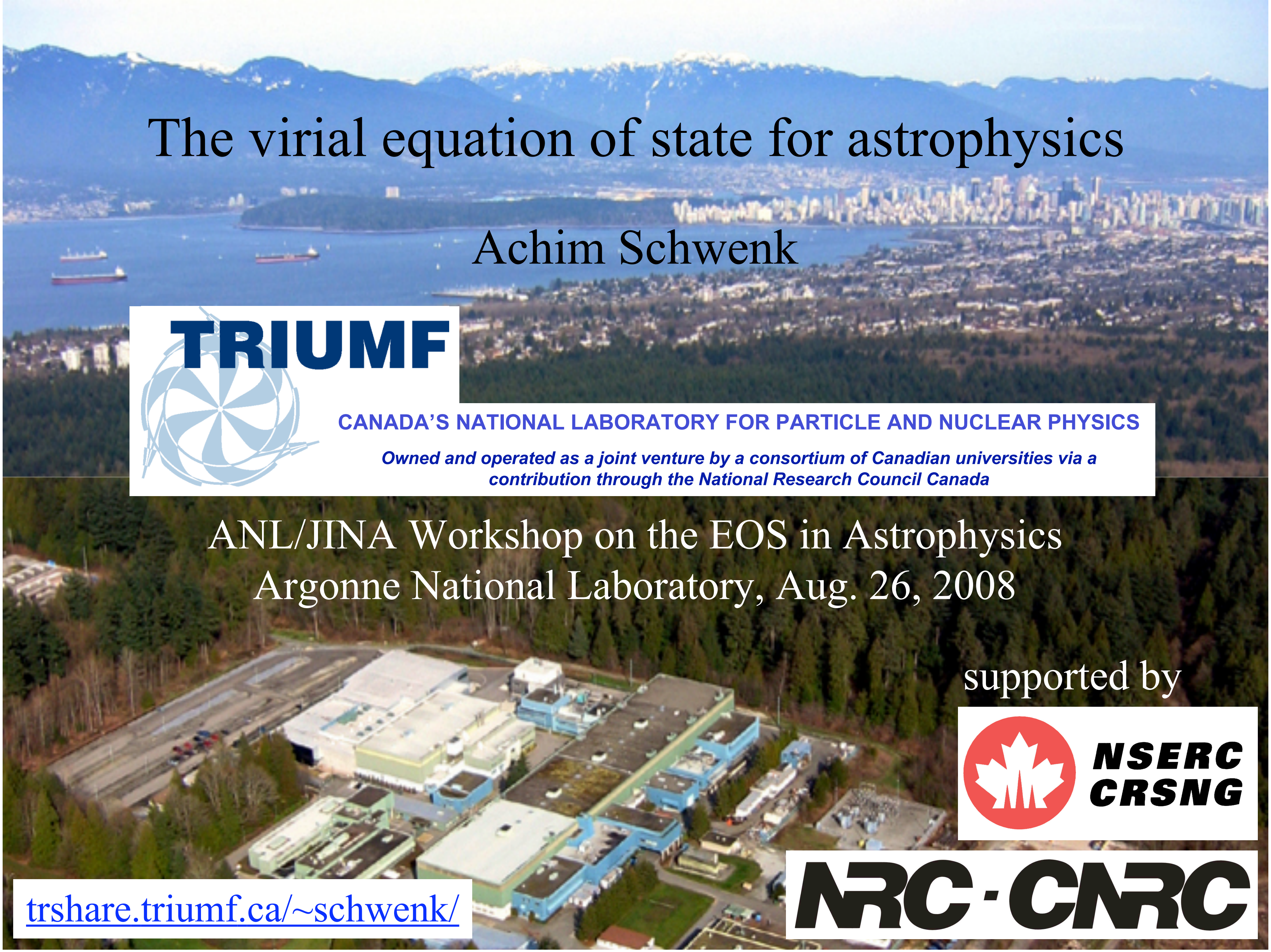

The virial equation of state for astrophysics

Achim Schwenk

TRIUMF

CANADA'S NATIONAL LABORATORY FOR PARTICLE AND NUCLEAR PHYSICS

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ANL/JINA Workshop on the EOS in Astrophysics
Argonne National Laboratory, Aug. 26, 2008

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Outline

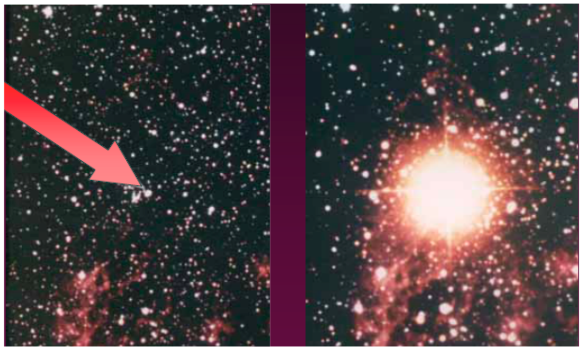
The virial equation of state of low-density nuclear matter
with C.J. Horowitz and E. O'Connor

Neutrino response from the virial expansion with C.J. Horowitz

Light nuclei A=2,3 and neutrino breakup
with E. O'Connor, D. Gazit, C.J. Horowitz, N. Barnea
and with A. Arcones, G. Martinez-Pinedo, T. Janka, C.J. Horowitz, K. Langanke

Equation of state from low-momentum interactions at intermediate densities with S.K. Bogner, R.J. Furnstahl, A. Nogga
with L. Tolos, B. Friman, and with C.J. Pethick

Motivation



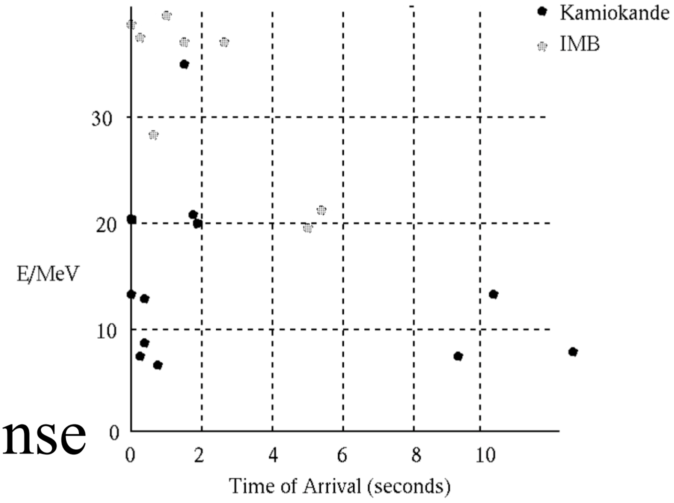
Before and after SN1987A

Core-collapse supernovae most sensitive to low-density nucleonic matter

Conditions at neutrinosphere (surface of last scattering of neutrinos):

T ~ 4 MeV from ~20 SN1987a events

n ~ 10^{11}-10^{12} g/cm^3 from nσ ~ n$(G_F E_\nu)^2$ ~ R^{-1}



What is the equation of state and neutrino response of nuclear matter near the neutrinosphere?

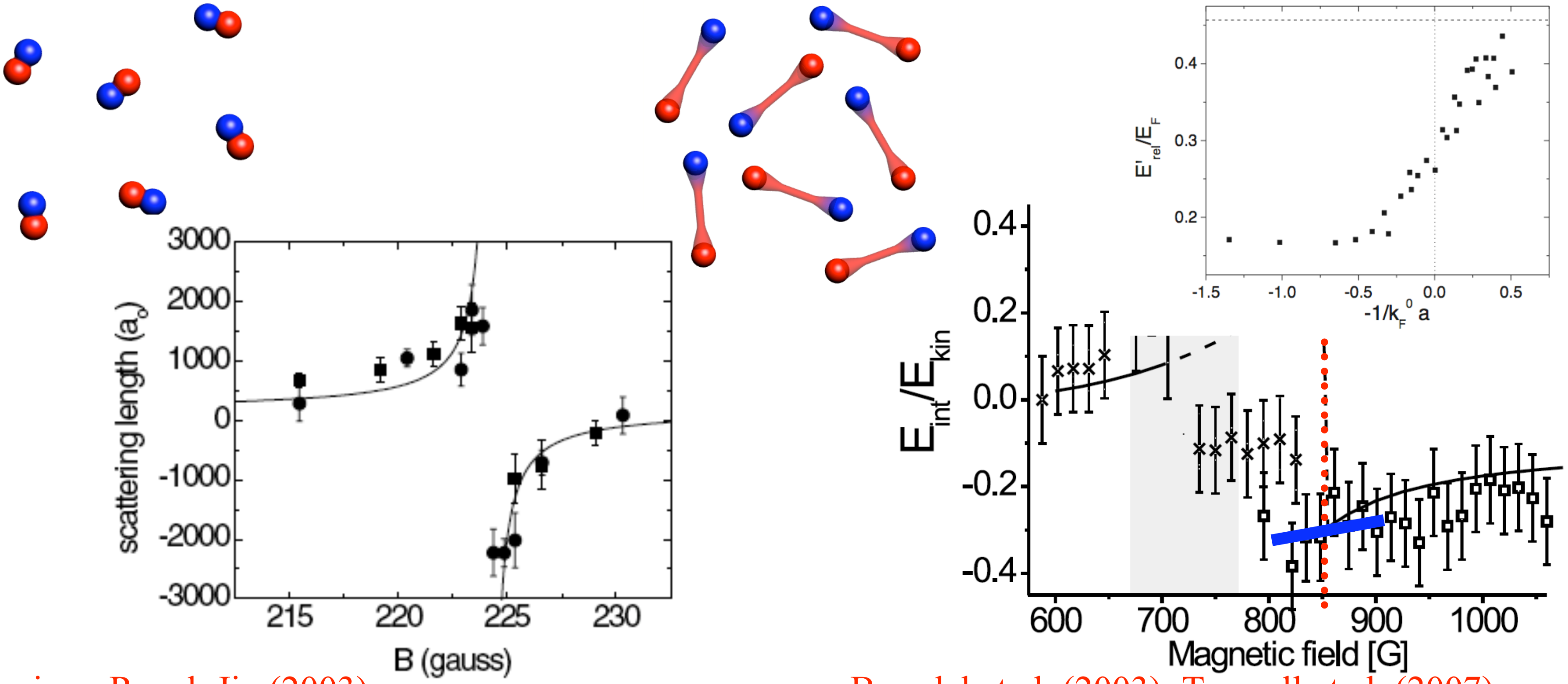
Fugacity small $z = e^{\mu/T} \lesssim 0.5$ for $n \lesssim 4 \cdot 10^{11}\,(T/\mathrm{MeV})^{3/2}\,\mathrm{g/cm}^3$

Virial exansion gives model-independent answers for SN neutrinosphere

Horowitz, AS (2006)

Very low-density physics is large scattering length physics

can tune scattering length of cold atoms via Feshbach resonances



Greiner, Regal, Jin (2003)

Bourdel et al. (2003); Tarruell et al. (2007)

Properties continuous across resonance, desire systematic approach that includes bound nuclei and resonant interactions on equal footing

Nuclear interactions/reactions have many large scattering lengths: all nucleon-nucleon, neutron-alpha $P_{3/2}$, alpha-alpha 0^+, 2^+,…

Virial expansion: general formalism for low n, high T

assumptions: gas phase, T > any T_{crit}, fugacity $z = e^{\mu/T}$ small

Neutron matter

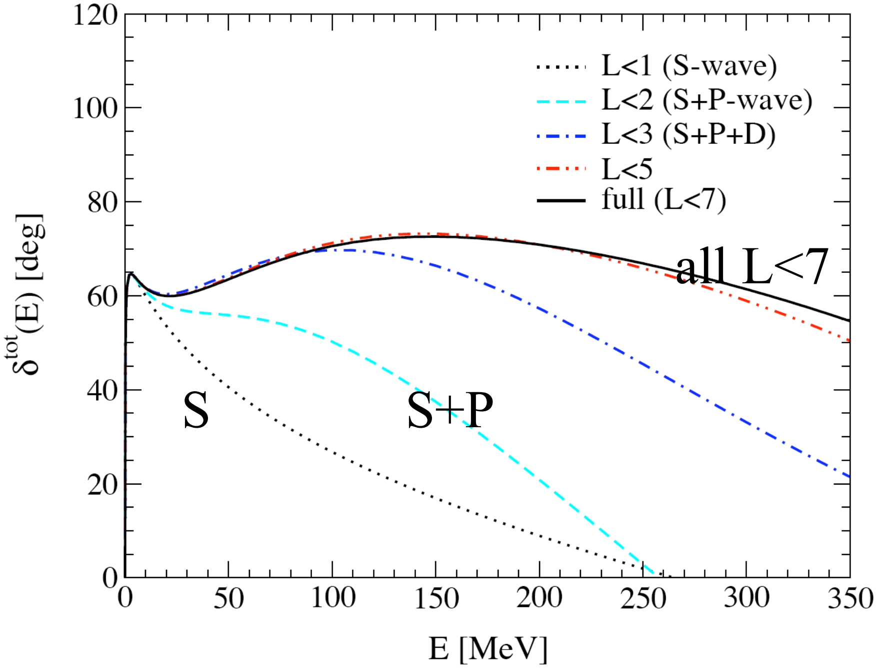
$$P = \frac{2T}{\lambda^3}\left(z + z^2 b_n + z^3 b_n^{(3)} + \mathcal{O}(z^4)\right) \qquad n = \frac{2}{\lambda^3}\left(z + 2z^2 b_n + 3z^3 b_n^{(3)} + \mathcal{O}(z^4)\right)$$

Second virial coefficient ~ 2-particle partition fn

$$b_n(T) = \frac{1}{2^{1/2}\,\pi T}\int_0^\infty dE\, e^{-E/2T}\,\delta^{\mathrm{tot}}(E) - 2^{-5/2}$$

For infinite scattering length a=±∞ $b_n = 3/2^{5/2} = 0.53$, not $k_F a$ expansion, tested in cold atoms Ho, Mueller (2004); Thomas et al. (2005)

Second virial coefficient for neutrons approx T independent b_n=0.30, leads to scaling $E/E_{\mathrm{free}} = P/P_{\mathrm{free}} = \xi(T/T_F)$

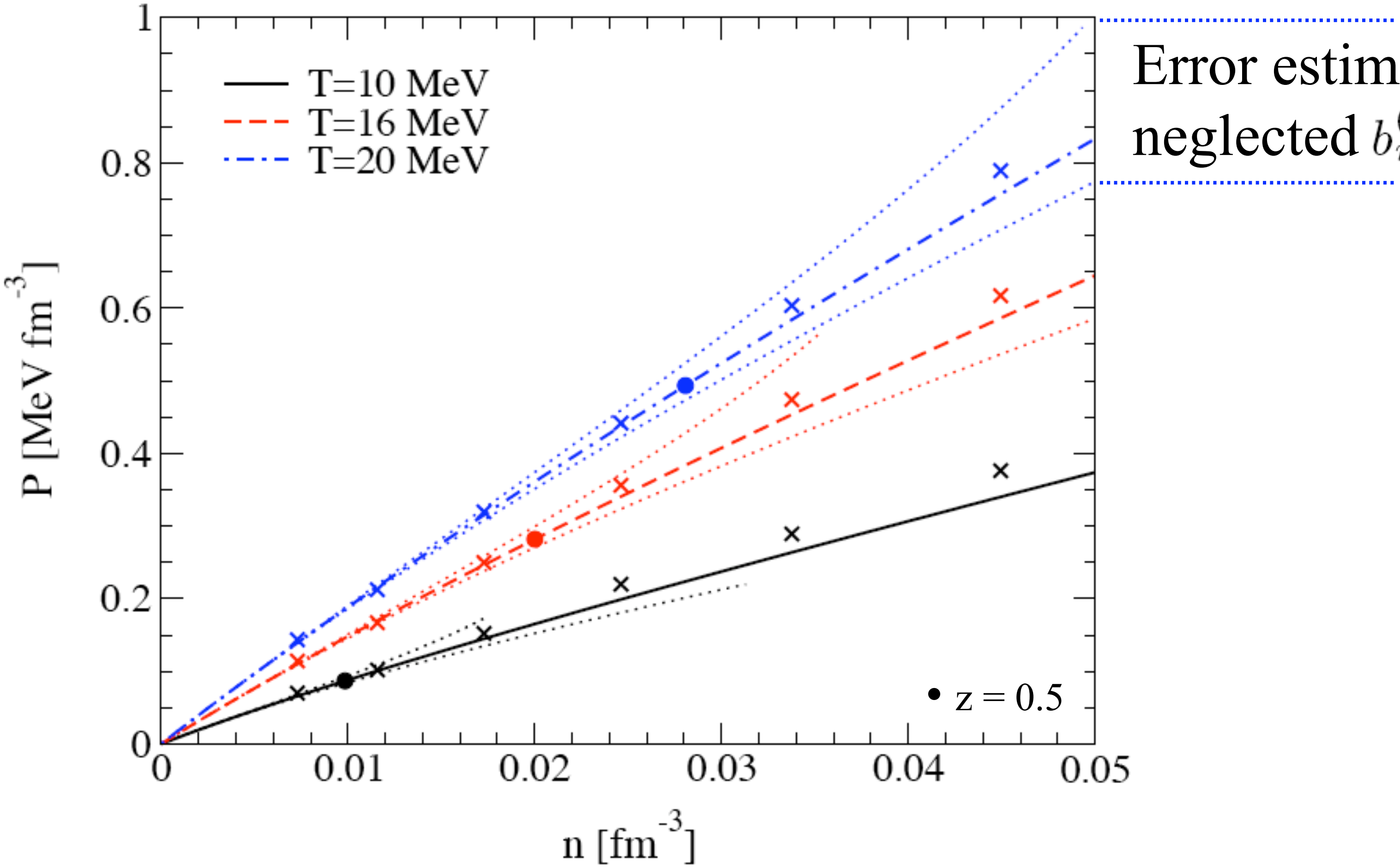


Previous work Buchler, Coon (1977); Pratt et al. (1987); Venugopalan, Prakash (1992); Roepke at al.

Neutron matter equation of state

Fugacity small for $n \lesssim 4 \cdot 10^{11} \, (T/\mathrm{MeV})^{3/2} \, \mathrm{g/cm^3}$

Comparison to Friedman, Pandharipande (×)



Error estimates due to neglected $b_n^{(3)} = \pm b_n/2$

Nuclear matter

deuterons enter as bound state contribution to $b_2 \sim e^{E_d/T}$
nuclei as bound state contributions to b_A, limits nucleon virial expansion

at low densities, nuclear matter mainly composed of n,p and α particles,
include α particles explicity, to second-order in fugacities z_n, z_p, z_α

$$\frac{P}{T} = \frac{2}{\lambda^3}\left(z_n + z_p + (z_n^2 + z_p^2)\enclose{circle}{b_n} + 2 z_p z_n \enclose{circle}{b_{pn}}\right) + \frac{1}{\lambda_\alpha^3}\left(z_\alpha + z_\alpha^2 \enclose{circle}{b_\alpha} + 2 z_\alpha (z_n + z_p) \enclose{circle}{b_{\alpha n}}\right)$$

second virial coefficients directly from NN, Nα, αα phase shifts and E_d

model-independent description of matter in thermal equilibrium

consider chemical equilibrium $z_\alpha = z_p^2\, z_n^2\, e^{E_\alpha/T}$
adjust z_n, z_p to reproduce desired baryon density and proton fraction

can include heavy nuclei at higher densities with z_A
virial b_{NA},… correct NSE models for strong interactions between nuclei

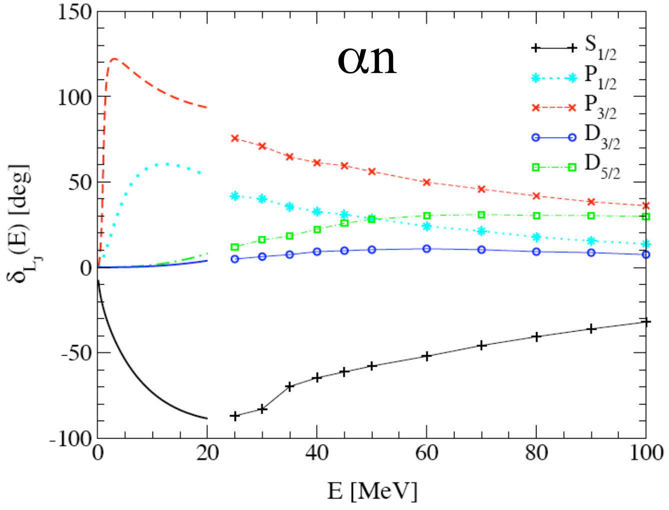
Virial coefficients

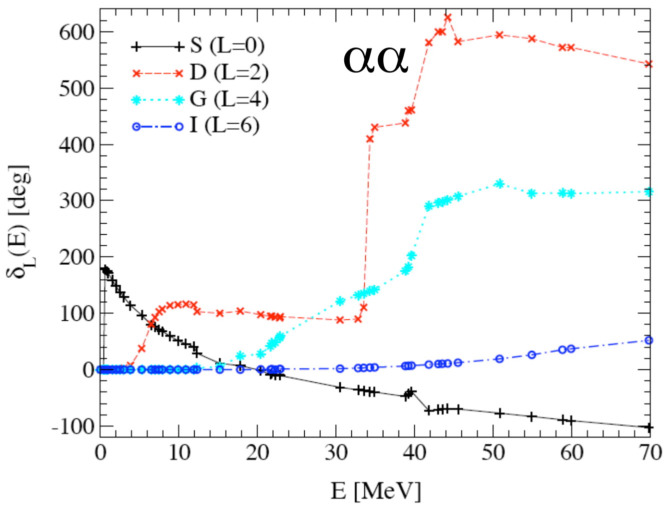
neglected Coulomb (use np,nα phase shifts; b_2 for plane wave bc), mixing parameters and inelasticities in scattering, can improve this

for b_{NN}: all L≤6 from Nijmegen PWA93, includes deuteron and large $^1S_0$ scattering lengths on equal footing

for $b_{\alpha n}$: all L≤3 from Arndt, Roper (1970) for E<20 MeV, Amos, Karataglidis (2005) optical model for higher E, includes $P_{3/2}$ resonance

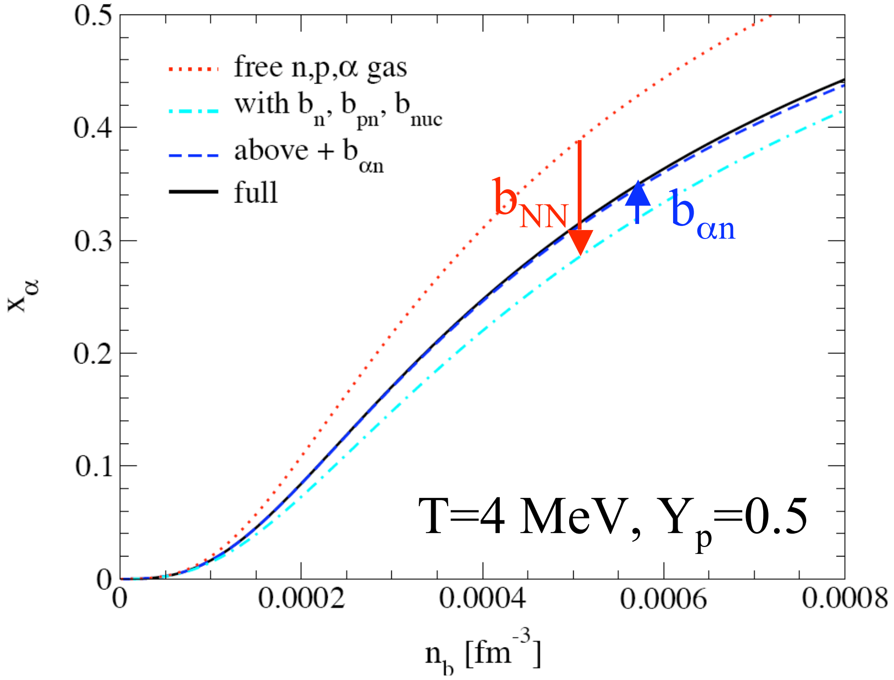
for $b_{\alpha\alpha}$: all L≤6 from Afzal et al. (1969) for E<30 MeV, Bacher et al. (1972) for 30<E<70 MeV includes 0^+,2^+ resonances





virial coefficients dominated by resonant (large a) interactions

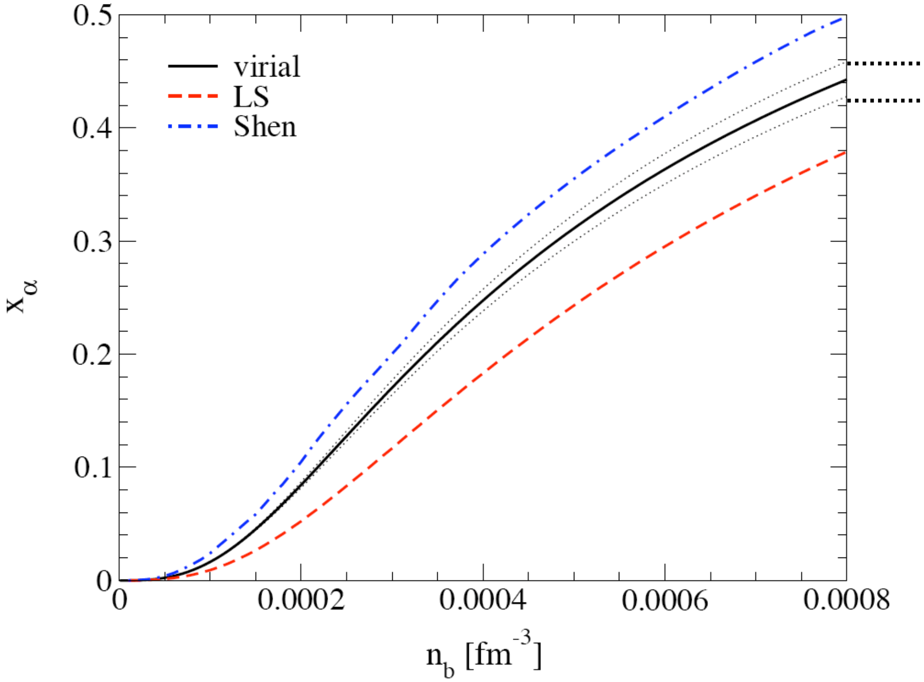
Composition: α mass fraction



Hierarchy of virial contributions:

b_{NN} more important than $b_{\alpha n}$, b_α

$b_{\alpha n}$ attractive due to $P_{3/2}$ resonance

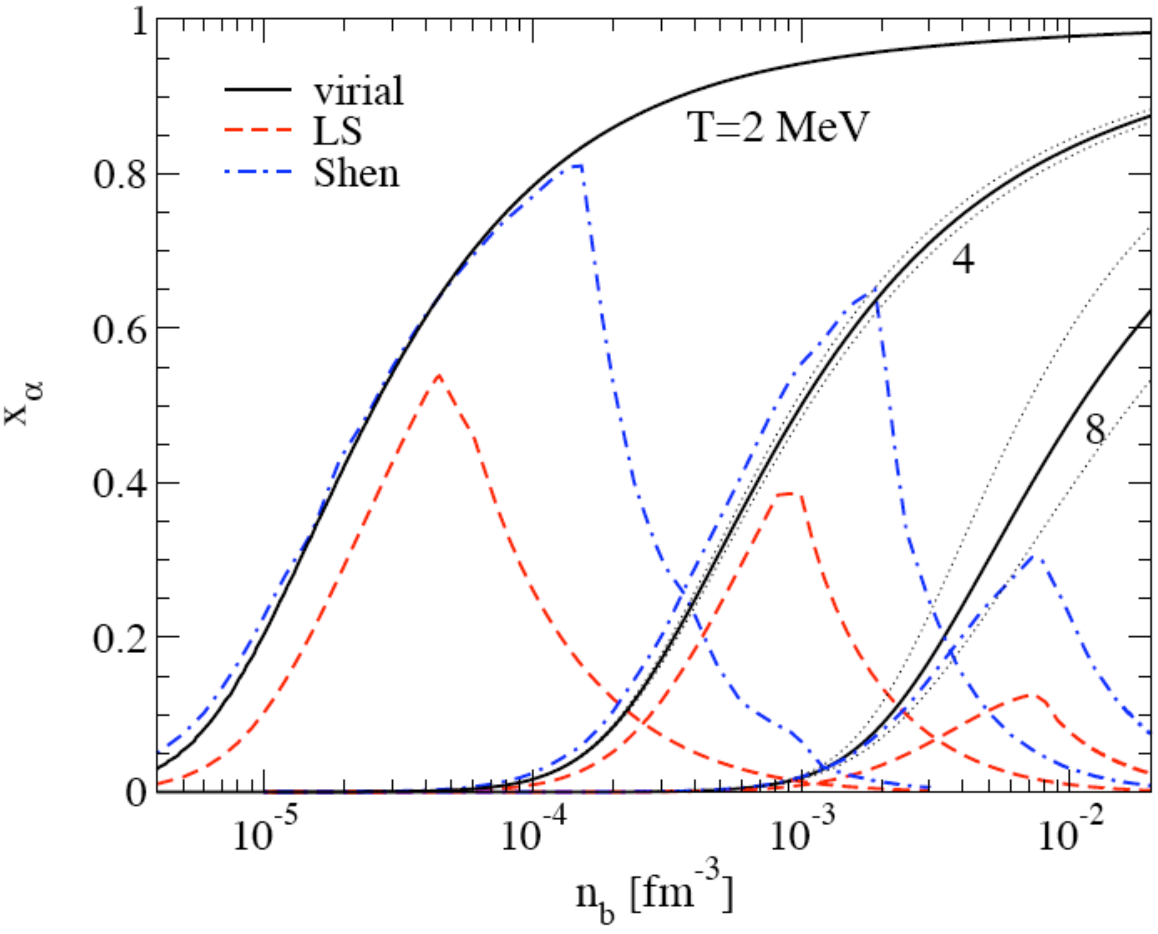


Estimate errors due to neglected third virial coefficient $b_3 \sim \pm 10$

α mass fraction differs from LS=Lattimer-Swesty, Shen et al. EOS used in SN simulations

LS models nα interaction with repulsive excluded volume

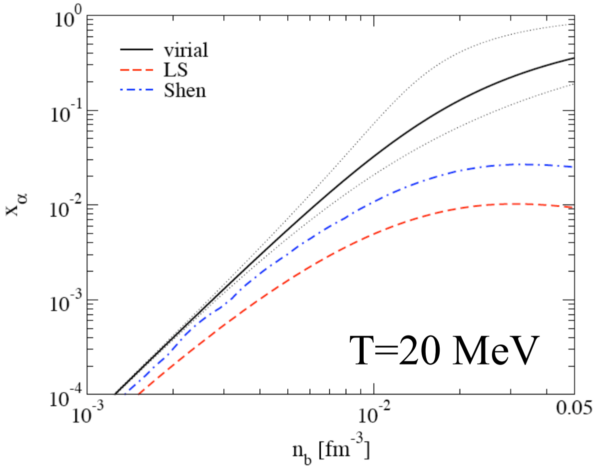
α mass fraction for various T



α fractions drop in LS/ Shen at high density due to formation of heavy A

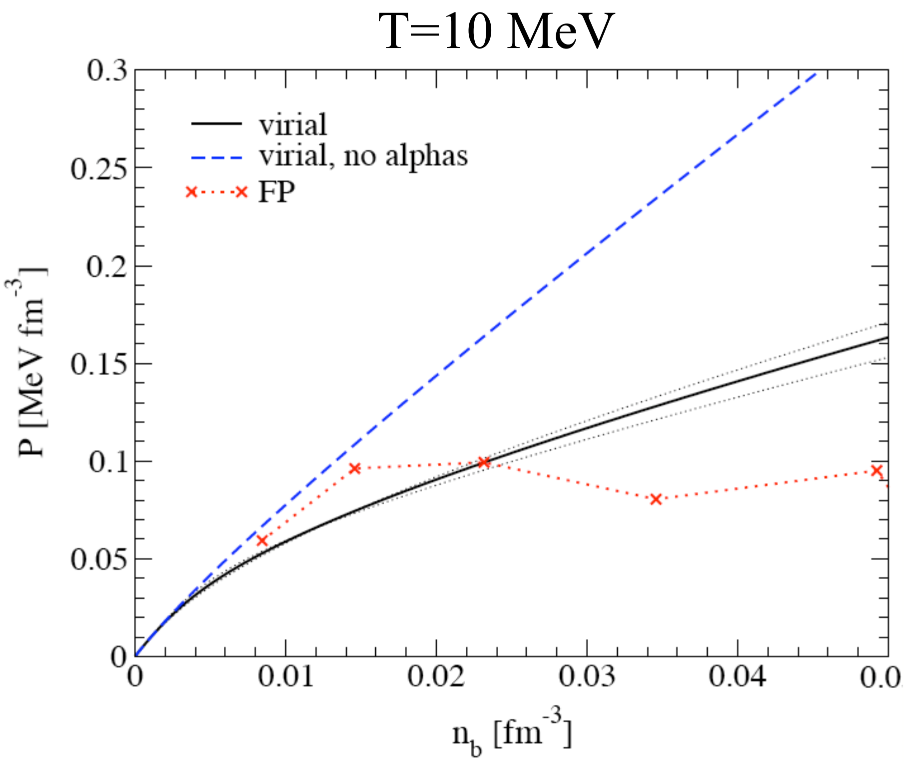
limits validity of n,p,α virial EOS

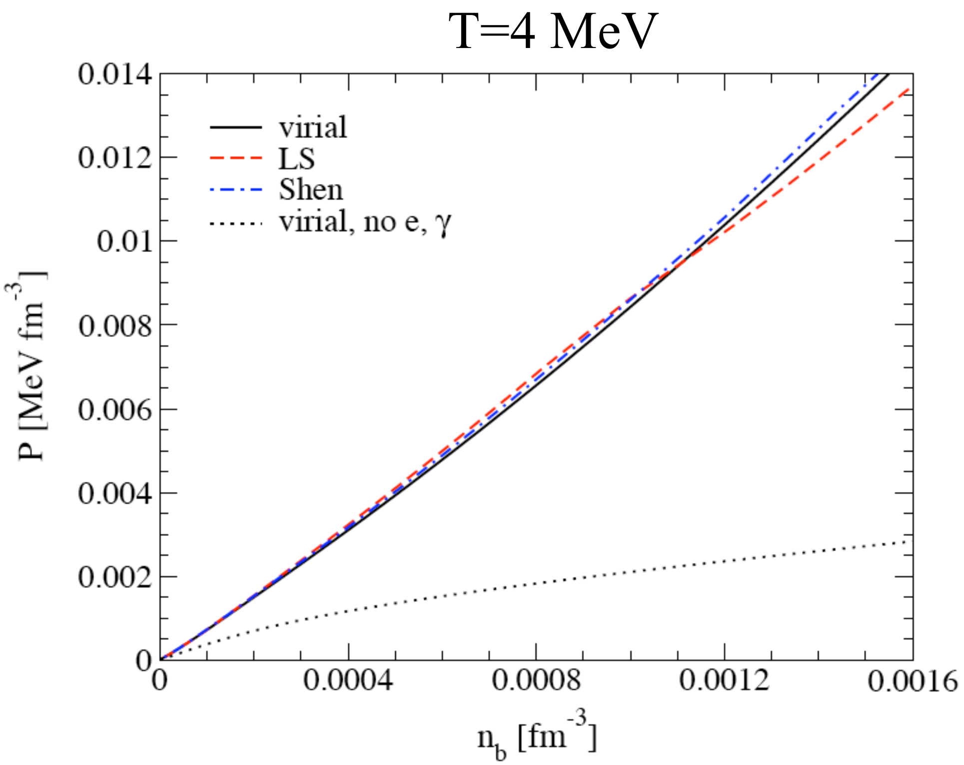
for T>10 MeV, models underestimate α fraction



x_α important for spin/neutrino response, since α particles have J=0

Pressure



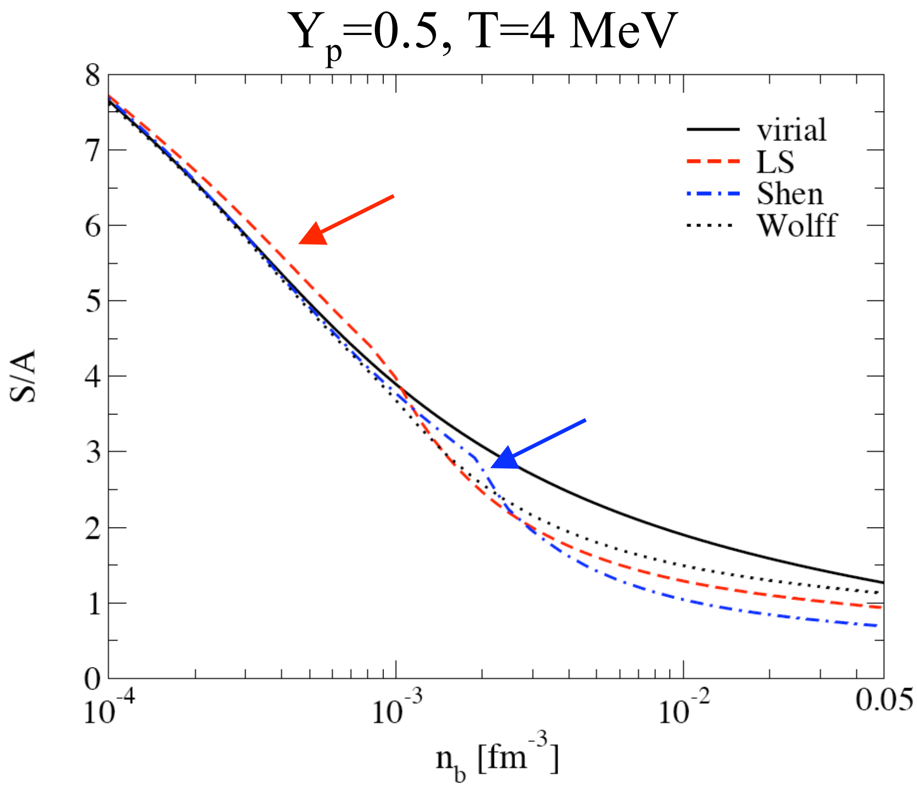


Variational calculations Friedman, Pandharipande fail to describe α contributions

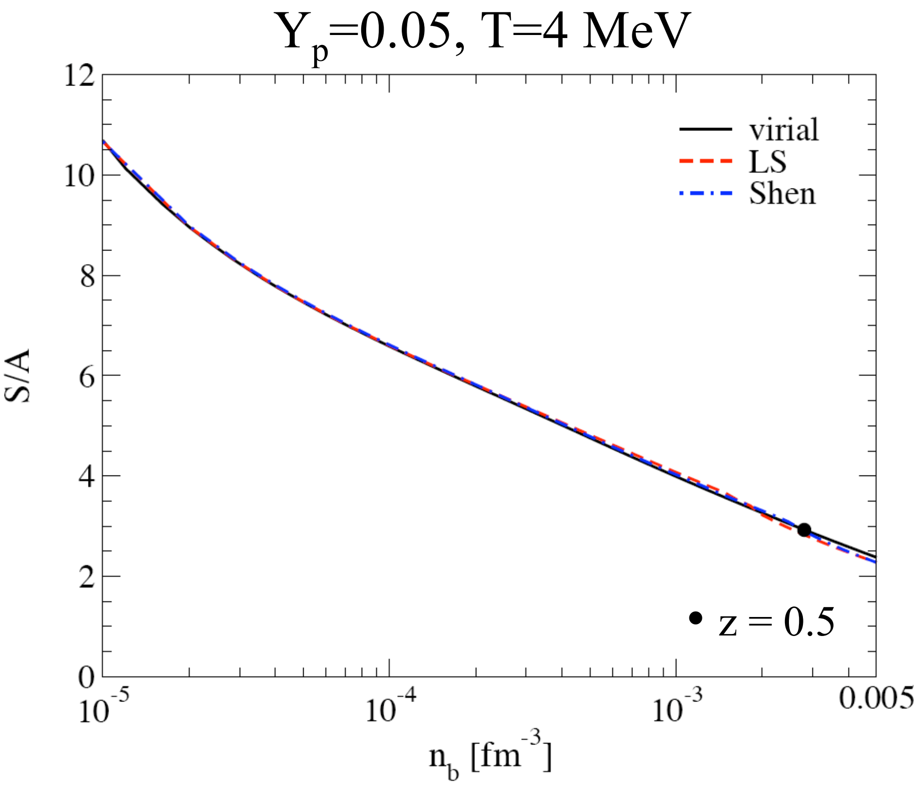
Pressure agrees well with LS, Shen et al. EOS

Entropy

Entropy reflects composition (proton and α fraction)



Y_p=0.5, T=4 MeV



Y_p=0.05, T=4 MeV

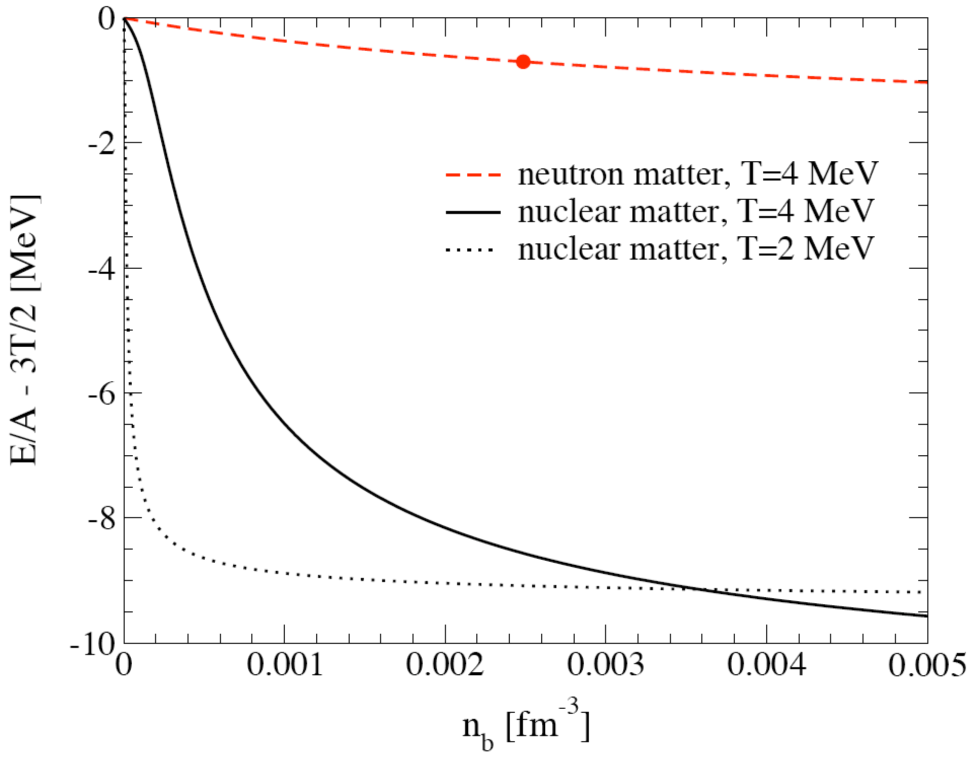
LS may predict too few α particles in nuclear matter

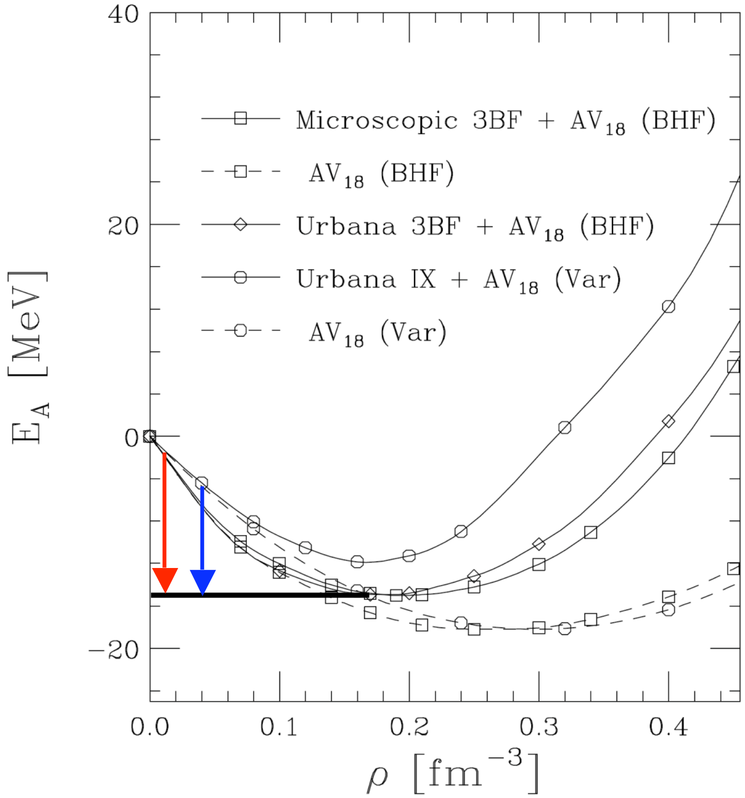
Good agreement for extremely neutron-rich matter

Breakdown of virial EOS due to heavy nuclei, while fugacities $z < 0.2$

Energy of low-density nuclear matter

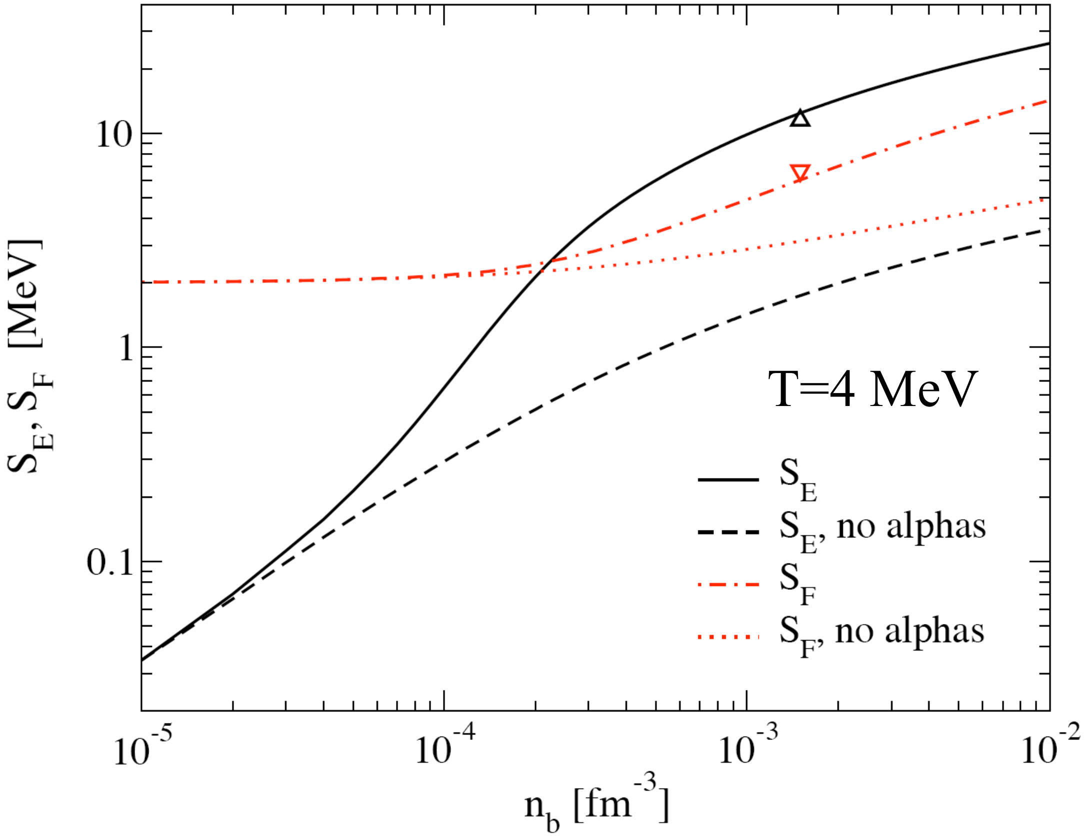
find large E/A at low densities due to clustering, α particles crucial
E/A ≈ const. even for 1/100 nuclear matter density





E/A ≈ const. requires α particles, heavy nuclei and larger clusters

Symmetry energy

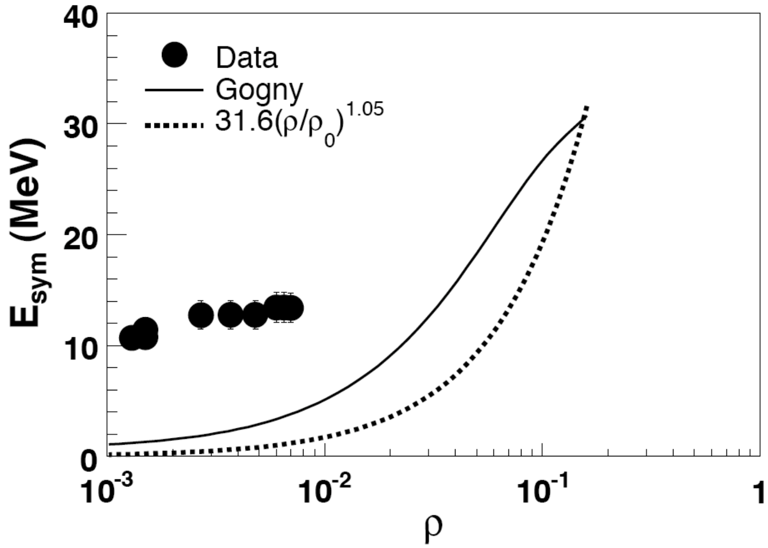


$$S_E = \frac{1}{8}\frac{\partial^2}{\partial {Y_p}^2}\frac{E}{A}\Big|_{Y_p=1/2}$$

large S_E at low density due to clustering

large S_E confirmed in near Fermi-energy HI collisions

triangles: $^{64}Zn+^{92}Mo/^{197}Au$ (Y_p=0.44) Kowalski et al. TAMU (2006)



Consistent neutrino response

Cross section for elastic νN scattering in n,p,α matter

$$\frac{1}{V}\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{16\pi^2}\left(g_a^2\,(3-\cos\theta)(n_n+n_p)S_a(q) + (1+\cos\theta)(n_n+4n_\alpha)S_v(q)\right)$$

S_a describes axial/spin response, S_v vector/density response

virial expansion provides consistent, model-independent response in long-wavelength limit. Neutron matter:

$$S_v(q=0) = \frac{T}{(\partial P/\partial n)_T} = \frac{1+4zb_n}{1+2zb_n}$$

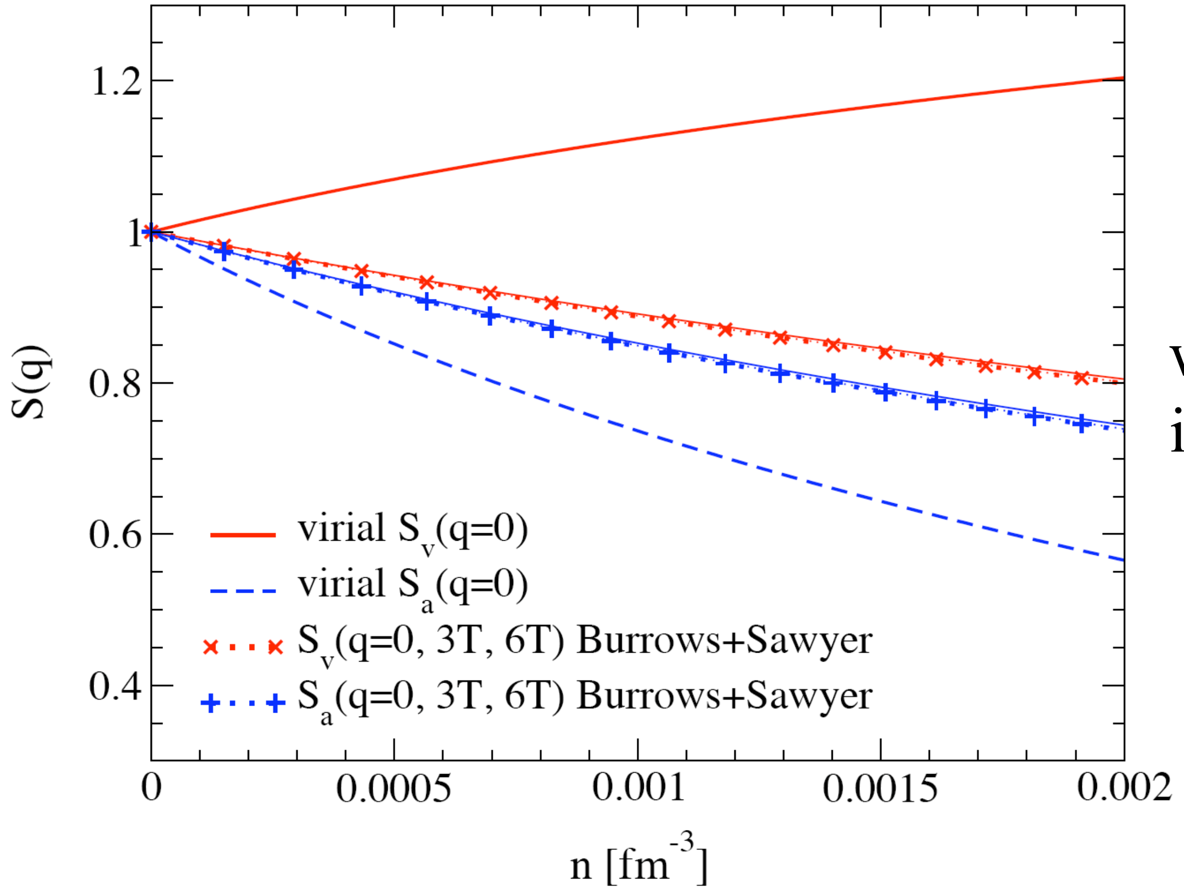
axial response from spin-polarized matter see Burrows, Sawyer (1998)

$z_{+/-}$ fugacity for spin ↑/↓, axial $z_a=(z_+/z_-)^{1/2}$

$$S_a(q=0) = \frac{1}{n}\frac{\partial}{\partial z_a}(n_+ - n_-)\bigg|_{z_a=1} = 1 + 2(b_+ - b_-)\frac{z}{1+2zb_n}$$

virial coeff $b_{+/-}$ for neutron-neutron with like ↑↑ / opposite ↑↓ spins

Response of neutron matter

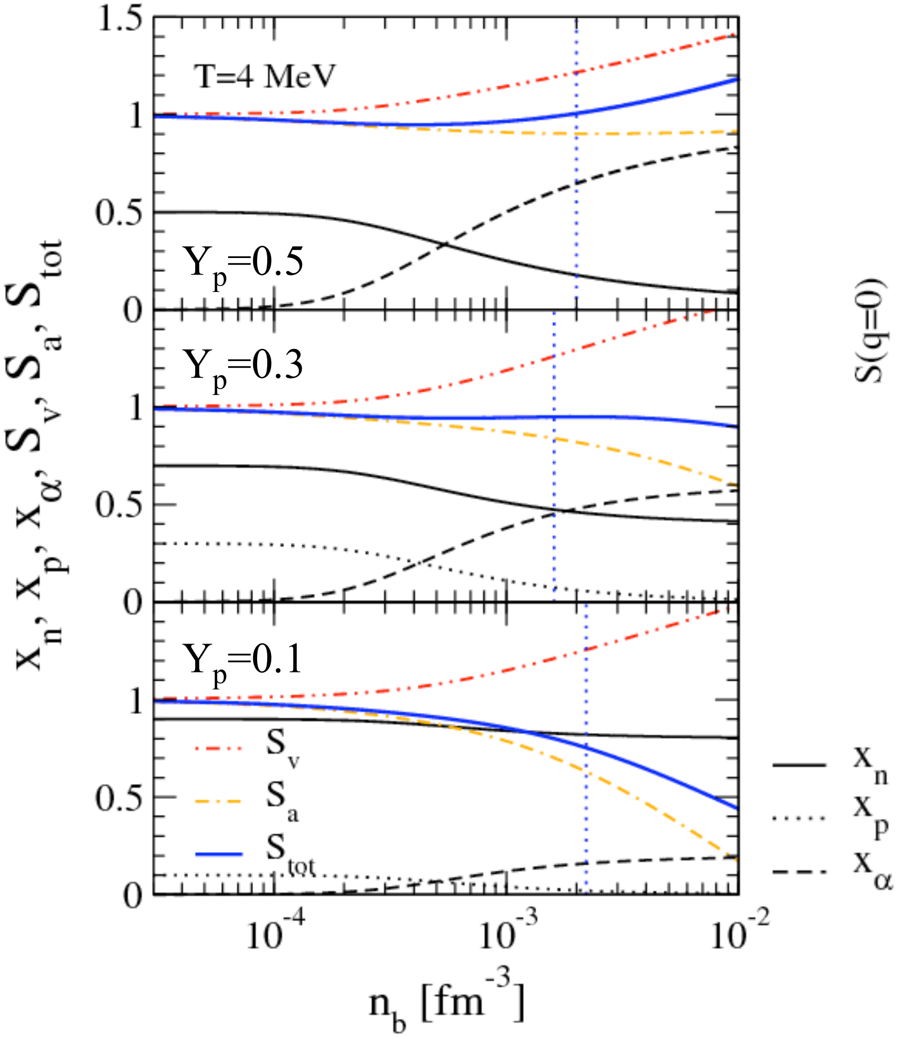


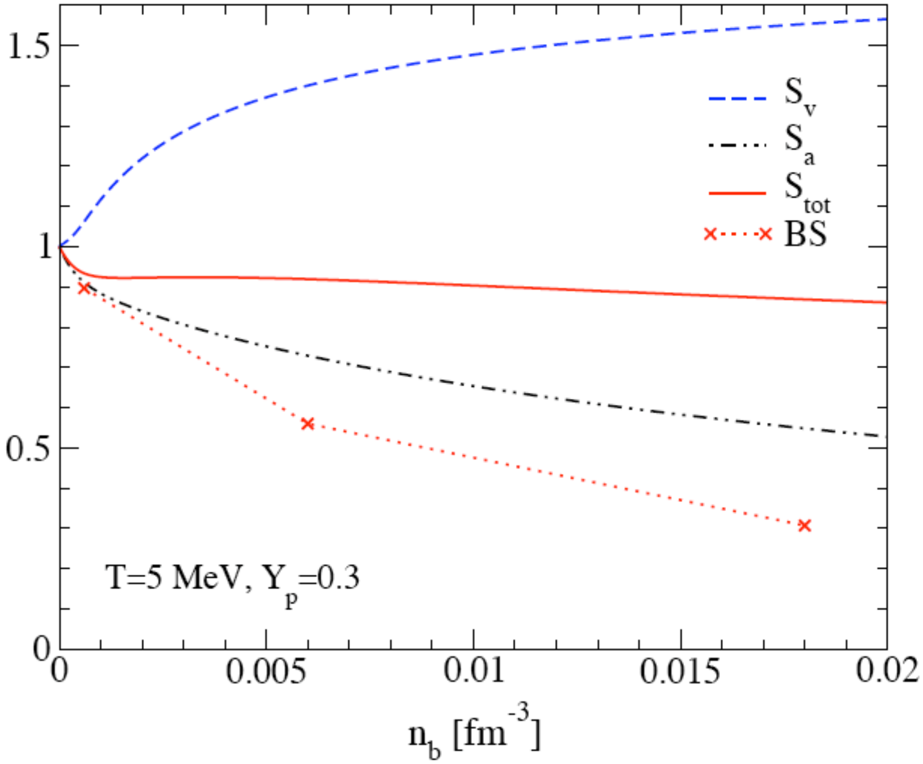
weak q dependence
in RPA response

virial vector response is attractive $S_v>1$, disagrees with RPA of Burrows, Sawyer (×) use Landau parameters of symmetric nuclear matter for all Y_p

virial axial response is repulsive $S_a<1$, follows from Pauli principle, qualitatively similar to Burrows, Sawyer (+)

Response of nuclear matter



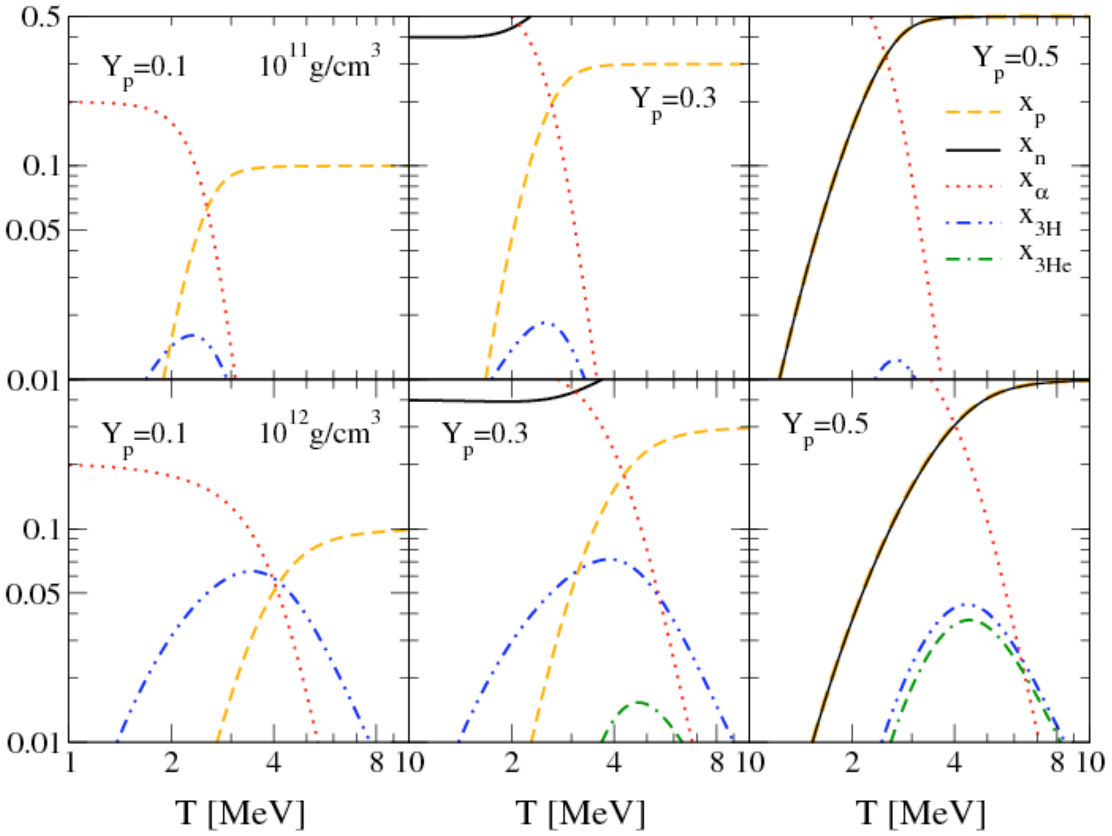


Horowitz, AS, in prep.

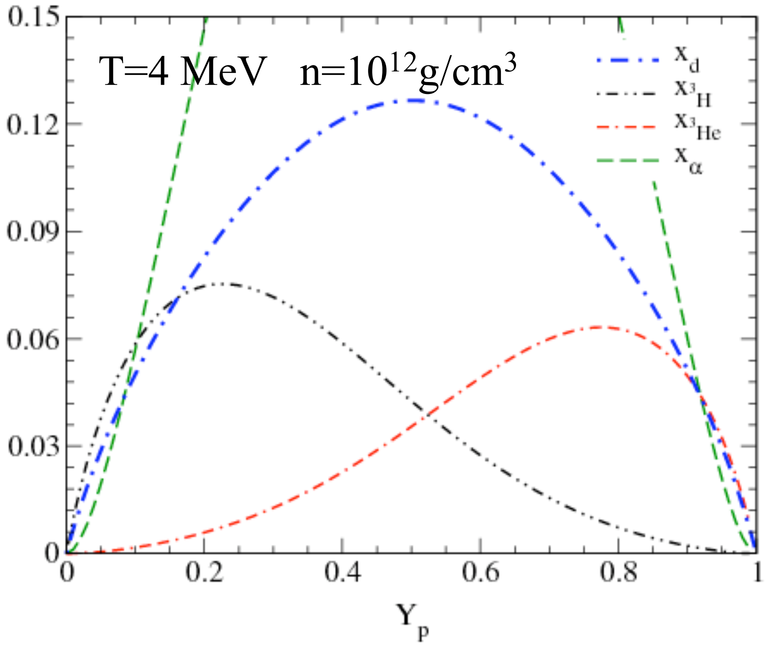
attractive nn, nα, αα interactions increase prob to find n or α particles close together, increase local weak charge, leads to attractive $S_v>1$

total virial response $\sigma=S_{tot}\sigma_0$ larger than RPA due to α contributions

Virial equation of state with light nuclei



O’Connor et al. (2007)



O’Connor, Horowitz, AS, in prep.

included A=3 nuclei and nucleon-A=3 virial coefficients

A=3 nuclei decrease alpha mass fraction, small effects of $b_{N\text{-}A=3}$

near neutrinosphere ~10% in A=3

d, ^{3}H, ^{4}He mass fractions can be comparable for neutron-rich matter

Neutrino breakup of A=3

T_ν [MeV]	^{3}H		^{3}He	
1	1.97×10^{-6}	1.68×10^{-5}	3.49×10^{-6}	2.76×10^{-5}
2	4.62×10^{-4}	4.73×10^{-3}	6.15×10^{-4}	5.94×10^{-3}
3	5.53×10^{-3}	6.38×10^{-2}	6.77×10^{-3}	7.41×10^{-2}
4	2.68×10^{-2}	3.37×10^{-1}	3.14×10^{-2}	3.77×10^{-1}
5	8.48×10^{-2}	1.14	9.70×10^{-2}	1.25
6	2.09×10^{-1}	2.99	2.35×10^{-1}	3.21
7	4.38×10^{-1}	6.61	4.87×10^{-1}	7.03
8	8.20×10^{-1}	13.0	9.03×10^{-1}	13.7
9	1.41	23.4	1.54	24.6
10	2.27	39.3	2.47	41.2

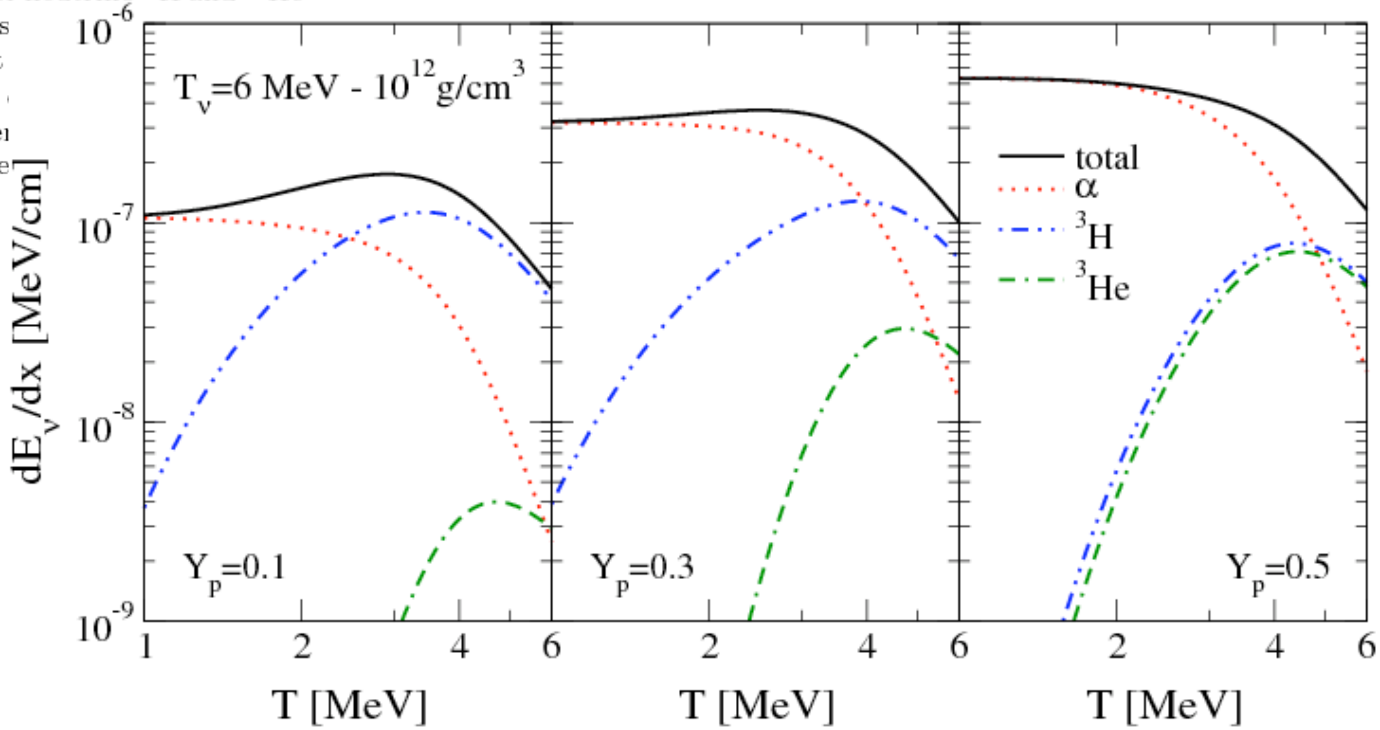
TABLE II: Averaged neutrino- and anti-neutrino-^{3}H and -^{3}He neutral-current inclusive inelastic cros (A=3), $\langle\sigma\rangle_{T_\nu} = \frac{1}{2A}\langle\sigma_\nu + \sigma_{\overline{\nu}}\rangle_{T_\nu}$ (left transfer cross-sections, $\langle\omega\sigma\rangle_{T_\nu} = \frac{1}{2A}$ columns), as a function of neutrino tei of 10^{-42} cm^2 and 10^{-42} MeVcm^2 respe

O'Connor et al. (2007)

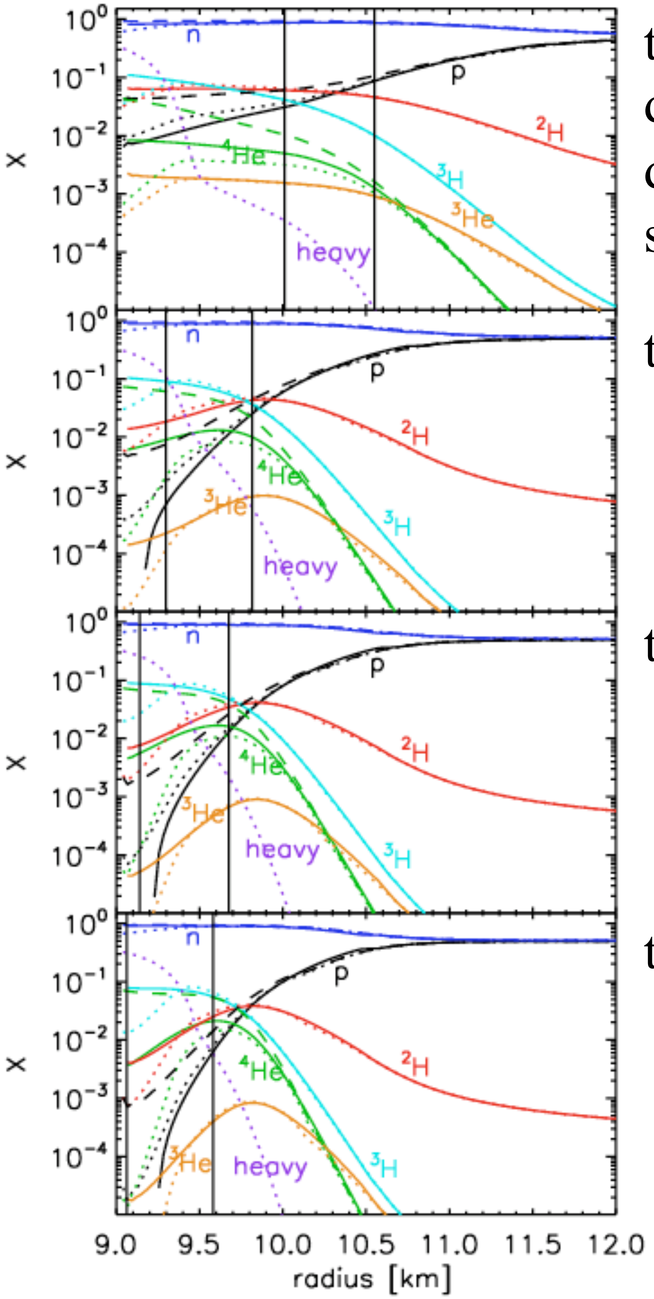
energy transfer

$$\frac{dE_\nu}{dx} = n_b \sum_{i={}^3\mathrm{H},\,{}^3\mathrm{He},\,{}^4\mathrm{He}} x_i \,\langle\omega\sigma\rangle_{i,\,T_\nu}$$

can be dominated by breakup of loosely-bound A=3 nuclei



Light nuclei and neutrino-driven supernova outflows



t=2s Arcones et al. (2008)

dashed=simulation →

dotted=NSE

solid=virial

t=5s

t=7s

t=10s

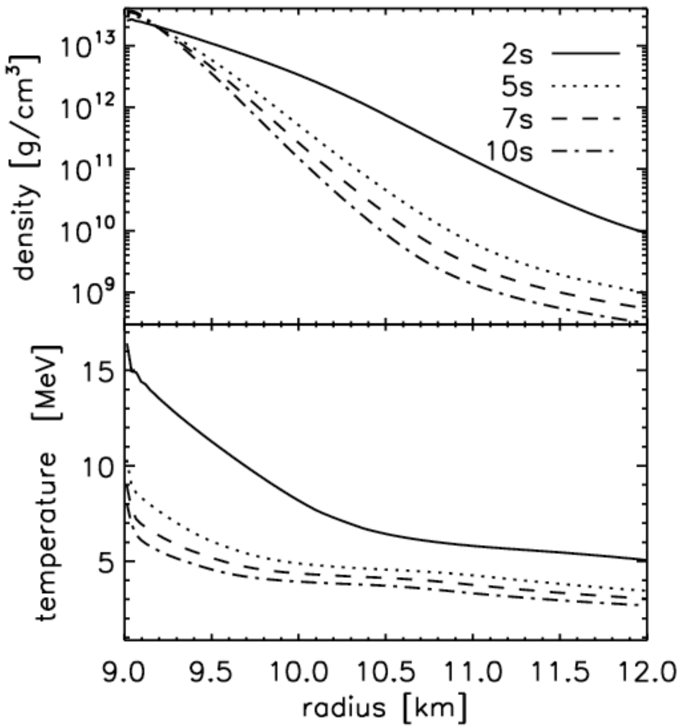
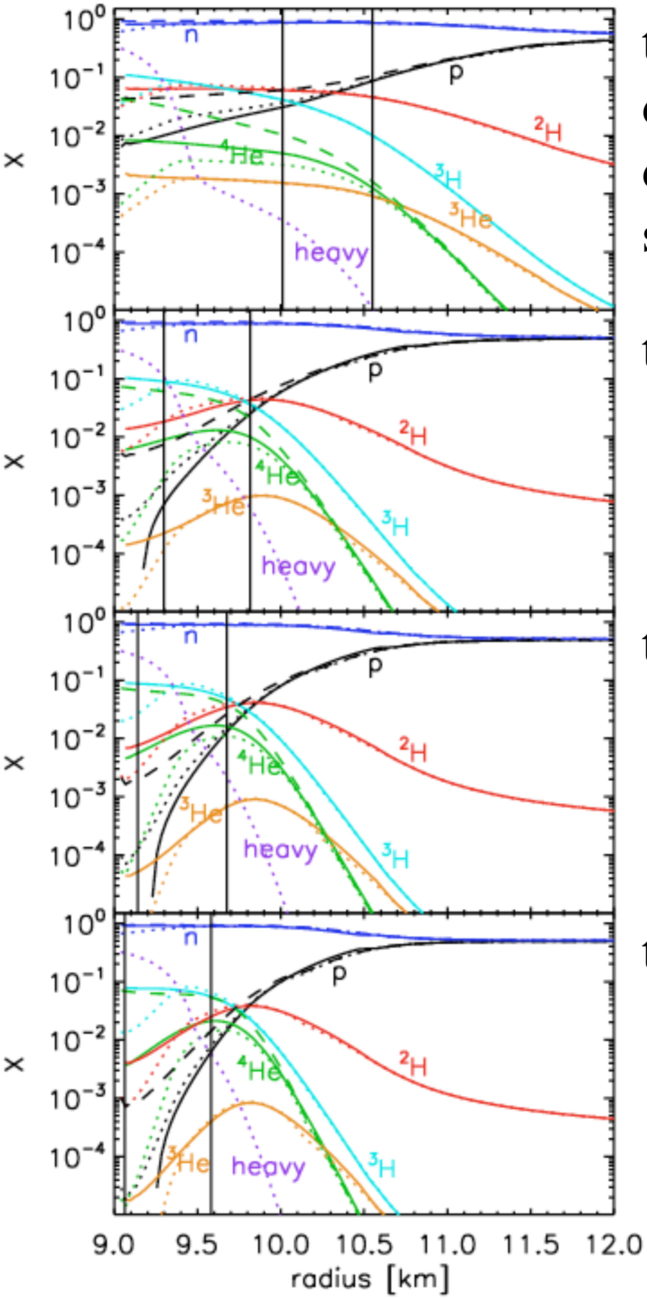


FIG. 1: Neutron star atmosphere profiles of density and temperature corresponding to model M15-l1-r1 of Ref. [7] for times $t = 2, 5, 7$ and 10 s post bounce.

Light nuclei present in substantial amounts in outer layers of protoneutron star

dominated by d, ^{3}H (and ^{4}He)

Light nuclei and neutrino-driven supernova outflows



t=2s Arcones et al. (2008)

dashed=simulation

dotted=NSE

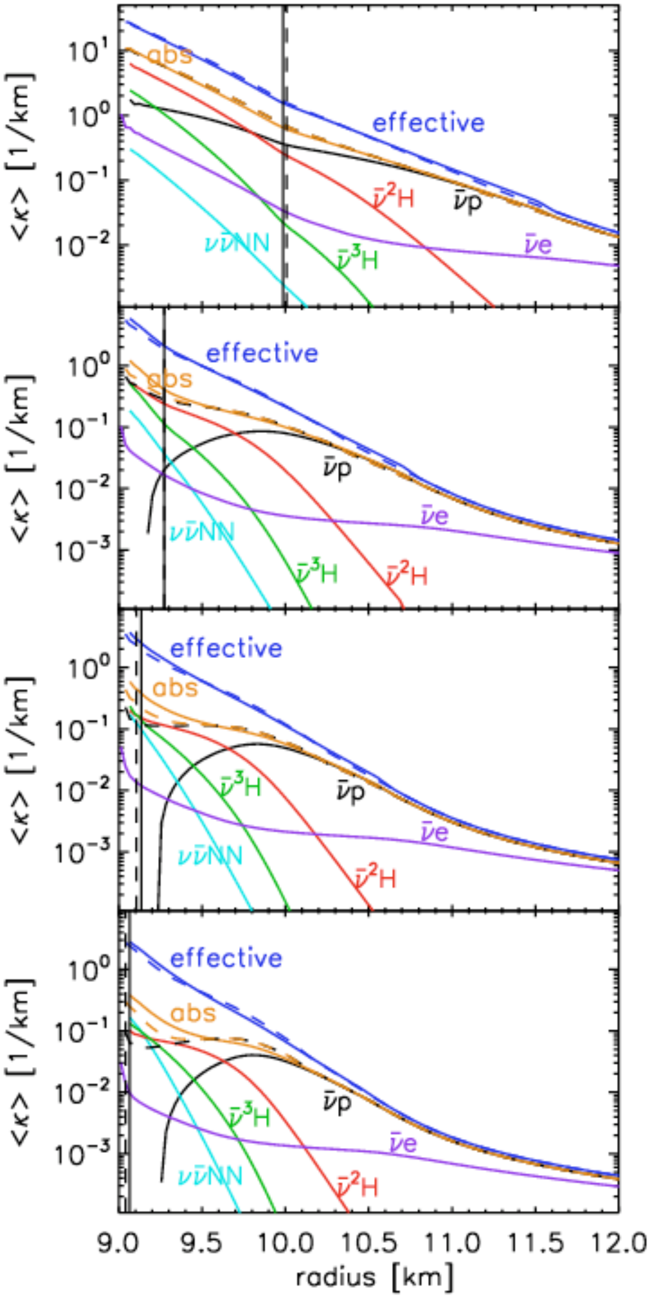
solid=virial

t=5s

t=7s

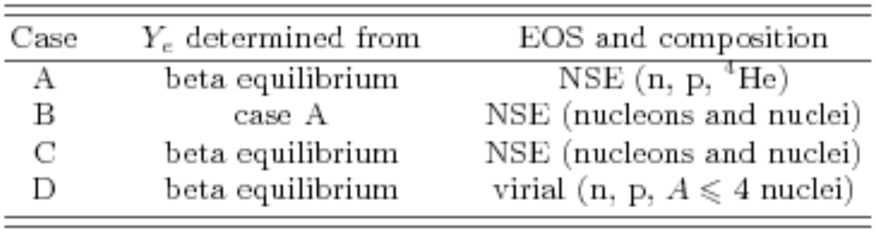
t=10s

absorption of electron antineutrinos on d, ^{3}H dominates over p



Light nuclei and neutrino-driven supernova outflows

TABLE II: Different cases explored in Sect. III.

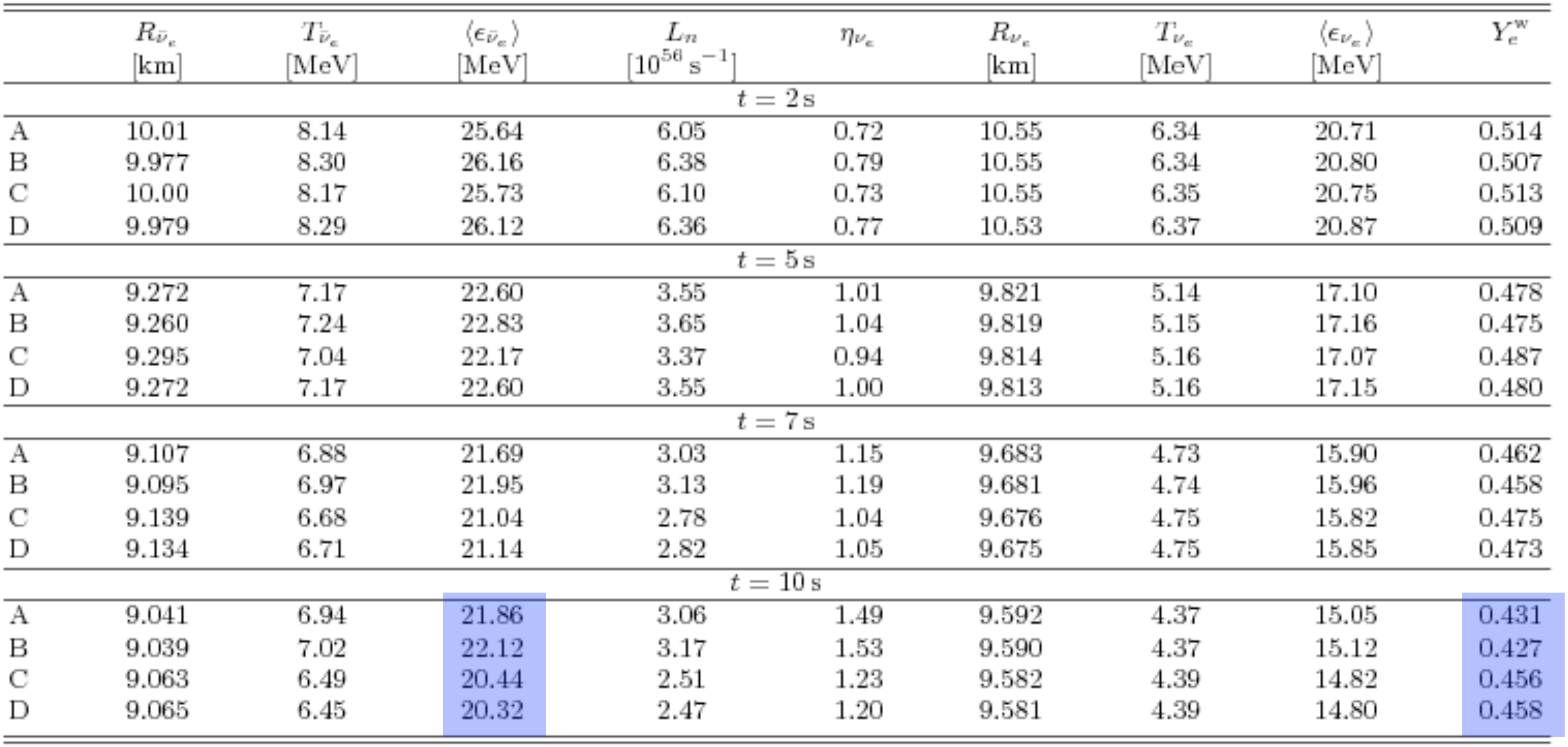


Case	Y_e determined from	EOS and composition
A	beta equilibrium	NSE (n, p, ^{4}He)
B	case A	NSE (nucleons and nuclei)
C	beta equilibrium	NSE (nucleons and nuclei)
D	beta equilibrium	virial (n, p, $A \leqslant 4$ nuclei)

Arcones et al. (2008)

impact of light nuclei on electron antineutrino emission and wind Y_e compared to reference case A

TABLE III: Neutrinosphere radii $R_{\bar{\nu}_e,\nu_e}$, neutrino spectral temperatures $T_{\bar{\nu}_e,\nu_e}$, and average energies $\langle\epsilon_{\bar{\nu}_e,\nu_e}\rangle$, as well as number luminosities L_n, spectral parameter η_{ν_e}, and wind electron fractions Y_e^w at four different times post bounce.



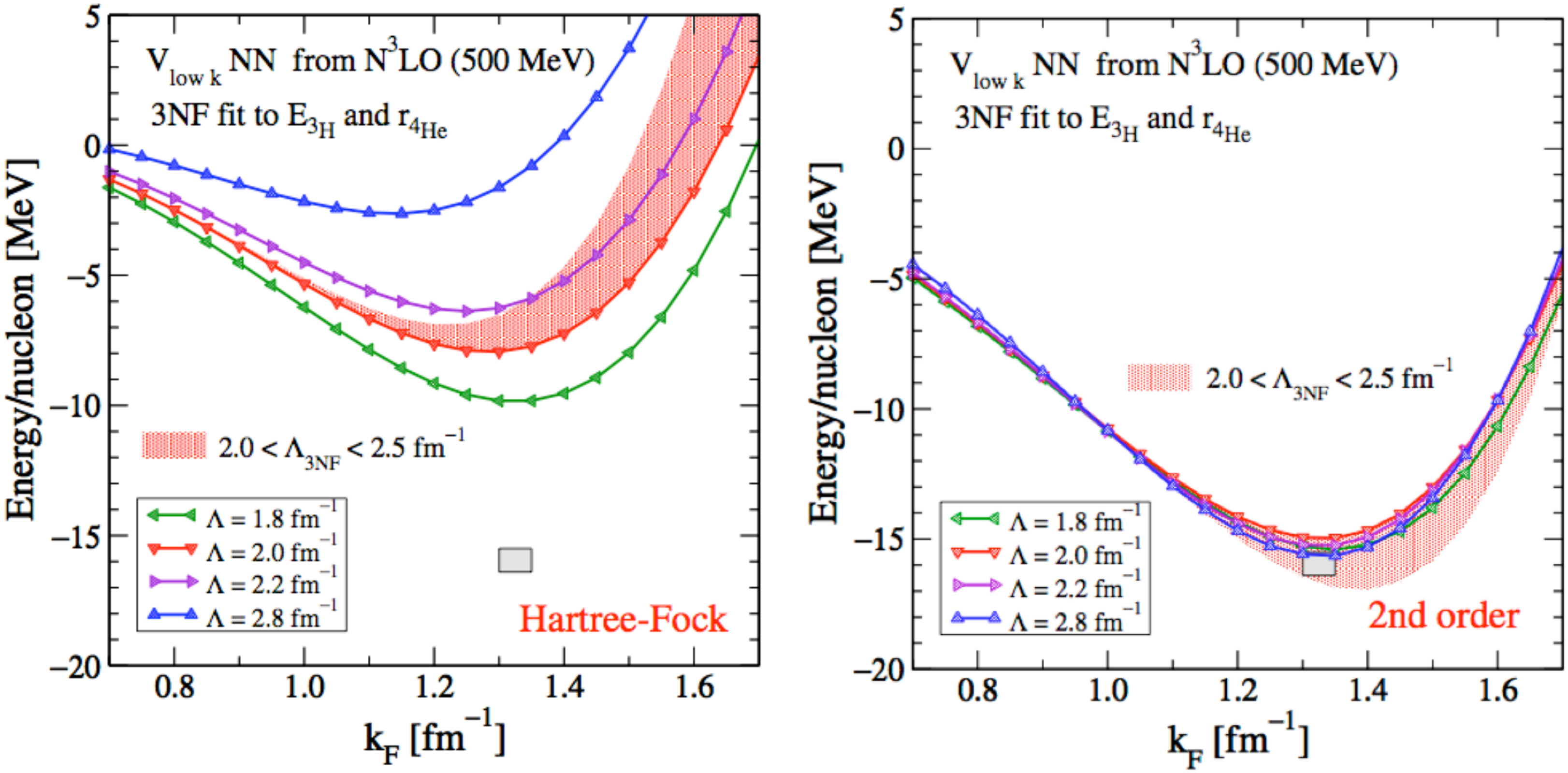
	$R_{\bar{\nu}_e}$ [km]	$T_{\bar{\nu}_e}$ [MeV]	$\langle\epsilon_{\bar{\nu}_e}\rangle$ [MeV]	L_n [10^{56} s^{-1}]	η_{ν_e}	R_{ν_e} [km]	T_{ν_e} [MeV]	$\langle\epsilon_{\nu_e}\rangle$ [MeV]	Y_e^w
$t = 2\,$s									
A	10.01	8.14	25.64	6.05	0.72	10.55	6.34	20.71	0.514
B	9.977	8.30	26.16	6.38	0.79	10.55	6.34	20.80	0.507
C	10.00	8.17	25.73	6.10	0.73	10.55	6.35	20.75	0.513
D	9.979	8.29	26.12	6.36	0.77	10.53	6.37	20.87	0.509
$t = 5\,$s									
A	9.272	7.17	22.60	3.55	1.01	9.821	5.14	17.10	0.478
B	9.260	7.24	22.83	3.65	1.04	9.819	5.15	17.16	0.475
C	9.295	7.04	22.17	3.37	0.94	9.814	5.16	17.07	0.487
D	9.272	7.17	22.60	3.55	1.00	9.813	5.16	17.15	0.480
$t = 7\,$s									
A	9.107	6.88	21.69	3.03	1.15	9.683	4.73	15.90	0.462
B	9.095	6.97	21.95	3.13	1.19	9.681	4.74	15.96	0.458
C	9.139	6.68	21.04	2.78	1.04	9.676	4.75	15.82	0.475
D	9.134	6.71	21.14	2.82	1.05	9.675	4.75	15.85	0.473
$t = 10\,$s									
A	9.041	6.94	21.86	3.06	1.49	9.592	4.37	15.05	0.431
B	9.039	7.02	22.12	3.17	1.53	9.590	4.37	15.12	0.427
C	9.063	6.49	20.44	2.51	1.23	9.582	4.39	14.82	0.456
D	9.065	6.45	20.32	2.47	1.20	9.581	4.39	14.80	0.458

Possibility of perturbative nuclear matter with NN and 3N

start from chiral EFT to given order, soften with RG

nuclear matter converged at ≈ 2nd order,
motivated by Weinberg eigenvalue analysis

reduced cutoff dependence at low densities, 3N drives saturation
Bogner, AS, Furnstahl, Nogga (2005) + improvements, in prep.



provides guidance to UNEDF http://unedf.org

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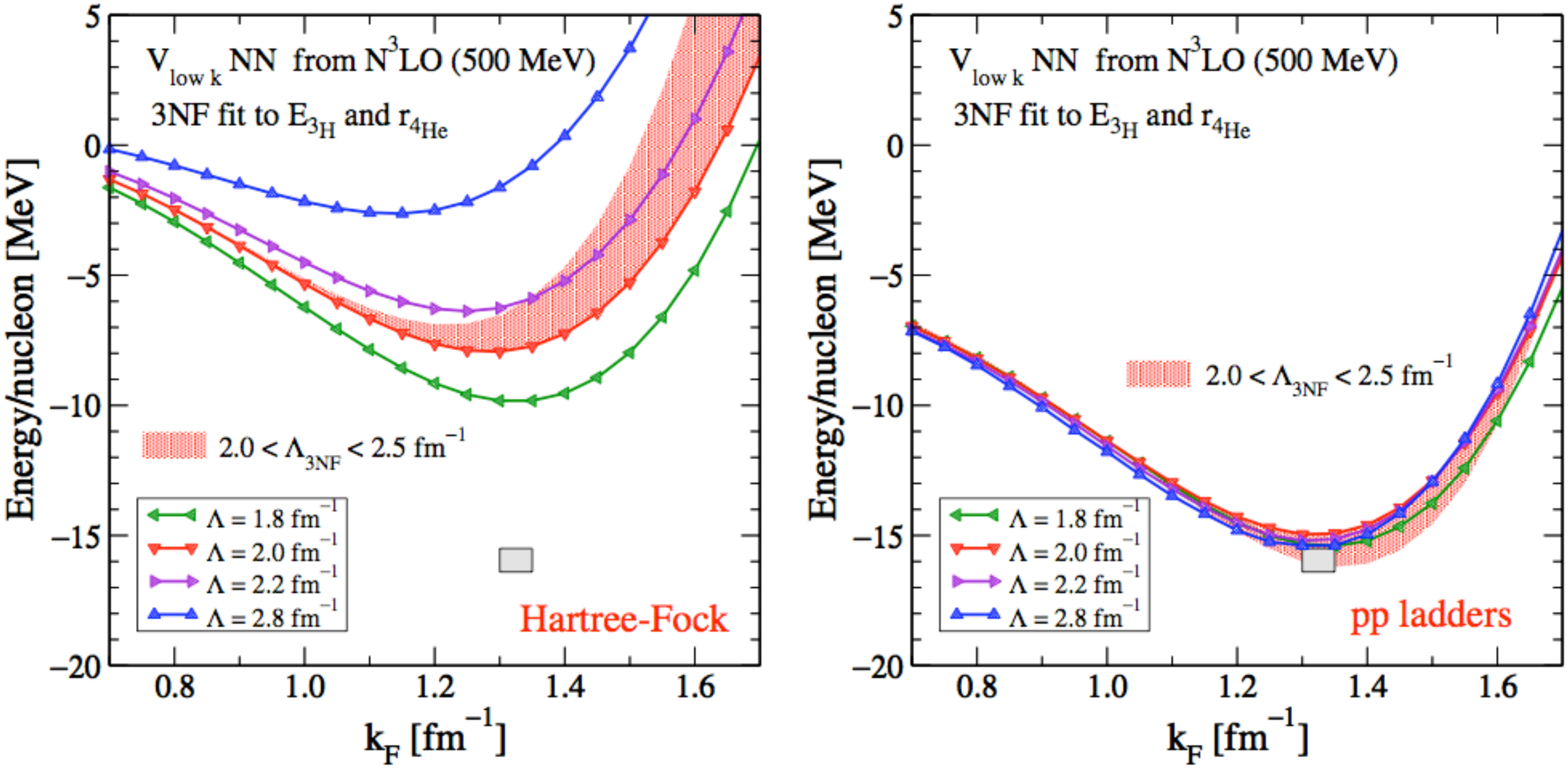
motivated by Weinberg eigenvalue analysis

reduced cutoff dependence at low densities, 3N drives saturation

Bogner, AS, Furnstahl, Nogga (2005) + improvements, in prep.

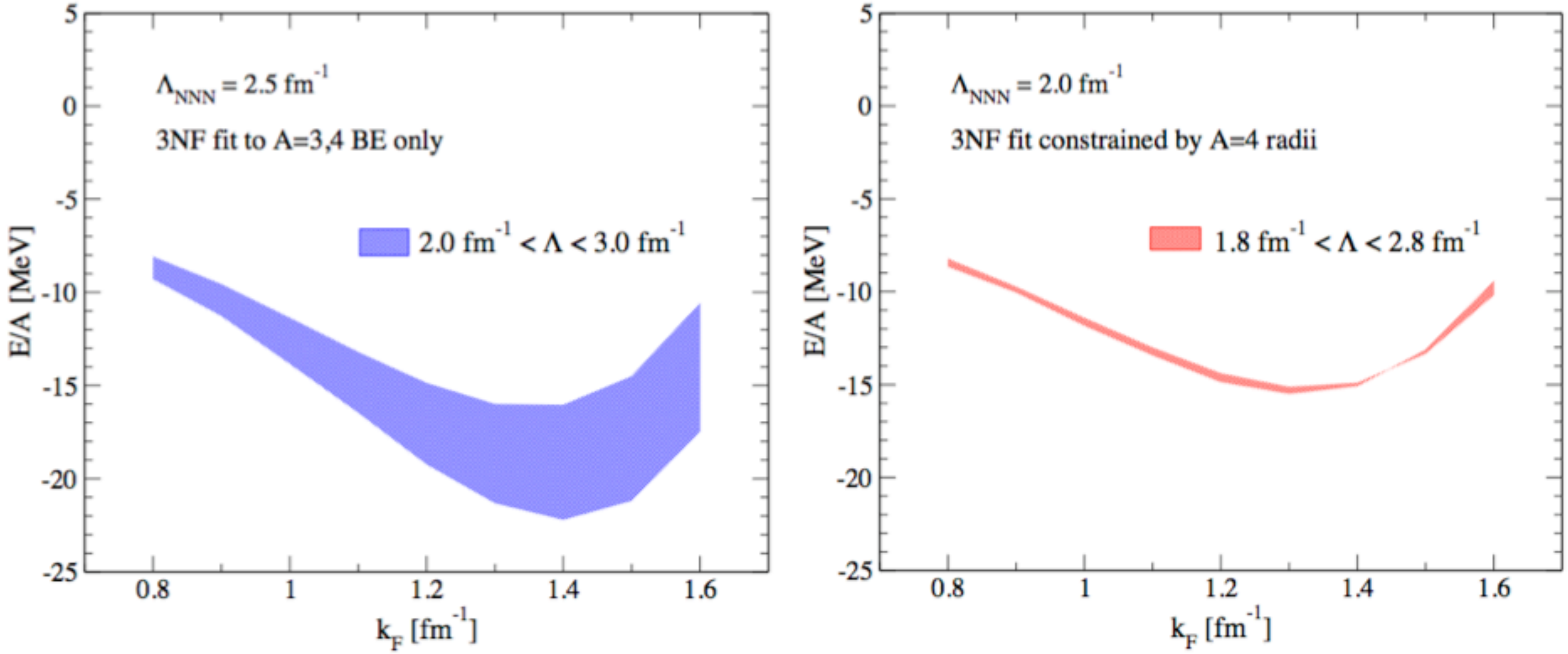


provides guidance to UNEDF http://unedf.org

Nuclear matter with NN and 3N

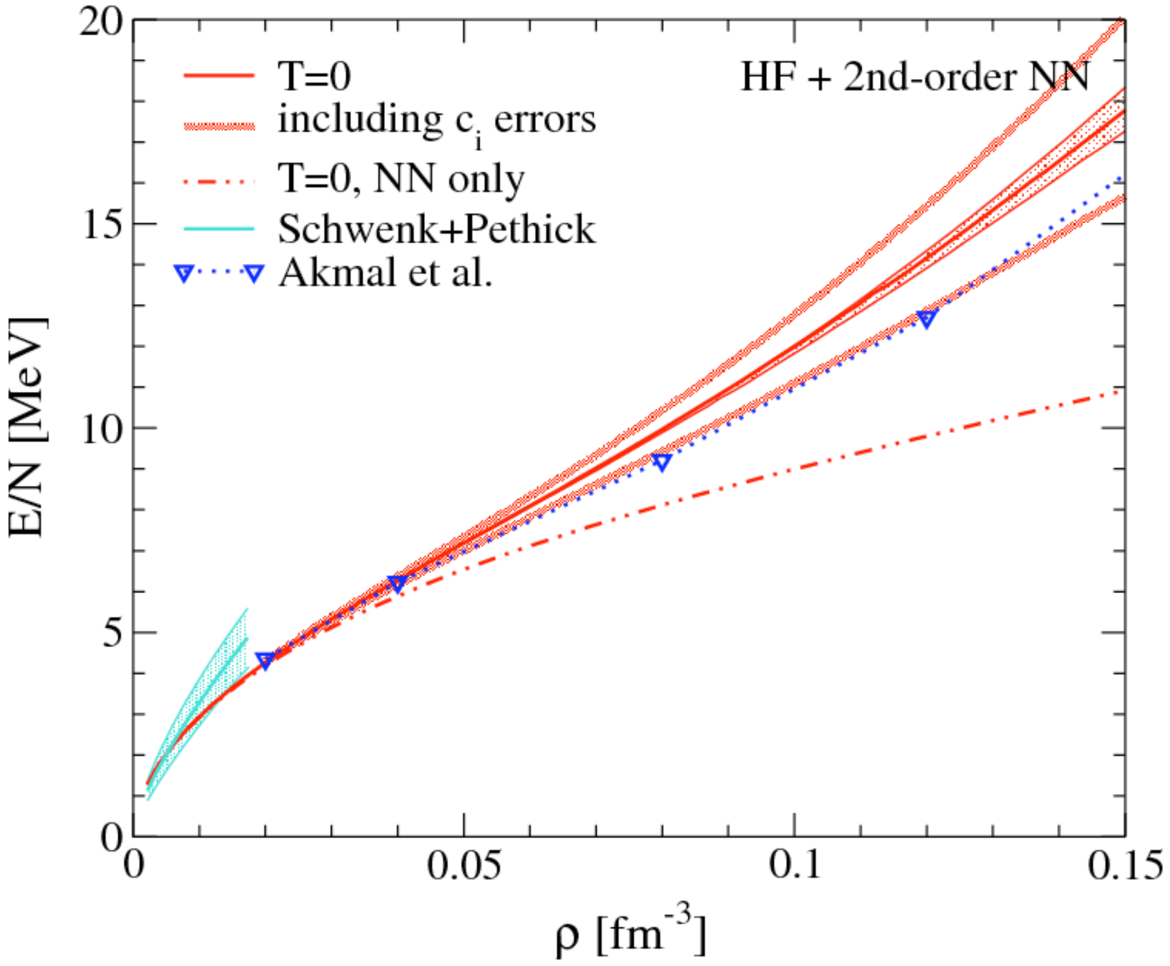
comparison of 3N fits to ^{3}H, ^{4}He binding energies vs. ^{3}H be, ^{4}He radius

Bogner et al., in prep.



radius constraint improves cutoff dependence

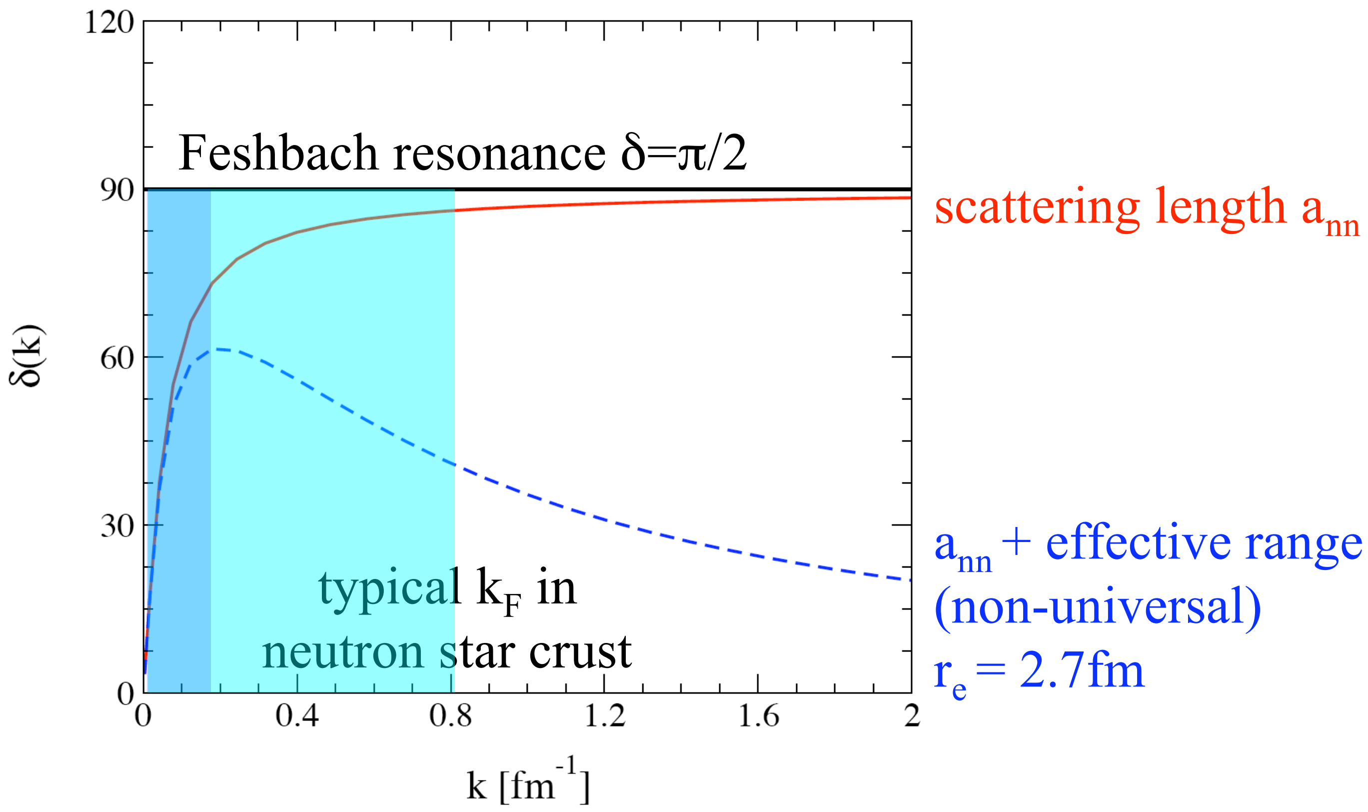
Neutron matter from NN and 3N



low densities from large scattering length and effective range

AS, Pethick (2005)

Neutron matter and non-universal corrections



phase shifts characterize strength of interaction

effective range important, weakens interactions at higher momenta

idea: large-N expansion, N = number of particles/resonantly-int. pairs

AS, Pethick (2005)

Neutron matter and non-universal corrections

di-fermion EFT for large scattering length and large effective range

Weinberg (1963), Kaplan (1997),…

$$\mathcal{L} \;=\; \psi^\dagger\left(i\partial_0 + \frac{\nabla^2}{2}\right)\psi - d^\dagger\left(i\partial_0 + \frac{\nabla^2}{4} - \Delta\right) d \;-\; g\left(d^\dagger\psi\psi + d\,\psi^\dagger\psi^\dagger\right)$$

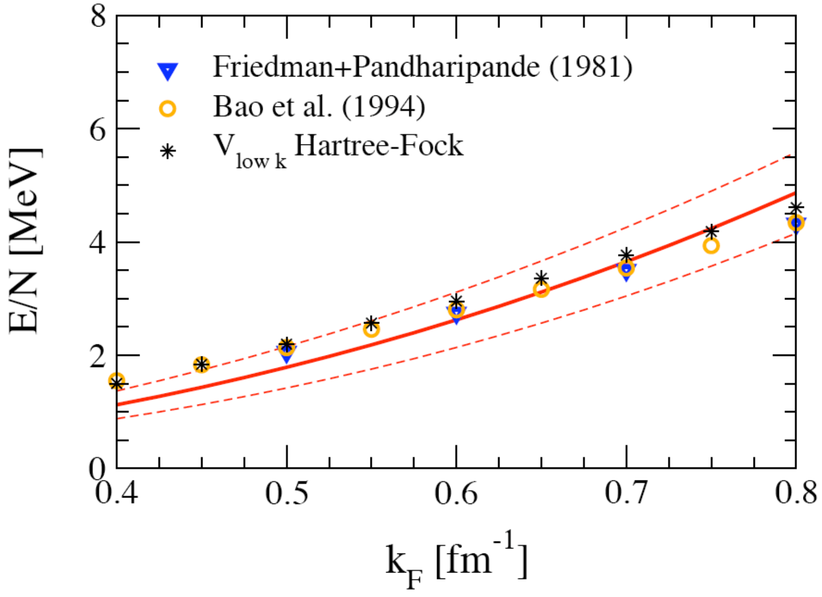
leading-order neutron matter E/N for $k_{\mathrm{F}} r_{\mathrm{e}} \lesssim 2$ or $\rho < 0.02\,\mathrm{fm}^{-3}$

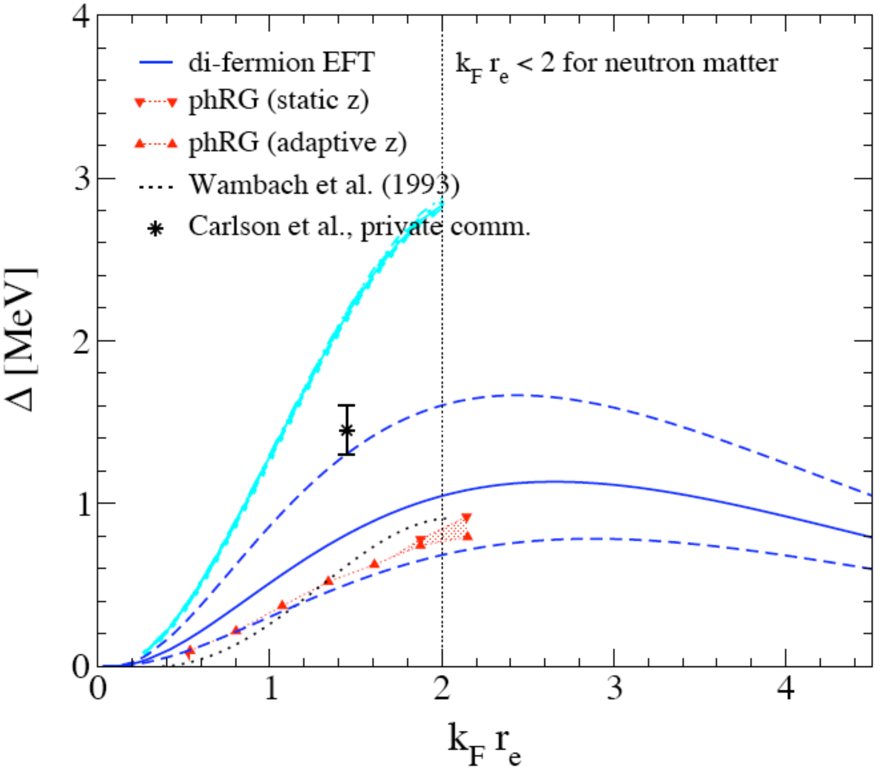
AS, Pethick (2005)

next-to-leading-order superfluid pairing gap

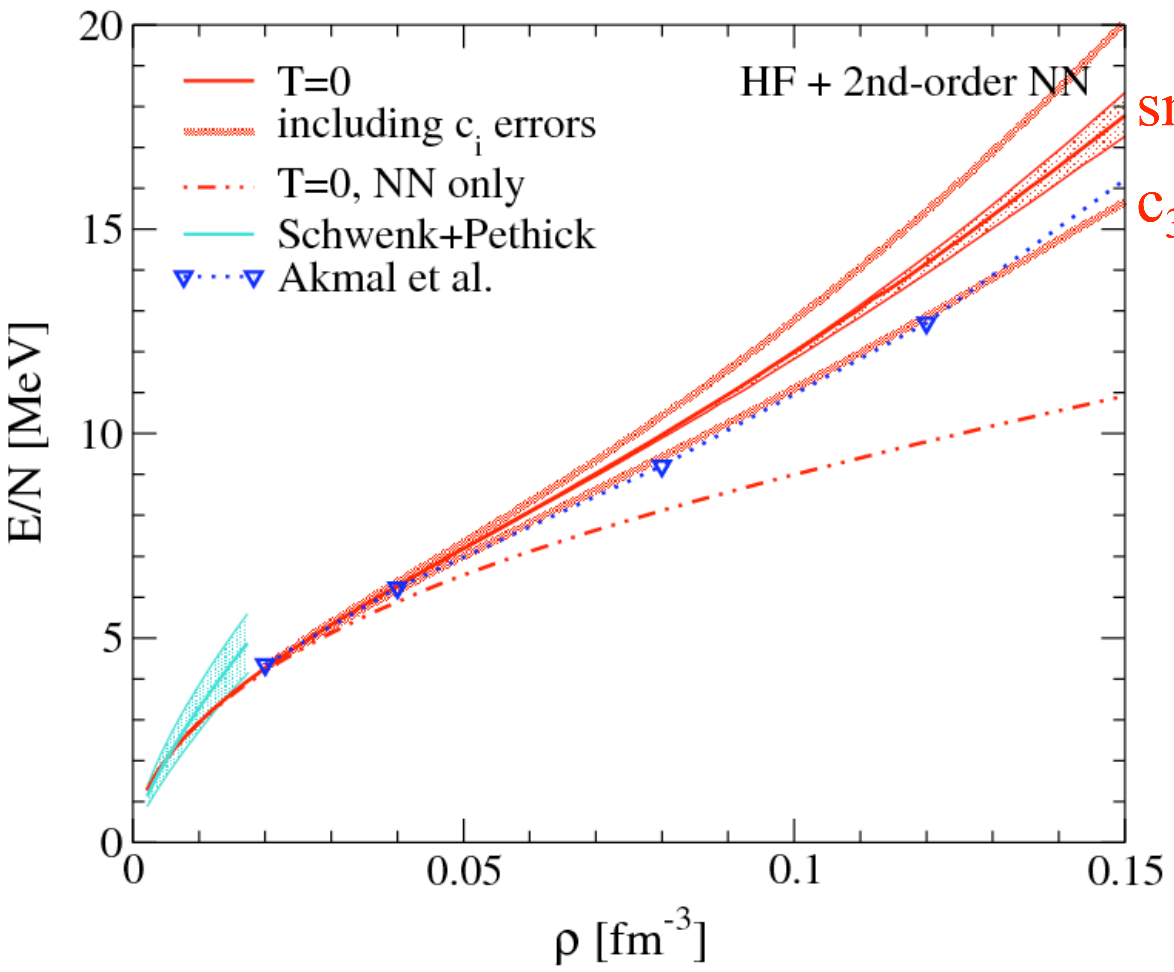
Reuter, AS, in prep.

microscopic calculations
within EFT error bands





Neutron matter from NN and 3N



small cutoff variation

c_3 uncertainties

Tolos, Friman, AS (2007)

uncertainties from c_i overwhelm errors due to cutoff variation, mainly c_3 for neutron matter

lower c_3 (Δ dominated): less repulsion, similar to results of AV18+UIX

Summary

virial equation of state provides model-independent constraints for low-density nuclear matter and neutrino response

based directly on scattering phase shifts, includes bound states and resonant interactions on equal footing

important for supernova neutrinosphere

light nuclei can be present in significant amounts,
d and ^{3}H favored for neutron-rich conditions

include light nuclei and interactions with neutrinos in supernova and neutrino-driven wind simulations

equation of state based on low-momentum interactions at intermediate densities, with neutrino response Lykasov, Pethick, AS